PHYSICAL REVIEW B VOLUME 34, NUMBER 4 15 AUGUST 1986

Two-dimensional density of states in the extreme quantum limit

T. P. Smith III and W. I. Wang
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

P. J. Stiles
Department of Physics, Brown University, Providence, Rhode Island 02912

(Received 21 April 1986)

We have measured the density of states of a two-dimensional electron gas from the weak-field limit to the extreme quantum limit and obtained quantitative information about the density of states at filling factors of $\frac{1}{3}$ and $\frac{2}{3}$. Our magnetocapacitance measurements were made at magnetic field strengths up to 29 T at temperatures between 0.4 and 4.2 K using modulation-doped GaAs-Al_xGa_{1-x}As heterostructures on conducting substrates.

Current theories[1-5] ascribe the fractional quantum Hall effect (FQHE) to the formation of an incompressible quantum-fluid state. This state has cusps in the total energy at fractional filling factors with odd denominators. The density of states (DOS) in this regime should also exhibit gaps or minima at these filling factors. Thus far, experimental studies of the FQHE have primarily focused on transport properties associated with this effect, and information about the energy gaps associated with these states has come from examining the temperature dependence of the diagonal resistivity. Fletcher, Maan, and Weimann[6] have studied the thermopower in the extreme quantum limit but no information concerning the DOS in the fractional quantum Hall regime could be extracted from their results.

In this Rapid Communication we present results of magnetocapacitance measurements from the weak-field limit to the extreme quantum limit. Using heterostructure capacitors with a conducting electrode near the two-dimensional electron gas (2DEG) we have avoided the series-resistance effects encountered in previous measurements and obtained in the DOS directly. At integer filling factors the DOS can be fit by a Gaussian model. However, there is a very large enhancement of the g factor in the lowest Landau level. While there is a large (as much as 70%) drop in the DOS at a filling factor of $\frac{1}{3}$, the DOS in the extreme quantum limit is larger than the zero-field DOS.

Capacitance measurements have been used to study two-dimensional electronic systems for a number of years.[7] Capacitance measurements can be used to study transport phenomena,[8,9] but they are particularly well suited to DOS measurements since the differential capacitance is directly related to the thermodynamic density of states, $dn/d\mu$ in the small-signal approximation. In previous experiments,[7] the conductivity in the plane of the 2DEG limited DOS measurements to lower magnetic fields. When the conductivity of the 2DEG becomes too small, the measured capacitance depends on the conductivity in a complicated way and the DOS is not easily extracted. Recently, Hickmott[10] has measured the capacitance of accumulation layers in GaAs-Al_xGa_{1-x}As capacitors relying on transport perpendicular to the 2DEG to modulate the carrier concentration, and observed structure at fractional filling factors. However, his samples are quite different than those used in these experiments, and series-resistance effects are not negligible. In order to measure the DOS both in-plane and series-resistance effects must be small.

To accomplish this, GaAs heterostructure capacitors were fabricated on conducting substrates. A 2DEG is located at the interface between an undoped GaAs layer and a modulation-doped Al_xGa_{1-x}As layer. The 2DEG is separated from the n^+-type GaAs by 50 nm of undoped GaAs. The proximity of the conducting substrate to the heterojuncton allows electrons to flow in and out of the 2DEG with little or no resistive loss. An aluminum electrode, 0.76 mm in diameter, was deposited on top of the stucture. The capacitance of the Al_xGa_{1-x}As layer between the electrode and the 2DEG is about 463 pF. This value agrees well with the insulator capacitance required to fit the data. The differential capacitance was measured using a phase-sensitive detector and the phase was set using a calibrated standard. In general, a 1-mV-rms signal was used to measure the capacitance. Measurements were made between 100 Hz and 100 kHz and at temperatures between 0.4 and 4.2 K. There was little or no frequency dependence below 10 kHz indicating that the series resistance of the 50-nm undoped GaAs layer between the 2DEG and the conducting substrate was much smaller than the capacitive impedance of the sample and could be ignored at these frequencies.

For the case of the metal-insulator-semiconductor and heterostructure capacitors the measured capacitance is the series combination of the capacitance of the 2DEG and the capacitance of the dielectric between the electrode and the 2DEG. If a variational approximation[11] is used for the wave function of an electron in the 2DEG, and image effects, many-body effects, penetration of the wave function into the barrier, and nonparabolicity of the band structure are ignored, the measured capacitance is given by

$$\frac{A}{C_{\text{meas}}}=\frac{A}{C_{\text{ins}}}+\frac{\gamma z_0}{\varepsilon_0\kappa_c}+\frac{1}{e^2 dn/d\mu}, \qquad (1)$$

where A is the area of the capacitor, C_{ins} is the capacitance of the modulation-doped layer between the 2DEG and the electrode, γ is a numerical constant between 0.5 and 0.7,[11]

z_0 is the average position of the electrons in the channel, κ_c is the relative dielectric constant of the channel material, and $dn/d\mu$ is the thermodynamic DOS at the Fermi energy. The first two terms on the right-hand side of the equation remain constant in experiments where the carrier concentration in the 2DEG is fixed and the magnetic field is swept. Thus, changes in the measured capacitance directly reflect changes in the DOS of the 2DEG.

We will first discuss the weak-field and integer quantum Hall regime. This will allow us to compare the results from our new samples with previous results and will provide a basis for interpreting our results as fractional filling factors. The measured and calculated magnetocapacitance are shown in Fig. 1. Although the mobility of the electrons in the 2DEG cannot be measured directly, the observation of quantum oscillations in the capacitance at fields as low as 0.4 T indicates that the mobility and homogeneity of the sample are better than 300000 cm^2/Vs and 1%, respectively. Below 0.7 T the capacitance oscillations are exponentially damped indicating that the Landau levels are not fully resolved. Between 0.7 and 4 T the amplitude of the oscillations increases approximately linearly with magnetic field. The capacitance oscillations are not symmetric about the zero-field value. In the center of a Landau level the DOS becomes larger than the zero-field value and the measured capacitance increases accordingly. However, since $e^2 dn/d\mu$ is 30 times C_{ins} at $B=0$, the measured capacitance [see Eq. (1)] can increase by 3% at most. Although the decrease in the capacitance between Landau levels is larger than the increase in the capacitance in the center of a Landau level, it is relatively small (~4% at $\nu=2$) indicating that the DOS in the gap between Landau levels is large (~40% of the zero-field DOS at $\nu=2$). If the density of states were zero, C_{meas}/C_{ins} would drop by about 25%. We obtained similar results on samples with different separations between the 2DEG and n^+-type GaAs layer, indicating that the small changes in the capacitance are not an artifact of the sample configuration.

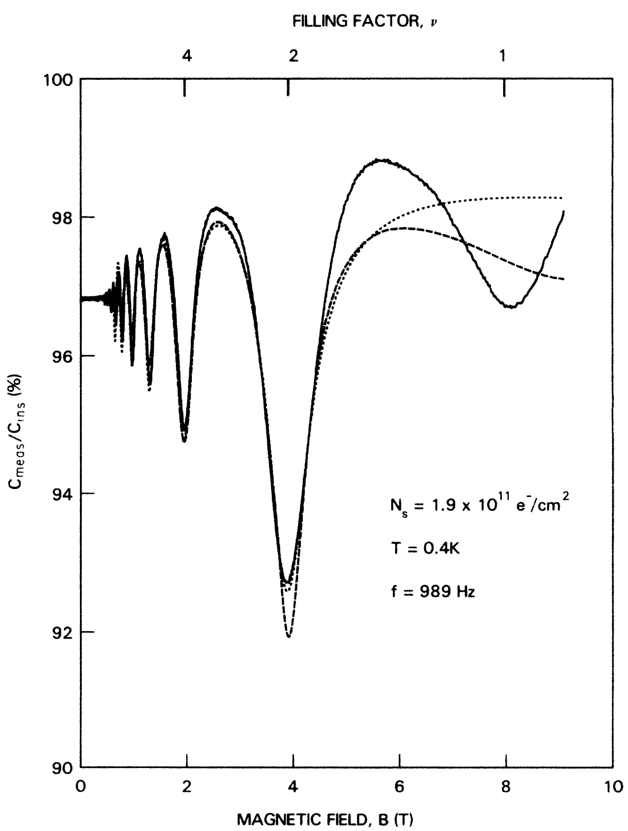

FIG. 1. The measured and calculated magnetocapacitance below the extreme quantum limit. The dashed line is the calculated capacitance using a monotonically increasing g factor and the dotted line is calculated using an oscillatory g factor.

In general, the DOS of a 2DEG in a quantizing magnetic field is calculated in terms of a Landau-level energy and width at a particular magnetic field. In order to determine these parameters, one assumes a particular form for the DOS, chooses a magnetic field, and then integrates the DOS with respect to energy until the appropriate carrier concentration is reached. This procedure essentially translates the DOS, which is measured as a function of magnetic field, into a DOS in energy space. To model our capacitance data we used a DOS of the form[12]

$$\frac{dn}{d\mu}=\frac{eB}{h}\sum_{Ns}\frac{1}{\Gamma}\sqrt{2/\pi}\exp\left[-2\frac{(E-E_{N,s})^2}{\Gamma^2}\right], \quad (2)$$

where B is the magnetic field strength, $E_{N,s}$ is the Landau-level energy, and Γ is the Landau-level width. For fitting the data we used $\Gamma=\Gamma_0 B^\alpha$, where Γ_0 and α are fitting parameters. To take into account the effects of spin splitting (observed at $B\simeq 8$ T) we used both a smoothly varying g factor g, of the form

$$g=g_1 B^\beta, \quad (3)$$

where g_1 and β are fitting parameters, and a self-consistent oscillatory g factor used previously by Englert, Tsui, Gossard, and Uihlein,[13] where g is given by

$$g=g_0+\frac{E_{ex}}{\mu_B B}\sum_N (n_{N\uparrow}-n_{N\downarrow}), \quad (4)$$

and g_0 is the free-electron g factor, μ_s is the Bohr magneton, E_{ex} is the exchange parameter, and $n_{N\uparrow(\downarrow)}$ are the occupation factors of the spin levels. The best fits for these two models are also shown in Fig. 2. For the first model the parameters are $\Gamma_0=1.17$ meV$\,T^{-\alpha}$, $\alpha=0.61$, $g_1=1.33$, and $\beta=0.68$. For the second model the best-fit parameters are $\Gamma_0=1.06$ meV$\,T^{-\alpha}$, $\alpha=0.80$, and $E_{ex}=12.8\times 10^{-12}$ meV$\,cm^2$. The least-squares fitting weighed each of the capacitance oscillations equally since the data are equally spaced as a function of $1/B$, not B. Both of the fits are good up to about 5 T. The magnetic field dependence of the Landau-level broadening in the first fit is close to the $\sqrt{B}$ dependence predicted by Ando and Uemura[14] but the level width for our fit is almost an order of magnitude larger than predicted (3.2 vs 0.6 meV at 5 T). However, our results agree fairly well with magnetization[15,16] ($\Gamma=4.4$ meV at 5 T) and specific-heat[17] ($\Gamma=2.1$ meV with a 20% background at 5 T) experiments.

The amplitude of the capacitance oscillation at $\nu=1$, and hence the g factor in the lowest Landau level, is much larger than either model predicts. This indicates that exchange effects may be much more important when only

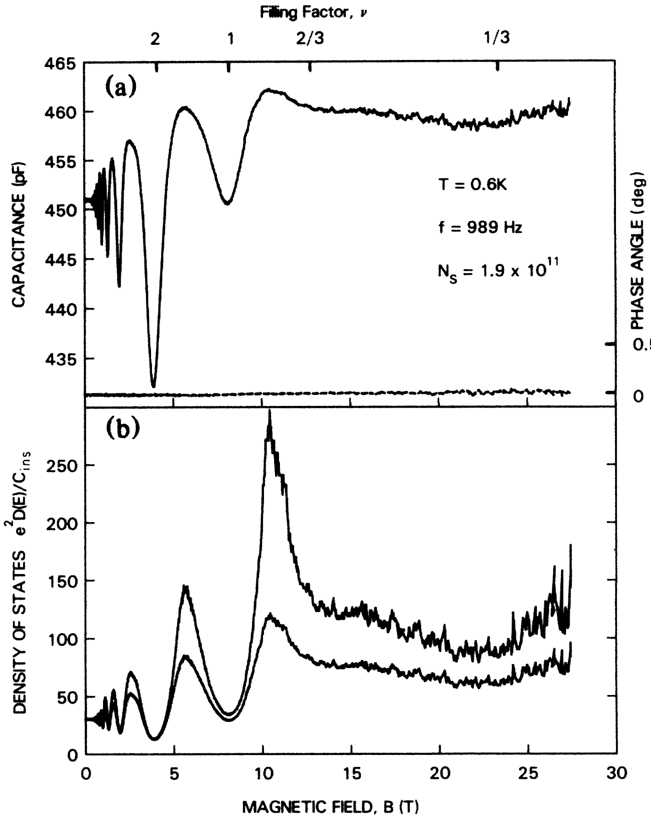

FIG. 2. (a) The measured magnetocapacitance and phase angle. (b) The measured density of states. The shape of the DOS is given by either the upper or lower curve but the magnitude lies between them. The upper curve is calculated using $C_{ins}=462$ pF and the lower curve is calculated using $C_{ins}=464$ pF.

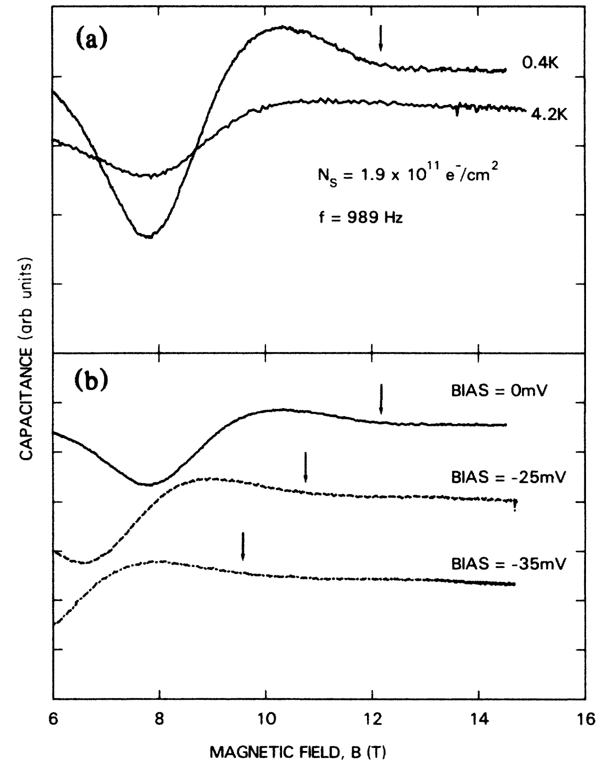

FIG. 3. The measured magnetocapacitance (a) at different temperatures and (b) under different bias conditions. Arrows indicate the structure at $\nu=\frac{2}{3}$.

the lowest Landau level is occupied. Although the change in capacitance due to spin splitting is much larger than predicted at $\nu=1$, it is much smaller than Landau-level splitting at $\nu=2$. This may account for the fact that spin splitting is not resolved in magnetization[15,16] and specific-heat[17] measurements.

Having analyzed our results at integer filling factors we now turn to the magnetocapacitance in the extreme quantum limit. Figure 2 shows the measured capacitance and DOS from 0 to 29 T. The DOS has the form shown by either the upper or lower curve in Fig. 2(b). However, the magnitude lies somewhere between these two curves. The uncertainty in the magnitude of the DOS corresponds to an uncertainty of 2.0 pF in the insulator capacitance. We also examined the magnetocapacitance up to 15 T as a function of temperature [Fig. 3(a)] and dc bias [Fig. 3(b)]. The structure at $\nu=\frac{2}{3}$ shifts appropriately as the bias increases and the carrier concentration decreases. The temperature dependence is also as expected: The change in the measured capacitance is small between 0.6 and 1.2 K but above 1.2 K the minima at $\nu=\frac{1}{3}$ and $\frac{2}{3}$ disappear rapidly. At 4.2 K, well above the temperature required for observation of the FQHE, there is still a gradual decrease in the capacitance above about 11 T. The origin of this decrease is not clear but the calculated DOS and capacitance exhibit the same behavior if the g factor is not enhanced ($g=0.52$) for $\nu<1$ and the Landau-level broadening is proportional to $\sqrt{B}$.

The most striking feature of our data is that the structure at fractional filling factors of $\frac{2}{3}$ and $\frac{1}{3}$ is extremely weak. Although these small changes in the capacitance reflect large changes in the density of states, the DOS is much larger than the zero-field DOS. This may be because these measurements were made at relatively high temperatures (0.6 K) and the DOS, which is increased by the Fermi function, may be smaller at lower temperatures. Another result of the finite measurement temperature is that the minima at $\nu=\frac{2}{3}$ and $\frac{1}{3}$ are shifted slightly toward a filling factor of $\frac{1}{2}$.

The large DOS in the fractional gaps is consistent with the results at integer filling factors. In samples with comparable carrier densities and mobilities, the $\nu=1$ Hall step is clearly observed at 0.6 K, where the measured DOS is also slightly greater than the zero-field DOS. As the magnetic field strength increases the degeneracy of each Landau level increases, as do the level width and the effects of sample inhomogeneity. These factors will all tend to increase the absolute number of states in the fractional gaps. Although the sample homogeneity is very good ($<1\%$), small-scale inhomogeneities of this magnitude can increase the number of localized states dramatically at 24 T.

The origin of the large number of states between Landau levels at integer filling factors is not yet well understood. Disorder, scattering, inhomogeneity, are all probably involved, but this has not been confirmed theoretically. Although scaling arguments involving disorder have been extended to the fractional quantum Hall regime,[18] these effects have not been examined rigorously. Because the FQHE arises from many-body interactions and is only observed in very high-quality samples, disorder, scattering, and inhomogeneity have been largely ignored. However, interpretation of these and future results may require just such treatment.

In summary, we have measured the DOS of a 2DEG from the weak-field limit to the extreme quantum limit. At the lowest fields, the density of states can be modeled with by a Gaussian DOS. Spin splitting is observed in the lowest Landau level and the g factor appears to be enhanced by more than an order of magnitude over the free-electron value. At fractional filling factors, the DOS is drastically reduced but still very large.

The authors are grateful to L. Rubin, B. Brandt, and the staff at the Francis Bitter National Magnet Laboratory for their support. We would also like to thank E. Calleja, L. L. Chang, L. Esaki, F. F. Fang, B. B. Goldberg, T. Jackson, and E. E. Mendez for their help and advice. This work was supported in part by the Army Research Office.

[1]R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).
[2]F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983).
[3]B. I. Halperin, Phys. Rev. Lett. **52**, 1583 (1984).
[4]D. Yoshioka, Phys. Rev. B **29**, 6833 (1984).
[5]F. D. M. Haldane and E. H. Rezayi, Phys. Rev. Lett. **54**, 237 (1985).
[6]R. Fletcher, J. C. Maan, and G. Weimann, Phys. Rev. B **32**, 8477 (1985).
[7]T. P. Smith III, B. B. Goldberg, P. J. Stiles, and M. Heiblum, Phys. Rev. B **32**, 2696 (1985), and references therein.
[8]V. Dolgopolov, C. Mazure, A. Zrenner, and F. Koch, J. Appl. Phys. **55**, 4280 (1984).
[9]V. T. Dolgopolov, S. I. Dorenzhkin, and A. A. Shashkin, Solid State Commun. **50**, 273 (1984).
[10]T. Hickmott, Phys. Rev. Lett. (to be published).
[11]F. Stern, Phys. Rev. B **5**, 4891 (1972).
[12]R. R. Gerhardts, Surf. Sci. **58**, 227 (1976).
[13]Th. Englert, D. C. Tsui, A. C. Gossard, and Ch. Uihlein, Surf. Sci. **113**, 295 (1982).
[14]T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **36**, 959 (1974).
[15]T. Haavasoja, H. L. Stormer, B. J. Bishop, V. Narayanamurti, A. C. Gossard, and W. Wiegmann, Surf. Sci. **142**, 294 (1984).
[16]J. P. Eisenstein, H. L. Stormer, V. Narayanamurti, A. Y. Cho, A. C. Gossard, and C. W. Tu, Phys. Rev. Lett. **55**, 875 (1985).
[17]E. Gornik, R. Lassnig, G. Strasser, H. L. Stormer, A. C. Grossard, and W. Wiegmann, Phys. Rev. Lett. **54**, 1820 (1985).
[18]R. B. Laughlin, M. L. Cohen, J. M. Kosterlitz, H. Levine, S. B. Libby, and A. M. M. Pruisken, Phys. Rev. B **32**, 1311 (1985).