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Transition to Ohmic conduction in ultrasmall tunnel junctions

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The effective dc conductance of an ultrasmall tunnel junction is calculated from the functional-integral description beyond the weak-coupling limit by employing the self-consistent harmonic approximation. With increase in the normal conductance R_N^{-1}, the effective conductance exhibits a crossover from the weak-coupling regime, dominated by thermal activation over the Coulomb gap to the Ohmic behavior in the strong-coupling limit. As the temperature and capacitance approach zero, the results indicate a possibility of a precipitous transition for R_N of the order of $\hbar/e^2$.

Recently, several authors[1,2] have pointed out that the effective conductance of a normal tunnel junction with low capacitance becomes temperature and frequency dependent as a result of quantum voltage fluctuations associated with the electron transfer. Previous calculations of the conductance[1,2] have been limited to weakly damped junctions, for which the nominal junction resistance R_N (defined by the Ohmic response of a large-capacitance junction) must satisfy the inequality

$$\frac{R_N}{R_0} \gg \frac{e^2}{k_B T C} , \tag{1}$$

where T is the temperature, C is the capacitance of the junction, and $R_0 = \hbar/e^2 = 4.11$ kΩ. For weakly damped junctions, the calculations[1,2] predict an activation-type T dependence of the dc conductance:

$$Y(\omega=0) = \frac{1}{R_N} \exp\left(-\frac{e^2}{2k_B T C}\right) . \tag{2}$$

Also, the frequency dependence of $\mathrm{Re}Y(\omega)$ is found to exhibit, at low temperatures, a threshold near the charging energy $\hbar\omega \simeq e^2/2C$, indicating a photon-assisted tunneling.

The present work attempts to answer the following question: How are the above weak-coupling effects modified as one decreases R_N so that Eq. (1) is satisfied no more? Specifically, we calculate $Y(\omega=0)$ for a wide range of the parameters $g = \pi R_0/2R_N$, and $a = e^2/2\pi^2 k_B TC$, well outside the weak-coupling regime. In a strongly damped junction, the charging energy rapidly decays during the electron transfer as a result of the capacitor discharge. Consequently, one expects the dc conductance to depend on g faster than indicated by Eq. (2). This nonlinear dependence of $Y(\omega=0)$ upon g is borne out by the present calculations. In particular, on increasing the parameter a, the conductance exhibits an interesting trend towards a precipitous onset at a certain critical value of g, suggestive of a cooperative breakdown of the Coulomb barrier in electron tunneling.

Our calculations start from the Kubo formula for the conductance

$$\mathrm{Re}Y(\omega) = \frac{1}{\omega} \mathrm{Im}\left[\lim_{i\omega_n \to \omega + i\delta} \int_0^\beta d\tau\, e^{i\omega_n \tau} \langle T_\tau I(\tau) I(0)\rangle\right] , \tag{3}$$

where $\omega_n = 2\pi/\beta n$ is the Matsubara frequency and $\beta = (k_B T)^{-1}$. The current-current correlation function is calculated from the generating functional[2]

$$Z[\xi] = \mathrm{tr}\left[T_\tau \exp\left(-\int_0^\beta d\tau [H - I\xi(\tau)]\right)\right] , \tag{4}$$

where H is the Hamiltonian of the junction, including the charging energy and $\xi(\tau)$ is an external potential. Following Ref. 2, the quantity $Z[\xi]$ can be expressed as a path integral over the phase variable $\theta(\tau)$, which is related to the potential difference V across the junction by $\theta = eV/\hbar$. Then the current-current correlation can be expressed also as a path integral[2]

$$\langle T_\tau I(\tau) I(0)\rangle = \frac{2e^2}{Z[0]} \int D\theta\, e^{-A[\theta]} \alpha(\tau) \cos[\theta(\tau) - \theta(0)] , \tag{5}$$

where $A[\theta]$ is the effective action of the junction of the form

$$A[\theta] = \frac{C\hbar}{2e^2} \int_0^\beta d\tau\, \dot\theta^2(\tau) + 2\int_0^\beta d\tau \int_0^\beta d\tau' \alpha(\tau - \tau') \times \sin^2\left(\frac{\theta(\tau) - \theta(\tau')}{2}\right) . \tag{6}$$

The function $\alpha(\tau)$ is given by[2]

$$\alpha(\tau)=\frac{1}{2\pi}\left(\frac{R_0}{R_N}\right)\frac{(\pi k_B T/\hbar)^2}{\sin^2(\pi k_B T\tau/\hbar)} . \qquad (7)$$

According to Eqs. (3) and (5), the calculation of the conductance involves the phase correlator, defined as a non-Gaussian path integral

$$\Gamma(\tau)=\frac{1}{Z[0]}\int D\theta e^{-A[\theta]}\cos[\theta(\tau)-\theta(0)] . \qquad (8)$$

In what follows, we employ the self-consistent harmonic approximation (SCHA) to evaluate $\Gamma(\tau)$. This approximation can be derived from the variational principle[3,4]

$$F\le F_0+\frac{1}{\beta Z_0}\int D\theta e^{-A_0[\theta]}(A-A_0)=F_t , \qquad (9)$$

where

$$Z_0=e^{-\beta F_0}=\int D\theta e^{-A_0[\theta]} . \qquad (10)$$

The trial action $A_0[\theta]$ is chosen in the form

$$A_0[\theta]=\frac{C\hbar}{2e^2}\int_0^\beta d\tau\dot{\theta}^2(\tau)+\tfrac{1}{2}\int_0^\beta d\tau\int_0^\beta d\tau' a(\tau-\tau')[\theta(\tau')]^2 . \qquad (11)$$

The function $K(\tau-\tau')$ plays a role of the variational parameter which is determined from Eq. (9) by requiring that $\delta F_t=0$ when $K\to K+\delta K$. This yields a self-consistent equation for K:

$$K(\tau-\tau')=\exp[-\tfrac{1}{2}D(\tau-\tau')] , \qquad (12)$$

where

$$D(\tau-\tau')=\frac{1}{Z_0}\int D\theta e^{-A_0[\theta]}[\theta(\tau)-\theta(\tau')]^2 . \qquad (13)$$

According to Eq. (8), the phase correlator $\Gamma(\tau)$, evaluated within the SCHA, is given by the Gaussian path integral

$$[\Gamma(\tau)]_{\mathrm{SCHA}}=\Gamma_0(\tau)=\frac{1}{Z_0}\int D\theta e^{-A_0[\theta]}\cos[\theta(\tau)-\theta(0)]=K(\tau) . \qquad (14)$$

The explicit solution of Eqs. (12) and (13) is done by expanding $\theta(\tau)$ into a Fourier series[3]

$$\theta(\tau)=\sum_{n=-\infty}^{\infty}\theta_n e^{-i\omega_n\tau} . \qquad (15)$$

Then the path integral (13) reduces to a multiple Gaussian integration over the coefficients θ_n. An integral equation for $\Gamma_0(\tau)$ is obtained in the form

$$\Gamma_0(\tau)=\exp\left(\frac{-e^2\beta}{2\pi^2 C}\sum_{n=1}^{\infty}\frac{1-\cos(\omega_n\tau)}{n^2+(e^2\beta^2/2\pi^2C)\int_0^\beta d\tau\alpha(\tau)\Gamma_0(\tau)[1-\cos(\omega_n\tau)]}\right) . \qquad (16)$$

Introducing the dimensionless parameters $a=e^2\beta/2\pi^2C$ and $g=\pi R_0/2R_N$, changing the variable τ to $x=(2\pi/\beta)\tau$, and using Eq. (7) for $\alpha(\tau)$ we obtain from Eq. (16)

$$\Gamma_0(x)=\exp\left(-a\sum_{n=1}^{\infty}\frac{1-\cos(nx)}{n^2+ag\int_0^\pi\Gamma_0(x)\frac{1-\cos(nx)}{1-\cos x}dx}\right) . \qquad (17)$$

This equation has been solved numerically for $\Gamma_0(x)$ by the method of successive approximations. This method exhibits, in the present case, a good convergence, allowing us to limit the number of iterations to 10. On the other hand, the Fourier series in the exponent of Eq. (17) shows a slow convergence, especially when the parameters a and g are large. Hence, the number of terms in the series was taken $n=10^3$, in which case the overall accuracy of $\Gamma_0(x)$ was of the order of 1%.

In order to test whether some sort of critical behavior takes place, as a result of the non-Gaussian nature of the effective action $A[\theta]$, we calculate the effective conductance at $\omega=0$, which according to Eqs. (3)–(14) is given by

$$Y_{\mathrm{SCHA}}(\omega=0)=\frac{1}{\pi R_N}\int_0^\pi dx\,\Gamma_0(x) , \qquad (18)$$

where $\Gamma_0(x)$ is the solution of Eq. (17). In Fig. 1, we plot the dimensionless conductance Y_{SCHA} defined as

$$Y_{\mathrm{SCHA}}=\frac{\pi^2R_0}{2}Y_{\mathrm{SCHA}}(\omega=0)=g\int_0^\pi dx\,\Gamma_0(x) \qquad (19)$$

as a function of the coupling constant g for three values of the parameter a.

For the sake of comparison, we also show, as dashed curves, the conductance $\overline{Y}_H$ calculated in the harmonic approximation, which is based on the Gaussian choice for the action (6):

$$A_H[\theta]=\frac{C\hbar}{2e^2}\int_0^\beta d\tau\dot{\theta}^2(\tau)+\tfrac{1}{2}\int_0^\beta d\tau\int_0^\beta d\tau' a(\tau-\tau')[\theta(\tau)-\theta(\tau')^2] . \qquad (20)$$

Y_H is then given by Eqs. (18) and (19), with $\Gamma_0(\tau)$ re-

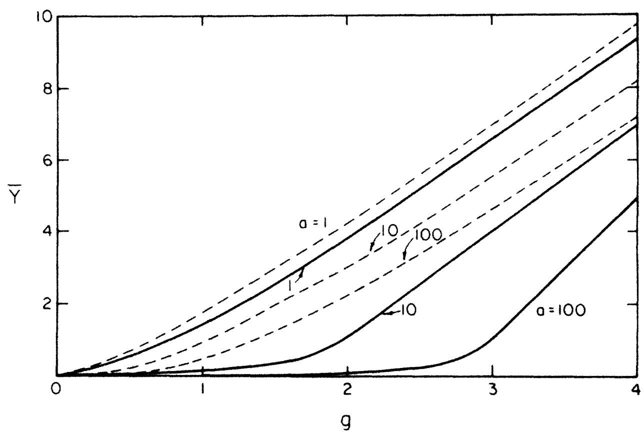

FIG. 1. Dimensionless dc conductance $\bar{Y}$ of a tunnel junction plotted as a function of the coupling constant $g = \pi\hbar/2e^2R_N$, where R_N is the nominal resistance of the junction. The parameter $a = e^2/2\pi^2k_BTC$, where C is the capacitance. The full lines represent $\bar{Y}_{\rm SCHA}$ given by Eq. (19); the dashed lines give the conductance $\bar{Y}_H$, based on the harmonic approximation of Eq. (20).

placed by

$$\Gamma_H(\tau) = \frac{1}{Z_h}\int D\theta e^{-A_H[\theta]}\cos[\theta(\tau)-\theta(0)] \ . \qquad (21)$$

We see that the values of $\bar{Y}_{\rm SCHA}$ are always smaller than those for $\bar{Y}_H$, indicating that the harmonic approximation overestimates the role of dissipation in the suppression of the phase fluctuations. The difference $\bar{Y}_H - \bar{Y}_{\rm SCHA}$ becomes more pronounced as the parameter a increases, showing the increasing role of the non-Gaussian phase fluctuations. We note that the "weak-coupling" conductance $\bar{Y}_{\rm wc}$, calculated from Eq. (2), yields straight lines (not shown in Fig. 1) with the slopes 0.8, 0.06, and 0.006 for $a = 1$, 10, and 100, respectively. For very large values of g, both the $\bar{Y}_H$ and $\bar{Y}_{\rm SCHA}$ curves tend to exhibit, for all values of a, the same asymptotic "strong-coupling" slope, equal to π. The most interesting feature of the SCHA calculations is the existence of the crossover between these weak-coupling and the strong-coupling regimes, which takes place, for large values of a, over a narrow region near some critical value of the coupling constant g_c. The values of g_c increase with the parameter a, but tend to saturate towards $g_c \simeq 3$. This indicates that, for large values of a, the weak-coupling approximation breaks down when $g \gtrsim 3$, which is consistent with the criterion $R_N/R_Q \gg 1$. The latter weak-coupling condition can be also written as $R_NC \gg \hbar/E_c$, where E_c is the charging energy (Coulomb gap). This implies that, in a weakly coupled junction, the time constant for the capacitor discharge is large enough so as not to smear the Coulomb gap.

Extrapolating the SCHA results to $a \to \infty$, one expects that in this limit the $\bar{Y}_{\rm SCHA}$ curve develops a precipitous onset, marking a transition from an insulating to conducting regime at some critical value of g_c. In this context, it is useful to remark that as $\beta = 1/k_BT$ goes to infinity, the action (6) is equivalent to a one-dimensional xy model with long-range interactions along the τ axis given by the kernel $\alpha(\tau) \propto \tau^{-2}$. According to the renormalization-group analysis of Fisher, Ma, and Nickel[5] the interaction decaying as τ^{-2} has a borderline range for the possibility of having a phase transition in a one-dimensional system. Although true phase transition with broken symmetry ($\langle\theta\rangle \neq 0$) is not expected in the problem of a tunnel junction, the present calculations indicate the possibility of a critical behavior in the phase correlation function.

It should be interesting to apply the functional integration method to granular metals consisting of arrays of ultrasmall tunnel junctions.[6] The crossover from activated to Ohmic conduction found in the present work for a single junction may also contribute to the metal-insulator transition in granular films.[7]

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