Exciton linewidth in semiconducting quantum-well structures

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The exciton linewidth has been calculated in semiconducting quantum-well structures as a function of the width of the well when the electron-hole pairs which are bound into the exciton are scattered by polar optical phonons. The linewidth for this scattering mechanism is found to increase as the width of the well decreases; therefore confinement of the exciton in a quantum-well structure will increase the linewidth of the exciton peaks when the exciton is scattered by polar optical phonons. The dominant contribution to the linewidth comes from processes in which the exciton is ionized into a free electron-hole pair by the absorption of a polar optical phonon. The numerical value we calculate for the linewidth is in good agreement with the experimentally observed value in GaAs/$Ga_{1-x}Al_xAs$ quantum-well structures.

I. INTRODUCTION

With the advance of crystal growth techniques such as molecular-beam epitaxy (MBE), it has become possible to grow high-quality superlattice and quantum-well structures.[1] These structures have become of great interest because of their enhanced mobilities when they are modulation doped[2–4] and the appearance of strong exciton peaks[5,6] which persist even at room temperature.[7,8] These excitonic effects have been observed in the optical absorption of these quantum-well structures,[5–8] in their photoluminescence,[9,10] and in Raman scattering.[11] The study of these exciton effects has evoked considerable interest because of the physical insight they provide about the optical properties of materials and their possible technological applications in low-power optical switching and signal-processing devices.[12–15] As a result of this interest, theoretical calculations have been performed to understand the contribution of the excitons to the optical absorption in quantum-well structures[16] and the effect of the confinement on the exciton binding energy.[17–19] Calculations have taken account of the effect of screening[20] and the application of strong electric fields[21] on the exciton binding energy. Work has also been done on the effect of the confinement of the bound electron-hole pair on the binding of excitons to impurities.[22,23]

In recent experiments,[7,8] evidence has been put forward which indicates that the exciton linewidth in a quantum-well structure does not change from its value in a bulk semiconductor. At certain photon energies, there is resonant absorption of light due to the creation of excitons. The photon energy at which this resonant absorption takes place is determined by the energy gap between the conduction and valence bands in the semiconductor and the binding energy of the exciton relative to the bottom of the conduction band. This exciton absorption peak is broadened because of the interaction of the exciton with acoustic and optical phonons and with various other imperfections in the crystal. However, theoretical work[24–27] indicates that for some mechanisms, the scattering of carriers should be greatly enhanced by their confinement in a quantum-well structure. Since the linewidth of the exciton will be determined by the scattering of the electron-hole pairs which are bound together into the exciton by various scattering mechanisms in the semiconductor,[28–31] it is of some interest to calculate the exciton linewidth in a quantum-well structure. Knox[28] has presented a theoretical calculation of the exciton linewidth using first-order perturbation theory. The linewidth of the exciton can be directly related to the exciton lifetime due to the interaction of the bound electron-hole pair with various imperfections in the semiconductor. This in turn can be calculated using time-dependent perturbation theory to find the transition probability between some initial state of the exciton and all final states for which the transition is allowed by the selection rules and energy conservation. Several authors have used this approach to calculate the contribution to the exciton linewidth due to scattering of the electron-hole pair from optical and acoustic phonons[29,30] and from other point imperfections in the semiconductor.[31] In polar semiconductors such as GaAs, a major contribution to the exciton linewidth is due to the interaction of the exciton with polar optical phonons. Alperovitch *et al.*[29] have performed calculations of the exciton linewidth for this mechanism in bulk semiconductors. Singh and Bajaj[32] have calculated the contribution to the excitonic photoluminescence linewidth due to statistical potential fluctuations caused by the alloy components in an alloy semiconductor. Iwanura *et al.*[33] have experimentally studied the linewidth by means of luminescence measurements and attributed the main contributions to the linewidth to interaction with polar optical phonons and to inhomogeneous fluctuations in the width of the well. Lee, *et al.*[34] have investigated

both theoretically and experimentally the luminescence linewidths in GaAs quantum wells at temperatures below 150 K and found the dominant contributions to the exciton linewidth at low temperatures are due to interaction with acoustic phonons and ionized impurities.

In this paper, we will use a similar approach to obtain the contribution to the exciton linewidth when the bound electron-hole pair is scattered by polar optical phonons in a quantum-well structure. The contributions to the linewidth which we will consider involve the scattering of the exciton by polar optical phonons from the initial exciton ground state in which the exciton is created without any center-of-mass motion to a final state in which the exciton either has been promoted to an excited state with center-of-mass motion due to its interaction with the polar optical phonons, remains in the ground state but with a gain of center-of-mass motion, or is ionized into a free electron-hole pair by the absorption of an optical phonon. The latter process can occur since the energy of a polar optical phonon in GaAs, which is 36 MeV, is much greater then the binding energy of the exciton even when the latter is enhanced by the confinement of the electron-hole pair in a quantum-well structure. There are other mechanisms, such as fluctuations in the layer thicknesss,[35,36] which can contribute to the exciton linewidth in quantum-well structures but these mechanisms should be temperature independent and would play an important role at low temperatures while phonons would be expected to give the dominant contribution to the exciton linewidth at higher temperatures. In Sec. II, we will present our formalism for calculating these contributions to the exciton linewidth and our numerical results for the effect of the confinement on the linewidth, while in Sec. III we will present a discussion of these results.

II. CALCULATION OF EXCITON LINEWIDTH

In this paper we will follow the same approach as Alperovitch, *et al.*[29] in calculating the exciton linewidth. For the case of interaction of the electron-hole pair with polar optical phonons, they have obtained the following expression for the linewidth:

$$\Gamma_{\rm op}=Ne^2(\epsilon_\infty^{-1}-\epsilon_0^{-1})\hbar\omega_{\rm L}\times\sum_f\int d^3q\,q^{-2}\,|v_{f,1s}(\mathbf{q})|^2\times\delta(E_f(\mathbf{q})-E_{1s}(0)-\hbar\omega_{\rm L})\,, \tag{1}$$

where

$$v_{f,1s}(\mathbf{q})=\int d^3r\,\Psi_f(\mathbf{r})[\exp(im_e\mathbf{q}\cdot\mathbf{r}/m)-\exp(im_h\mathbf{q}\cdot\mathbf{r}/m)]\Psi_{1s}(\mathbf{r})\,. \tag{2}$$

Here N is the number of thermal phonons, which is given by the Bose-Einstein factor, ϵ_0 and ϵ_∞ are the static and optical dielectric constants, respectively, $\omega_{\rm L}$ is the optical phonon frequency, m_e and m_h are the electron and hole masses, $m=m_e+m_h$, and f represents the quantum numbers of the final state of the exciton. Here, $\Psi_\alpha(\mathbf{r})$ is the part of the exciton wave function in the relative coordinates of the electron-hole pair, the integration over the center-of-mass coordinates having given the selection rule that the final state of the exciton must have a center-of-mass momentum $\hbar\mathbf{q}$.

For an electron-hole pair which is confined to move in a quantum well, the problem cannot be separated into relative and center-of-mass coordinates in the direction perpendicular to the plane of the well although such a separation can take place in the plane of the well. This is because both the electron and hole are separately confined by the potential barriers in the direction perpendicular to the plane of the well. In our calculations, we will use a wave function of the type used by Bastard *et al.*[17] in a variational calculation of the binding energy of excitons in a quantum well. This wave function has cylindrical symmetry with respect to the plane of the well and gives binding energies in agreement with those obtained using a somewhat better trial wave function for narrow quantum wells. This wave function is of the form

$$|\alpha)=(2/L)\sin(\pi z_e/L)\cos(\pi z_h/L)\exp(i\mathbf{K}\cdot\mathbf{R})\phi_\alpha(\mathbf{r})\,, \tag{3}$$

where L is the width of the well, z_e and z_h are the electron and hole coordinates in the direction perpendicular to the plane of the well, $\mathbf{K}$ is the center-of-mass momentum of the electron-hole pair, and $\mathbf{R}$ and $\mathbf{r}$ are the center-of-mass and relative coordinates of the electron-hole pair in the plane of the well.

For an electron-hole pair interacting with polar optical phonons in a weakly polar material like GaAs, there are many different final states to which the transition can take place. For example, in an optical transition, the electron-hole pairs are created with equal and opposite momentum so that the center-of-mass momentum of the exciton is zero. The exciton can then gain center-of-mass momentum by absorbing an optical phonon. This can occur with the exciton either remaining in its ground $1s$ state, as far as the relative motion of the electron-hole pair is concerned, or the exciton can be promoted to one of its excited states. In this case, the relative part of the wave function is hydrogenic in form for both the initial and final states while the center-of-mass part of the wave function differs in the final state, being of the plane-wave form with vector $\mathbf{q}$. Since the energy of the optical phonons, which is 35 meV in GaAs, is larger than the binding energy of the exciton in either the bulk or in the quantum well, transitions can also take place in which the final state is in the continuum, i.e., the exciton is disassociated into a free electron-hole pair. In this case, the final-state wave function is a product of plane waves for the electron and hole in the plane of the quantum well. It is these kinds of transitions that we will consider in our calculations of the exciton linewidth due to the interaction with polar optical phonons. For the first type of transition, in which the final state is one in which the exciton does remain in a bound state with only a change in the center-of-mass motion, we will consider situations in which the final state is a $1s$ state, $2s$ state, $2p$ state, or $3s$ state. For the case where both the initial and final states are $1s$ states, our relative wave function used is

$$\phi_{1s}=(\beta/2\pi^{1/2})\exp(-\beta r/2)\,, \tag{4}$$

where β is the variational parameter in the trial wave function used in the variational calculation of the exciton binding energy.[17] Using this wave function in Eqs. (1) and (2), we obtain the following expression for the exciton linewidth in a quantum well:

$$\Gamma_{\rm op}=(4Ne^2m\omega_{\rm L}\beta^6/\hbar\epsilon' L^2) \times I(2\pi/L,(2m\omega_{\rm L}/\hbar)^{1/2})f_0(2m\omega_{\rm L}/\hbar)\ , \quad (5)$$

where

$$I(x,y)=(\pi^2/xy^2) \times\{1+y^2/2(x^2+y^2) -x^5/2\pi y(x^2+y^2)^2[1-\exp(-2\pi y/x)]\} \quad (6)$$

and

$$f_0(z)=\{[\beta^2+(2m_h^2z/m^2)]^{-3/2} -[\beta^2+(2m_e^2z/m^2)]^{-3/2}\}^2\ . \quad (7)$$

For a bulk semiconductor, the equivalent result for the linewidth is given by

$$\Gamma_{\rm op}=(Ne^2/\epsilon')(2m\omega_{\rm L}/\hbar)^{1/2}\beta^8h_0(2m\omega_{\rm L}/\hbar)\ , \quad (8)$$

where

$$h_0(z)=\{[\beta^2+(m_h/m)^2z]^{-2}-[\beta^2+(m_e/m)^2z]^{-2}\}^2\ . \quad (9)$$

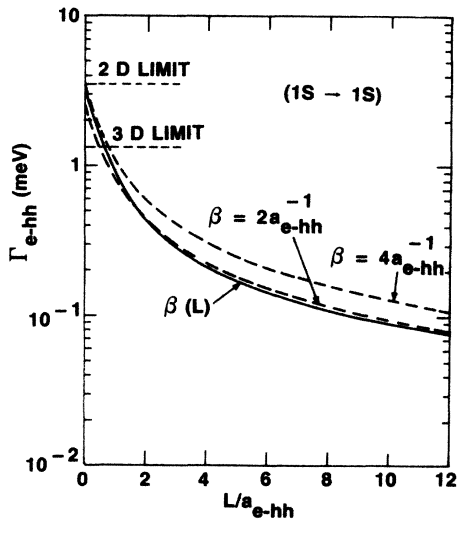

FIG. 1. The contribution to the exciton linewidth due to transitions in which the exciton is scattered by polar optical phonons from the $1s$ state without center-of-mass motion to the same state with center-of-mass motion is shown as a function of the well for the heavy hole exciton in GaAs. The solid line is the result using the value of the variational parameter β given by Bastard *et al.* (Ref. 17). The dashed curves represent the results obtained using the 2D and 3D values of β. The 2D and 3D limits of this contribution to the linewidth are also shown. The parameters used in the calculations are those characteristic of GaAs, i.e., the optical phonon energy is 36 meV, the Bohr radius for the heavy hole exciton is 118 Å and the temperature is taken to be 300 K.

Here, $\epsilon'^{-1}=\epsilon_\infty^{-1}-\epsilon_0^{-1}$ and we have used a three-dimensional hydrogenic wave function for the calculation of the linewidth in the bulk. Therefore, in the bulk, $\beta=(2/a)$, where a is the Bohr radius of the exciton in the bulk while for a quantum-well structure, β lies between this bulk value and its purely two-dimensional limit value $(4/a)$. In Fig. 1, this contribution to the exciton linewidth is shown as a function of the width of the quantum well. The value of the variational parameter β as a function of well width for the $1s$ state is taken from the variational calculation of Shinozuka and Matsuura.[18] The three-dimensional (3D) limit of the linewidth given by Eq. (6) is also shown in the figure. The quantum-well results using the wave function given in Eq. (4) fall below the 3D results in the limit of thick quantum wells but this is just a reflection of the fact that this type of variational wave function only gives good agreement with the results of a better trial wave function for narrow wells. This better wave function was not used in our calculations because analytic results for the linewidth could not be obtained using this wave function (Bastard's type-II trial function[17]). However, our results do show that the confinement of the carriers does lead to an increase of this contribution to the exciton linewidth.

For our calculations of the contributions of transitions to the $2s$ and $2p$ states to the exciton linewidth, we used the quasi-two-dimensional wave functions of Shinozuka and Matsuura[18]

$$\phi_{2s}(\mathbf{r})=(\alpha/\pi^{1/2})(1-\alpha r)\exp(-\alpha r/2) \quad (10)$$

and

$$\phi_{2p}(\mathbf{r})=\gamma^2/(6\pi)^{1/2}r\exp(-\gamma r/2)\ . \quad (11)$$

The results we obtain for the contribution to the exciton linewidth due to transitions to these states in a quantum well are

$$\Gamma_{\rm op}(1s\text{-}2s)=(4\alpha^2\beta^2Ne^2\omega_{\rm L}m/3L^2\hbar\epsilon') \times I(2\pi/L,[(2m/\hbar^2)(\hbar\omega_{\rm L}+E_{1s}-E_{2s})]^{1/2}) \times f_1((2m/\hbar^2)(\hbar\omega_{\rm L}+E_{1s}-E_{2s})) \quad (12)$$

and

$$\Gamma_{\rm op}(1s\text{-}2p)=(3Ne^2\omega_{\rm L}/\epsilon\hbar mL^2)\beta^2\gamma^4(\gamma+\beta)^2 \times I(2\pi/L,[(2m/\hbar^2)(\hbar\omega_{\rm L}+E_{1s}-E_{2p})]^{1/2}) \times f_2((2m/\hbar^2)(\hbar\omega_{\rm L}+E_{1s}-E_{2p}))\ , \quad (13)$$

where

$$f_1(z)=\{\tfrac{1}{2}(\alpha+\beta)[\tfrac{1}{4}(\alpha+\beta)^2+(m_h/m)^2z]^{-3/2}-\tfrac{1}{2}(\alpha+\beta)[\tfrac{1}{4}(\alpha+\beta)^2+(m_e/m)^2z]^{-3/2}$$
$$+\alpha[\tfrac{1}{2}(\alpha+\beta)^2-(m_h/m)^2z][\tfrac{1}{4}(\alpha+\beta)^2+(m_h/m)^2z]^{-5/2}$$
$$-\alpha[\tfrac{1}{2}(\alpha+\beta)^2-(m_e/m)^2z][\tfrac{1}{4}(\alpha+\beta)^2+(m_e/m)^2z]^{-5/2}\}^2 \quad (14)$$

and

$$f_2(z)=z\{m_h[\tfrac{1}{4}(\gamma+\beta)^2+(m_h/m)^2z]^{-5/2}$$
$$-m_e[\tfrac{1}{4}(\gamma+\beta)^2+(m_e/m)^2z]^{-5/2}\}^2 . \quad (15)$$

Here α and γ are the variational parameters for the $2s$ and $2p$ hydrogenic wave functions in a quantum well and E_{1s}, E_{2s}, and E_{2p} are the energies of the exciton in the $1s$, $2s$, and $2p$ states, respectively. Using the 3D hydrogenic wave functions for the $2s$ and $2p$ states, we get the following results for the contribution to the exciton linewidth due to transitions to these states:

$$\Gamma_{\rm op}(1s\text{-}2s)=(Ne^2\omega_L\beta^8/128\epsilon')(2m)^{1/2}$$
$$\times(\hbar\omega_L+E_{1s}-E_{2s})^{-1/2}$$
$$\times h_1((2m/\hbar^2)(\hbar\omega_L+E_{1s}-E_{2s})) \quad (16)$$

and

$$\Gamma_{\rm op}(1s\text{-}2p)=(Ne^2\beta^8\omega_L/384\epsilon')(2m)^{1/2}$$
$$\times(\hbar\omega_L+E_{1s}-E_{2p})^{-1/2}$$
$$\times h_2((2m/\hbar^2)(\hbar\omega_L+E_{1s}-E_{2p})) , \quad (17)$$

where

$$h_1(z)=16z\{(m_h/m)^2[(9\beta^2/16)+(m_h/m)^2z]^{-3}$$
$$-(m_e/m)^2[(9\beta^2/16)+(m_e/m)^2z]^{-3}\}^2 \quad (18)$$

and

$$h_2(z)=\{[(27\beta^2/16)-(m_h/m)^2z]$$
$$\times[(9\beta^2/16)+(m_h/m)^2z]^{-3}$$
$$-[(27\beta^2/16)-(m_e/m)^2z]$$
$$\times[(9\beta^2/16)+(m_e/m)^2z]^{-3}\}^2 . \quad (19)$$

Unfortunately, although the energies of the $2s$ and $2p$ states of the confined exciton have been calculated,[18] the variational parameters as a function of the width of the quantum well have not been published. Therefore, the only comparison we are able to show here is the results for a purely 2D exciton versus the 3D results for an exciton in a bulk semiconductor. These are shown in Figs. 2 and 3 where we have used the values of the parameters α and γ in both the 2D and 3D limits to bracket the behavior of the linewidth as a function of the well width. As can be seen from these figures, the contribution to the linewidth from transitions to these states is much smaller by many orders of magnitude than that due to transitions to the $1s$ state with center-of-mass motion. We have also calculated the contribution to the linewidth due to transitions to the $3s$ state. For the quasi-two-dimensional results, we use the 2D wave function for the $3s$ hydrogenic state. The result for this case is

$$\Gamma=(4Ne^2\omega_L\gamma^2\beta^2m/5\epsilon'L^2\hbar)$$
$$\times I(2\pi/L,[(2m/\hbar^2)(\hbar\omega_L+E_{1s}-E_{3s})]^{1/2})$$
$$\times f_3(2m(\hbar\omega_L+E_{1s}-E_{3s})/\hbar^2) \quad (20)$$

with

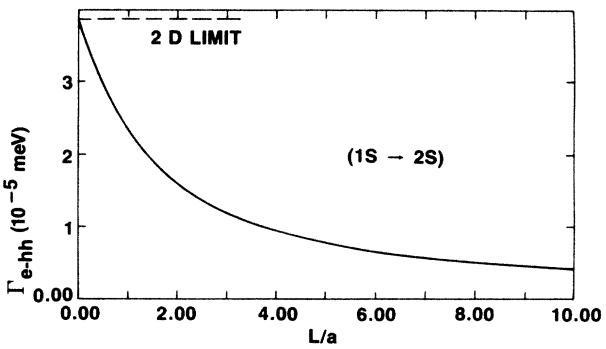

FIG. 2. The contribution to the linewidth due to transitions from the $1s$ to $2s$ state is shown as a function of well width for the same situation as that considered in Fig. 1.

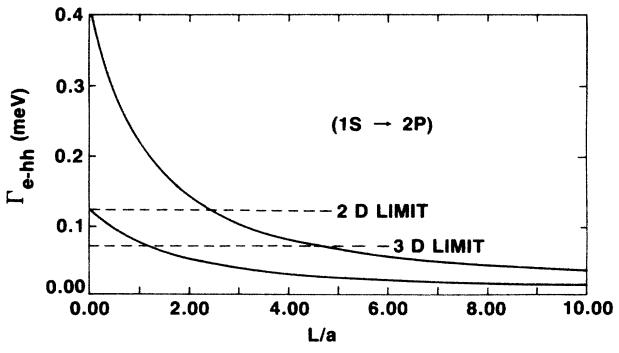

FIG. 3. The contribution to the linewidth due to transitions from the $1s$ to $2p$ state is shown as a function of well width for the same situation as that considered in Fig. 1.

$$f_3(x)=(\gamma+\beta)\{[\tfrac{1}{4}(\gamma+\beta)^2+(m_h/m)^2x]^{-3/2}-[\tfrac{1}{4}(\gamma+\beta)^2+(m_e/m)^2x]^{-3/2}\}$$
$$+4\gamma\{[\tfrac{1}{2}(\gamma+\beta)^2-(m_e/m)^2x][\tfrac{1}{4}(\gamma+\beta)^2+(m_e/m)^2x]^{-5/2}$$
$$-[\tfrac{1}{2}(\gamma+\beta)^2-(m_h/m)^2x][\tfrac{1}{4}(\gamma+\beta)^2+(m_h/m)^2x]^{-5/2}\}$$
$$+\tfrac{3}{2}\gamma^2(\gamma+\beta)^2\{[3(m_h/m)^2x-\tfrac{1}{2}(\gamma+\beta)^2][\tfrac{1}{4}(\gamma+\beta)^2+(m_h/m)^2x]^{-7/2}$$
$$-[3(m_e/m)^2x-\tfrac{1}{2}(\gamma+\beta)^2][\tfrac{1}{4}(\gamma+\beta)^2+(m_e/m)^2x]^{-7/2}\})^2 . \quad (21)$$

For the 3D results, we use the 3D hydrogenic $3s$ wave function and obtain the result

$$\Gamma=(Ne^2\omega_L\beta^8/81\epsilon')[2m/(\hbar\omega_L+E_{1s}-E_{3s})]^{1/2}$$
$$\times h_3(2m(\hbar\omega_L+E_{1s}-E_{3s})/\hbar^2) . \quad (22)$$

Our numerical calculations show that this contribution to the exciton linewidth is also much smaller than that due to transitions to the $1s$ state.

Finally, for the case where the final state of the exciton consists of a free electron-hole pair, the relative wave function of the exciton in the final state is given by

$$\Phi=\exp(i\mathbf{k}\cdot\mathbf{r})/A^{1/2} , \quad (23)$$

where $\mathbf{k}=(m_e\mathbf{k}_e-m_h\mathbf{k}_h)/m$ and $\mathbf{k}_e$ and $\mathbf{k}_h$ are the electron and hole wave vectors, respectively, in the plane of the quantum well. Using this wave function, we find that the contribution to the linewidth due to processes in which the exciton is disassociated into a free electron-hole pair by the absorption of a polar optical phonon in a quantum well is given by

$$\Gamma=(32Ne^2\beta^4\omega_L m_e/\pi\epsilon' L^2\hbar)\int_0^{k_1} dk\, k f(k,k_0)\int_0^{2\pi} d\phi\, I(2\pi/L,(k^2+k_0^2+2kk_0\cos\phi)^{1/2}) , \quad (24)$$

where

$$f(k,k_0)=(\beta^2+4k^2)^{-3/2}-(\beta^2+4k_0^2)^{-3/2} \quad (25)$$

and

$$k_0=(2m/\hbar^2)^{1/2}(E_{1s}+\hbar\omega_L-\hbar^2k^2/2m_h)^{1/2} , \quad (26a)$$
$$k_1=(2m/\hbar^2)^{1/2}(E_{1s}+\hbar\omega_L)^{1/2} . \quad (26b)$$

In Fig. 4, the contribution to the linewidth due to the ionization of the exciton into a free electron-hole pair is shown as a function of the well width. In the limit of very narrow wells, the numerical result we obtain from our calculation approaches the value for the linewidth of about 7 meV estimated from the experiments of Ref. 8.

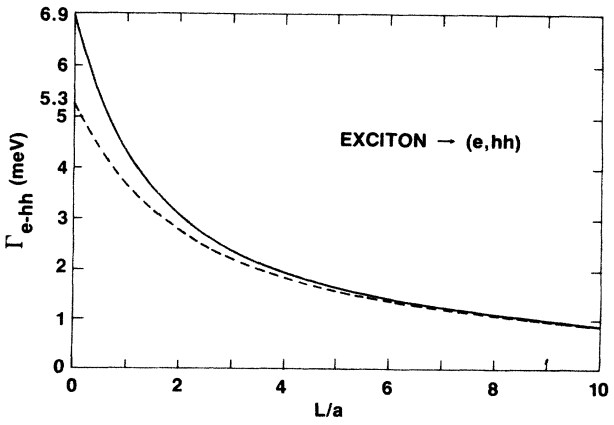

FIG. 4. The contribution to the linewidth due to transitions to a final state in which the exciton is ionized into a free electron-hole pair is shown as a function of well width for the same situation as that considered in Fig. 1.

To obtain the 2D limits of our expressions for the exciton linewidths, we use the result

$$I(2\pi/L,[(2m/\hbar^2)(\hbar\omega_L-\Delta E)]^{1/2})$$
$$=(\pi L^2/4)[(2m/\hbar^2)(\hbar\omega_L-\Delta E)]^{-1/2} \quad (27)$$

as L goes to zero where ΔE is the energy difference between the excited states and the $1s$ ground state of the exciton. In the purely 2D limit, this energy is $32/9R$ for the transition from the $1s$ to $2s$ or $2p$ states and the parameters α and γ have the values of $4/3a$. Also, in this limit, $\beta=4/a$. Here R and a are the Rydberg unit and Bohr radius for the exciton in the bulk semiconductor. In the 2D limit, Eqs. (5) and (12)–(14) reduce to

$$\Gamma(1s\text{-}1s)=(\pi Ne^2\beta^6/2\epsilon')$$
$$\times(2m\omega_L/\hbar)^{1/2}f_0(2m\omega_L/\hbar) , \quad (28)$$
$$\Gamma(1s\text{-}2s)=(\pi Ne^2\omega_L\alpha^2\beta^2/6\epsilon')(2m)^{1/2}$$
$$\times(\hbar\omega_L-32R/9)^{-1/2}$$
$$\times f_1(2m(\hbar\omega_L-32R/9)/\hbar^2) , \quad (29)$$
$$\Gamma(1s\text{-}2p)=[3\pi Ne^2\beta^2\gamma^4(\gamma+\beta)^2/2\epsilon'](2m)^{-3/2}$$
$$\times(\hbar\omega_L-32R/9)^{-1/2}$$
$$\times f_2(2m(\hbar\omega_L-32R/9)/\hbar^2) . \quad (30)$$

For the $3s$ state, the linewidth in the 2D limit is given by

$$\Gamma = (\pi N e^2 \gamma^2 \beta^2 / 10\epsilon')[2m(\hbar\omega_L + E_{1s} + E_{3s})]^{1/2} \times f_3(2m(\hbar\omega_L + E_{1s} - E_{3s})/\hbar^2) . \quad (31)$$

In the 3D limit, the energy difference between the $2s$ or $2p$ state and the $1s$ state is $3R/4$ while β is $2/a$. Our results show a definite enhancement of the exciton linewidth with decreasing well width. The 2D values of the linewidth are always much higher than the 3D values which indicates that the confinement of the exciton in the quantum well increases the exciton linewidth due to scattering from optical phonons. We do not present here the 2D limit's results for the linewidth due to the ionization of the exciton since they are not really any simpler in form than the quasi-two-dimensional results. Here too, this contribution due to the linewidth decreases with increasing width of the quantum well.

III. DISCUSSION

Our calculations indicate that the main contributions to the exciton linewidth due to scattering by polar optical phonons arise from transitions to the states where the exciton is ionized into a free electron-hole pair or the exciton is excited into the $1s$ state with center-of-mass motion. The other contributions we have calculated to the exciton linewidth are negligibly small by comparison which means that it may be valid to completely neglect contributions due to the transitions to the other excited bound states of the exciton. The numerical results we obtain for the exciton linewidth taking just these two contributions into account are very close to those reported experimentally.[8] Therefore, our theoretical results seem to be in agreement with experimental observations. Our results also indicate that the exciton linewidth due to scattering from polar optical phonons is enhanced by the confinement of the exciton in the quantum-well structure. Finally, we have obtained results for the various contributions to the exciton linewidth in the limit of a two-dimensional electron-hole gas. Our results should be valid for very narrow quantum wells where the exciton's behavior is close to that for 2D excitons. However, for wider wells, our results for the exciton linewidth drop below their values in the 3D limit. This is because the exciton wave functions we used in our calculations are only good in the limit of very narrow wells.

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[1]R. Dingle, in *Advances in Solid State Physics*, edited by H. J. Queisser (Pergamon, Vieweg, Braunschweig, 1975), Vol. XV, p. 21.
[2]H. L. Stormer, R. Dingle, A. C. Gossard, W. Wiegmann, and M. D. Sturge, J. Vac. Sci. Technol. **16**, 1517 (1979).
[3]T. S. Drummond, H. Morkoc, K. Hess, and A. Y. Cho, J. Appl. Phys. **52**, 5231 (1981).
[4]M. A. Paalenen, D. C. Tsui, A. C. Gossard, and J. C. M. Hwang, Phys. Rev. B **29**, 6003 (1984).
[5]R. Dingle, W. Wiegmann, and C. H. Henry, Phys. Rev. Lett. **33**, 827 (1974).
[6]R. C. Miller, D. A. Kleinman, W. T. Tsang, and A. C. Gossard, Phys. Rev. B **24**, 1134 (1981).
[7]D. A. B. Miller, D. S. Chemla, D. J. Eilenberger, P. W. Smith, A. C. Gossard, and W. T. Tsang, Appl. Phys. Lett. **41**, 679 (1982).
[8]D. S. Chemla, D. A. B. Miller, P. W. Smith, A. C. Gossard, and W. Wiegmann, IEEE J. Quantum Electron. **QE-20**, 265 (1984).
[9]R. C. Miller, D. A. Kleinman, and W. A. Norland, and A. C. Gossard, Phys. Rev. B **22**, 863 (1980).
[10]H. Kawai, K. Kaneko, and N. Watanabe, J. Appl. Phys. **56**, 463 (1984).
[11]J. E. Zucker, A. Pinczuk, D. S. Chemla, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. **51**, 1293 (1983).
[12]T. H. Wood, C. A. Burrus, D. A. B. Miller, D. S. Chemla, T. C. Damen, A. C. Gossard, and W. Wiegmann, Appl. Phys. Lett. **44**, 16 (1984).
[13]D. A. B. Miller, D. S. Chemla, T. C. Damen, A. C. Gossard, W. Wiegmann, T. H. Wood, and C. A. Burrus, Appl. Phys. Lett. **45**, 13 (1984).
[14]A. Miller, D. A. B. Miller, and S. D. Smith, Adv. Phys. **30**, 679 (1982).
[15]P. W. Smith, Bell Syst. Tech. J. **61**, 1975 (1982).
[16]S. Satpathy and M. Altarelli, Phys. Rev. B **23**, 2977 (1981).
[17]G. Bastard, E. E. Mendez, L. I. Chang, and L. Esaki, Phys. Rev. B **26**, 1974 (1982).
[18]Y. Shinozuka and M. Matsuuara, Phys. Rev. B **28**, 4878 (1983); **29**, 3717 (1984).
[19]R. L. Greene, K. K. Bajaj, and D. E. Phelps, Phys. Rev. B **29**, 1807 (1984).
[20]H. N. Spector, J. Lee, and P. Melman, Superlattices Microstructures **1**, 149 (1985); J. Lee, H. N. Spector, and P. Melman, J. Appl. Phys. (in press).
[21]D. S. Chemla, T. C. Damen, D. A. B. Miller, A. C. Gossard, and W. Wiegmann, Appl. Phys. Lett. **42**, 864 (1983).
[22]G. Bastard, C. Delalande, M. H. Meynadier, P. M. Frijink, and M. Voos, Phys. Rev. B **29**, 7042 (1984).
[23]J. M. Rorison and D. C. Herbert, Superlattices Microstructures **1**, 423 (1985).
[24]K. Hess, Appl. Phys. Lett. **35**, 484 (1979).
[25]P. J. Price, Ann. Phys. (N.Y.) **133**, 217 (1981); Surf. Sci. **113**, 199 (1982).
[26]V. K. Arora and F. G. Awad, Phys. Rev. B **23**, 5570 (1981).
[27]R. K. Ridley, J. Phys. C **15**, 5899 (1982).
[28]R. Knox, *Theory of Excitons*, Suppl. 5 of *Solid State Physics* (Academic, New York, 1963), pp. 141–147.
[29]V. I. Alperovitch, V. M. Zaletin, A. F. Kravchenko, and A. S. Terekhov, Phys. Status Solidi B **77**, 465 (1976).
[30]Y. Toyozawa, Prog. Theor. Phys. (Kyoto) **20**, 53 (1958); **27**, 89 (1962); J. Phys. Chem. Solids **25**, 59 (1964).
[31]V. L. Bonch-Bruevich and V. D. Iskra, Phys. Status Solidi B **68**, 369 (1975).
[32]J. Singh and K. K. Bajaj, Appl. Phys. Lett. **44**, 1075 (1984).

[33]H. Iwamura, H. Kobayashi, and H. Okamota, Jpn. J. Appl. Phys. **23**, L795 (1984).

[34]J. Lee, E. S. Koteles, and M. O. Vassell, Phys. Rev. B **33**, 5512 (1986).

[35]C. Weisbuch, R. Dingle, and A. C. Gossard, Sol. State Commun. **37**, 219 (1981).

[36]J. Singh, K. K. Bajaj, and S. Chaudhuri, Appl. Phys. Lett. **44**, 805 (1984).