Mobility in a quasi-one-dimensional semiconductor: An analytical approach

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The electron mobility in a quasi-one-dimensional semiconductor is theoretically investigated when the scattering is due to ionized donors in an approximation where (i) the envelope wave function is assumed to be constant inside a cylindrical wire and zero outside and (ii) the finite-temperature effect is taken into account in the static dielectric function. It is shown that for the one-band case (intraband scattering only) this leads to an entirely analytical formula for the mobility at low temperature, for both uniform and modulation doping. For modulation doping, the theoretical mobility is much larger than that obtained in a two-dimensional semiconductor for comparable buffer-layer thicknesses. The main differences between the one- and the two-band cases (intraband and interband scattering) are pointed out. The localization effect is also briefly discussed.

I. INTRODUCTION

The problems related to mobility in semiconductors have drastically changed since it became possible to place donors outside of the region where electrons can move. This occurs in so-called two-dimensional semiconductors (2D SC), where the electrons can move within a "plane" whose thickness is of the order of some tens or hundreds of angstroms and where donor location can *a priori* be arbitrarily fixed.[1] For example a buffer layer can be inserted between electrons and donors.

When donors and electrons are in the same region of space in a 2D SC the mobility is essentially unaltered compared to the situation of a bulk semiconductor. However, the existence of modulation doping and more particularly of a buffer layer changes this aspect of the problem and indeed increases the maximum mobility by a considerable amount. If the electrons are confined in a wire as described in Ref. 2, one obtains what may be called a one-dimensional semiconductor (1D SC). Some features of the 2D SC and more particularly the effects of modulation doping are preserved. This type of wire made up of a square section of GaAs, 200×200 angstroms, embedded in $Ga_{1-x}Al_xAs$ was first proposed by Sakaki[3,4] who carried out a calculation of mobility in approximations which will be reviewed below. Several papers which deal with various problems related to 1D SC have recently been published,[5–7] illustrating an increasing interest in 1D SC.

A calculation of the 1D SC mobility limited by ionized donors has been reported for zero temperature by Sakaki.[3] That limit is in fact quite realistic at low temperature in the best 2D SC samples. In his model Sakaki assumed that (i) the electrons are exactly in the center of the wire, meaning that the (square of the) envelope function is merely a Dirac function, (ii) all the donors are located at the same distance from the center of the wire, and (iii) the dielectric function is equal to the background dielectric constant.

The purpose of the present paper is essentially the same, but our approximations are completely different. We use a model where (i) the envelope function describes fairly well the electronic distribution in a wire of finite section and is sufficiently simple to allow analytical calculation, (ii) donors are arbitrarily distributed which means that by analogy to 2D SC the doping can be either uniform or modulated, and (iii) the dependence on the wave vector of the dielectric function is explicit. It is well known that the one-dimensional dielectric function diverges when the wave vector is equal to twice the Fermi wave vector,[5] which is precisely the wave vector of interest. We shall thus be obliged to perform a calculation at finite temperature, at least for the part where the dielectric function is needed. Nevertheless we shall see that this model leads to an analytical formulation of the mobility in 1D SC.

The outline of this paper is as follows. In Sec. II A we present the general framework of the paper and in Sec. II B we specify the consequences of the approximations. The explicit calculation is performed in Sec. III. The numerical results (mainly given in several figures) are presented for the case of interest in Sec. IV. A discussion is given in Sec. V, where problems related to interband scattering and localization are tackled. Section VI is devoted to concluding remarks. Some details about the dielectric function are given in the Appendix.

II. GENERAL FRAMEWORK

A. Mobility formulation in a one-dimensional semiconductor

Within the effective mass approximation, which is assumed to be valid throughout this paper, the mobility can be written as

$$\mu=\frac{e}{m^*}\tau\ , \tag{1}$$

where m^* is the effective mass ($m^*=0.067$ a.u. in GaAs)

and τ the relaxation time. In the following we consider a degenerate electron gas with a Fermi level E_F. Here the only wave vectors of interest are the Fermi wave vectors k_F and $2k_F$, which is the difference between the initial wave vector and the final wave vector after any scattering process.

The electron energies are

$$E_{nk}=E_n+\hbar^2k^2/2m^*\ , \tag{2}$$

with the corresponding wave functions:

$$|\,n,k\,\rangle=|\,\zeta_n(x,y)e^{i\bar{k}z}\rangle=|\,\zeta_n(\rho)e^{i\bar{k}z}\rangle\ . \tag{3}$$

z is parallel to the wire axis and $\bar{k}$ is the one-dimensional wave vector. $\rho=(x,y)$. We take $E_0=0$ and assume that $E_F<E_1$ so that $E_F=\hbar^2k_F^2/2m^*$.

In the Born approximation the relaxation time τ due to ionized impurities scattering is inversely proportional to the cross section, i.e., to

$$\frac{\left|\left\langle 0,-k_F\left|\frac{e^2}{|\,\mathbf{r}-\mathbf{r}_d\,|}\right|0,k_F\right\rangle\right|^2_{\rm av}}{\left|\epsilon(2k_F)\right|^2}\ , \tag{4}$$

where $\mathbf{r}$ and $\mathbf{r}_d$ are the positions of the electron and of a donor respectively: $\mathbf{r}=(x,y,z)=(\rho,z),\mathbf{r}_d=(x_d,y_d,z_d)=(\rho_d,z_d)$, $\rho_d=(x_d,y_d)$. Subscripts av denote the usual average of impurities assumed to be randomly distributed. $\epsilon(k)$ is the dielectric function. Finally the mobility is

$$\mu=\mu_c\frac{|\,\epsilon(2k_F)\,|^2}{f}\ , \tag{5}$$

with

$$\mu_c=[\pi e\hbar^3/16(m^*)^2](4\pi\epsilon_0/e^2)^2 \tag{6}$$

and

$$f=\left|\frac{1}{2}\left\langle 0,-k_F\left|\frac{1}{|\,\mathbf{r}-\mathbf{r}_d\,|}\right|0,k_F\right\rangle\right|^2_{\rm av}\ , \tag{7}$$

where the linear donor density N_d is assumed to be equal to the linear electron density N (no compensation) so that N_d determines k_F, and where $(\ )_{\rm av}$ indicates the average over the distribution of donors.

In two- or three-dimensional semiconductors, an average over the angles between initial and final wave vector is needed; since the dielectric function depends on the wave vector, there is no way to separate the part due to the Coulomb interaction and the part due to the screening in the cross section. In 1D SC all this is much simpler, and f is directly proportional to the Coulomb cross section so that it is easy to distinguish between what is due to Coulomb interaction (f) and what is due to screening [$\epsilon(2k_F)$].

More explicitly

$$\left\langle\zeta_0(\rho)e^{i\bar{k}z}\left|\frac{1}{|\,\mathbf{r}-\mathbf{r}_d\,|}\right|\zeta_0(\rho)e^{i(\bar{k}+\bar{q})z}\right\rangle$$
$$=2\int d^2\rho\,\zeta_0^2(\rho)K_0(q\,|\,\rho-\rho_d\,|)\ , \tag{8}$$

where $K_n(x)$ is the modified Bessel function of the second kind of order n,[8] and

$$f=\frac{Nd}{N}\int\frac{d^2\rho_d}{A_d}\left|\int d^2\rho\,\zeta_0^2(\rho)K_0(2k_F\,|\,\rho-\rho_d\,|)\right|^2\ . \tag{9}$$

A_d is the area of the section (normal to the wire) defining the place where the donors are randomly distributed. In Eq. (9) we have taken into account the possibility that the electron density N can be different from the donor density N_d (for example, if there is some compensation). Equations (5) and (9) summarize what is to be calculated when only ionized donors play a part in the scattering (and when only intraband scattering is taken into account which is assumed up to Sec. V).

B. Approximations

In general, Eqs. (5) and (9) can be solved by a computer only when the envelope function is known. The goal of this paper being to calculate orders of magnitude, it is equally useful to make approximations to obtain envelope functions as to pursue the mobility calculation.

Approximations made by Sakaki (and recalled in the Introduction) lead directly to (i) $|\,\zeta_0(\mathbf{r})\,|^2=\delta(\mathbf{r})$, (ii) $|\,\rho_d\,|=d=$const, and (iii) $\epsilon(2K_F)=\epsilon_{\rm BG}$, where $\epsilon_{\rm BG}$ is the background dielectric constant so that

$$\mu=\mu_c\frac{N}{N_d}[\epsilon_{\rm BG}/K_0(2k_Fd)]^2 \tag{10}$$

which is Eq. (5) of Ref. 3.

Now we define our approximations.

(i) *Wave functions.* General features of the wave functions are given in Fig. 1. The exact shape depends on the dimensions and on the depth of the well defined by the wire and by the surroundings. The wave function is "flatter" when the electron density is larger because electrons repel each other. In the end we take

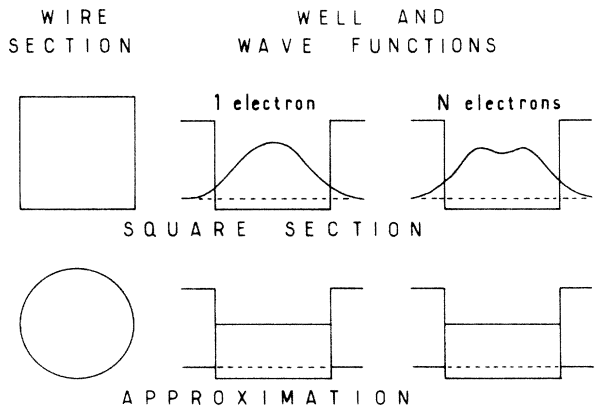

FIG. 1. Qualitative explanation of the approximations used in this paper. We take a circular section for the wire which gives a simpler cylindrical symmetry. The wave function is approximately constant inside and zero outside such a cylinder when the electron density is sufficiently high. We take this approximation for all electron densities.

$$|\zeta_0(\rho)|^2=(\pi R^2)^{-1}Y(R-|\rho|)\,, \tag{11}$$

where $Y(x)$ is the step function. This function, also used in Ref. 5, means that the square wire is approximated by a cylindrical wire of radius R. Furthermore, the approximation is likely better for high rather than low electron density. We shall use it for all densities.

(ii) *Donor distribution.* Donor distributions which are realized in 2D SC and which could be realized in 1D SC are described in Fig. 2. It is not interesting to overly increase the buffer-layer thickness, because beyond a thickness of several hundred angstroms the donor energy levels are below the Fermi level and donors cannot transfer their electrons to the "plane" mentioned in the Introduction. In the following we shall call U1 the case where the donors are (randomly distributed) inside the (cylindrical) wire, $U2$ the case where the donors are inside and outside the wire up to a given distance (which we shall take as twice the wire radius), $M1$ the case where all the donors are outside the wire (we shall take donors at a distance between one and two times the radius), and $M2$ the case where the donors are outside the wire but with a "buffer layer" (we shall take donors between two and three times the wire radius). For convenience we shall refer to cases $U1$ and $U2$ as uniform doping cases and $M1$ and $M2$ as modulated doping cases. Let d_m and d_M be the smallest and the largest radius of the cylinder where donors are randomly distributed: $U1$, $d_m=0$ and $d_M=R$; $U2$, $d_m=0$ and $d_M=2R$; $M1$, $d_m=R$ and $d_M=2R$; $M2$, $d_m=2R$ and $d_M=3R$.

With the approximations (i) and (ii), f can be written as

$$f=\int \frac{d^2\rho_d}{\pi(d_M^2-d_m^2)}\left|\int_{\rho<R}\frac{d^2\rho}{\pi R^2}K_0(2k_F|\rho-\rho_d|)\right|^2 . \tag{12}$$

(iii) *Dielectric function.* At zero temperature, the dielectric function is equal to $\epsilon_0(q,E_F)$, given by[5]

$$\epsilon_0(q,E_F)=\epsilon_{BG}+\frac{4e^2m^*}{\pi\hbar^2}\frac{F(q)}{q}\ln\left|\frac{q+2k_F}{q-2k_F}\right|\,, \tag{13}$$

with

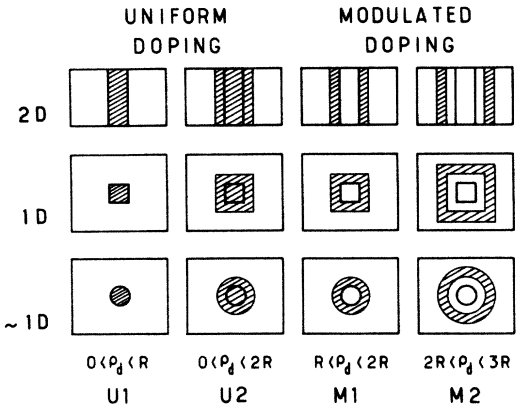

FIG. 2. Kinds of doping that can exist in a two-dimensional semiconductor (2D), in a one-dimensional semiconductor (1D), and the corresponding approximation ($\sim$1D).

$$F(q)=\int d^2\rho\int d^2\rho'\,\zeta_0^2(\rho)\zeta_0^2(\rho')K_0(q|\rho-\rho'|)\,. \tag{14}$$

In the approximation given by Eq. (11),

$$F(q)=\int_{\rho<R}d^2\rho\int_{\rho'<R}d^2\rho'K_0(q|\rho-\rho'|)$$
$$=\frac{2}{(qR)^2}[1-2K_1(qR)I_1(qR)]\,, \tag{15}$$

where $I_n(x)$ is the modified Bessel function of the first kind of order n. In any case, whatever the approximation on the envelope function, the dielectric function diverges for $q=2k_F$ at zero temperature so that we must look for another approximation. The simplest one is to consider the dielectric function at nonzero temperature:[9] $\epsilon_0(q,E_F)$ being the zero temperature dielectric function given by Eq. (13), the dielectric function $\epsilon_T(q,E_F)$ at finite temperature T [which will be written $\epsilon_T(q)$ for convenience] is[9]

$$\epsilon_T(q)=\int_0^\infty dE\,\epsilon_0(q,E)\left[4k_BT\cosh^2\left[\frac{E-E_F}{2k_BT}\right]\right]^{-1} . \tag{16}$$

For $q=2k_F$, $\epsilon_T(q)$ is given by

$$\epsilon_T(2k_F)=\epsilon_{BG}+\frac{4e^2m^*}{\pi\hbar^2}\frac{F(2k_F)}{2k_F}S(E_F/k_BT)\,, \tag{17}$$

with

$$S(x)=\tfrac{1}{2}\int_0^\infty dt\ln\left|\frac{\sqrt{t}+\sqrt{x/2}}{\sqrt{t}-\sqrt{x/2}}\right|\left[\cosh^2\left[t-\frac{x}{2}\right]\right]^{-1}\,, \tag{18}$$

which does not diverge any more. Making use of Eqs. (12), (15), (17), and (18), Eq. (5) is written as

$$\mu=\mu_c\frac{|\epsilon_T(2k_F)|^2}{f}\,. \tag{19}$$

Briefly, we take into account the finite temperature only when this leads to drastic modifications of a given quantity. In other words, if a quantity is finite at zero temperature the alteration introduced by the finite temperature is neglected.

III. EXPLICIT CALCULATION

Now we successively use the three approximations defined above

(i) *Calculation of scattering amplitude.* With the approximation defined by Eq. (12) we have to calculate the matrix element $A(\rho_d)$ defined by

$$A(\rho_d)=\tfrac{1}{2}\left\langle 0,-k_F\left|\frac{1}{|\mathbf{r}-\mathbf{r}_d|}\right|0,k_F\right\rangle$$
$$=\int_{\rho<R}\frac{d^2\rho}{\pi R^2}K_0(2k_F|\rho-\rho_d|)\,. \tag{20}$$

We have to distinguish between two situations: $\rho_d<R$ and $\rho_d>R$. We write the corresponding results $A(\rho_d<R)$ and $A(\rho_d>R)$, which are, respectively, equal to[10,11]

$$A(\rho_d<R)=\frac{2}{\alpha^2}[1-\alpha K_1(\alpha)I_1(\alpha)] \tag{21}$$

and

$$A(\rho_d>R)=\frac{2}{\alpha}I_1(\alpha)K_0(\delta)\ , \tag{22}$$

with

$$\alpha=2k_FR,\ \ \delta=2k_Fd\ . \tag{23}$$

(ii) *Calculation of f.* We have to calculate (with $N=N_d$)

$$f=|A(\rho_d)|^2_{\rm av}=\int_{d_m<\rho_d<d_M}\frac{d^2\rho_d}{\pi(d_M^2-d_m^2)}|A(\rho_d)|^2\ . \tag{24}$$

It is useful to distinguish between two cases:

$$d_m=0,\ \ d_M=R,\ \ f=f_U\ ; \tag{25}$$

$$R\le d_m<\rho_d<d_M,\ \ f=f_M\ . \tag{26}$$

After some algebraic manipulations, we get

$$f_U=\left[\frac{2}{\alpha^2}\right]^2\{1-4K_1(\alpha)I_1(\alpha)+\alpha^2K_1^2(\alpha)[I_0^2(\alpha)-I_1^2(\alpha)]\}\ , \tag{27}$$

$$f_M=\left[\frac{2I_1(\alpha)}{\alpha}\right]^2\frac{\delta_M^2[K_0^2(\delta_M)-K_1^2(\delta_M)]-\delta_m^2[K_0^2(\delta_m)-K_1^2(\delta_m)]}{\delta_M^2-\delta_m^2}\ , \tag{28}$$

with

$$\delta_M=2k_Fd_M,\ \ \delta_m=2k_Fd_m\ , \tag{29}$$

(We note that the limit of f_M is equal to $[2I_1(\alpha)/\alpha]^2[K_0(2k_Fd)]^2$ when d_M tends toward d_m, which allows comparison with Eq. (10) if necessary.) f_M is useful for cases $M1$ and $M2$ defined in approximation (ii) of Sec. II B, f_U is useful for case $U1$; both f_M and f_U are needed for case $U2$.

(iii) *Dielectric function.* The numerical integration of Eq. (18) is straightforward. However, two approximations are very useful. In the first one, $E_F\gg k_BT$ or $x\gg1$:

$$S(x)=\ln\left[\frac{8e^\gamma}{\pi}\right]x=\ln(4.535)x\ , \tag{30}$$

where γ is Euler's constant $\gamma=0.577$. Equation (30) gives back a well-known formula for the dielectric function in metal at low temperature.[12]

In the second approximation $E_F\ll k_BT$ or $x\ll1$:

$$S(x)=1.346\sqrt{x}\ . \tag{31}$$

Indeed it can be shown that Eq. (30) is valid for $x\ge0.638$, and Eq. (31) for $x\le0.638$ [see the Appendix where the accuracy of Eqs. (30) and (31) is discussed].

This last remark shows that with the set of Eqs. (15), (17), (27), (28), (30), and (31), Eq. (19) now provides an analytical result whatever the case of interest. For example, in the modulated case, which is the most interesting since it gives the largest mobility, and for the case which is the likely situation, where $T=4.2$ K, $N=10^6$ cm^{-1}, $E_F=14$ meV, i.e., $E_F\gg k_BT$, where Eqs. (28) and (30) are valid, one has

$$\frac{\mu}{\mu_c}=\frac{\left[\epsilon_{\rm BG}+\dfrac{4e^2m^*R}{\pi\hbar^2}\dfrac{2[1-K_1(\alpha)I_1(\alpha)]}{\alpha^3}\ln\left[\dfrac{8e^\gamma E_F}{k_BT}\right]\right]^2}{\left[\dfrac{2I(\alpha)}{\alpha}\right]^2\dfrac{\delta_M^2[K_0^2(\delta_M)-K_1^2(\delta_M)]-\delta_m^2[K_0^2(\delta_m^2)-K_1^2(\delta_m)]}{\delta_M^2-\delta_m^2}}\ . \tag{32}$$

IV. NUMERICAL RESULTS

We have to calculate $\epsilon_T(2k_F)$ and f. To give some explicit examples we have taken $R=50$ Å and $R=100$ Å with $T=4.2$ K. We have chosen to study the mobility as a function of the electron density N for 10^5 cm$^{-1}<N<10^6$ cm^{-1}, which is the range of interest. The variation of the dielectric function with electron density is given in Fig. 3. As in 2D SC,[1] the higher the electron density the lower the dielectric function for wave vector equal to $2k_F$.

Figure 4 gives the variation of f_U and f_M as a function of electron density. We can note that for a given geometry, i.e., once the donor distribution is known, f (f_U or f_M) depends only on the product k_FR, i.e., on NR: $f=f(NR)$. Now if we write $f=f^{(R)}(N)$, we see that

$$f^{(pR)}(N)=f^{(R)}(pN)\ , \tag{33}$$

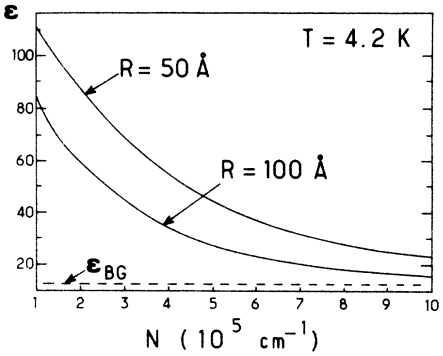

FIG. 3. Dielectric function $\epsilon_T(2k_F)$ at $T=4.2$ K as a function of electron density N for wire radius $R=50$ and 100 Å.

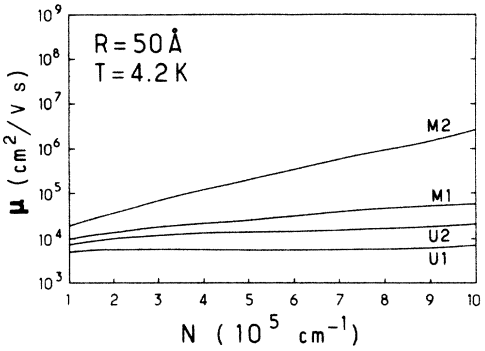

FIG. 5. Mobility in a cylindrical wire of radius $R=50$ Å as a function of electron or donor density. $U1$, $U2$, $M1$, and $M2$ are defined in the text and in Fig. 2.

where p is a positive number. In Fig. 4, $p=2$ which explains the values given on the abscissa. Contrary to the case of Fig. 3 where a linear scale is sufficient to describe the variation of the dielectric function a logarithmic scale is needed for the Coulomb cross section.

Figures 5 and 6 give the mobility for cylindrical wires of radius $R=50$ and 100 Å, respectively. In a geometry with a buffer layer (M2), the smallest distance between electrons and donors is proportional to the radius which explains the large difference between the two mobilities in this case. Due to the relatively smooth variation of the dielectric function, the mobility depends mainly on the Coulomb cross section given by f: the variation of f is more abrupt than the one of $\epsilon_T(2k_F)$ so that the mobility is an increasing function on the electron density [strictly speaking, the variation of f_U for $N\approx 5\times 10^5$ cm^{-1} is as smooth as the one of $\epsilon_T(2k_F)$ so that in this range the variation of the mobility becomes very flat]. It is worth noting that in the $M2$ case for conditions *a priori* comparable[13] to those of 2D SC, we obtain a huge mobility for N of the order of 10^6 cm^{-1}. This is the most striking difference between the 1D SC and the 2D SC.

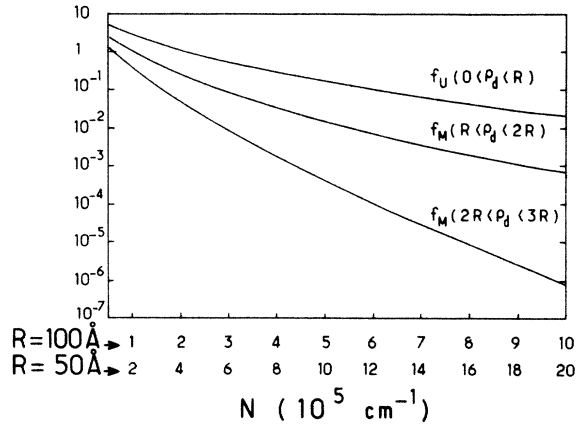

FIG. 4. f is directly proportional to the Coulomb cross-section. ρ_d gives the donor position with respect to the center of the cylindrical wire. The greater the average distance between electrons and donors, the weaker the Coulomb cross section becomes.

V. DISCUSSION

Up to now we have restricted our interest to the range 10^5 cm$^1 < N < 10^6$ cm^{-1}. We can wonder what the limits are if the electron density is very small or very large. A straightforward calculation (see Appendix) shows that in these two limiting cases the mobility tends to infinity as in 2D SC,[13] with the same interpretation: when $N=N_d$ is decreasing, the mobility is increasing for very low concentration because electrons see a decreasing number of scattering centers. For very large concentration the efficiency of scattering centers becomes very weak due to the increase of the Fermi level and more precisely to the increase of electron velocity.

In the above considerations we have not taken into account the interband scatterings which play a part if the Fermi level is higher than the bottom E_1 of the second conduction subband ($n=1$). In 2D SC the ratio between the mobilities corresponding to a Fermi level just below or above E_1 depends both on the ratio of the densities of states and on the ratio of intraband and interband scattering probabilities which are respectively noted B_{00} and D_{01} in the notation of Ref. 13, which we use here for convenience. d being the dimension of the semiconductor of interest ($d=1,2,3$), a tedious but straightforward calculation shows that for a Fermi level just below E_1:

$$\mu = C_d (B_{00})^{-1} \tag{34}$$

and for a Fermi level just above E_1:

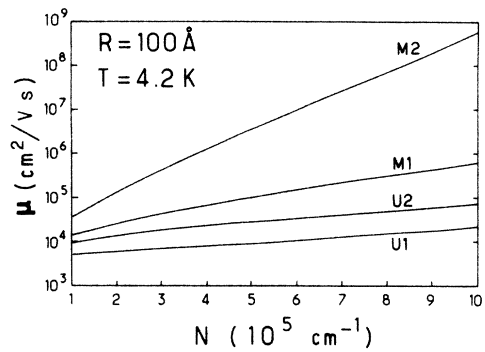

FIG. 6. Same as in Fig. 5 but for $R=100$ Å.

$$\mu = C_d \left[B_{00} + \left[\frac{E_F - E_1}{E_F - E_0} \right]^{(d-2)/2} D_{01} \right]^{-1}, \tag{35}$$

where to simplify the notation we have written B_{00} and D_{01} instead of $B_{00}(E_F)$ and $D_{01}(E_F)$. C_d depends on the dimension d of the semiconductor of interest. The results are summarized Table I. This shows that the mobility collapses when $E_F = E_1$ in 1D SC, contrary to the 2D SC case. This results merely from the density of states behavior. Of course any source of broadening will prevent the mobility from becoming equal to zero. For instance in our calculation we have assumed that the electron distribution can be described by Fermi-Dirac statistics reduced to a step function. This is certainly not the case for the electrons in the second subband $n = 1$, and this is sufficient to give a broadening. Indeed the main result is that the mobility in 1D SC will be be much more sensitive to the position of the Fermi level with respect to E_1 than in 2D SC.

We can also wonder about the influence of some imperfections on the mobility. Many kinds of imperfection can be considered but to give a likely example here we shall look at an "imperfect" case $M2$, i.e., a case where a weak percentage of donors are randomly distributed inside the wire as in the $U1$ case. The calculation is similar to the $U2$ case. Let us consider the case where $N = N_d = 10^6$ cm^{-1} with pN_d donors inside the wire: If $p = 0\%$, we are in the $M2$ case and $\mu = 5 \times 10^8$ cm^2/V s, if $p = 100\%$ we are in the $U1$ case and $\mu = 2 \times 10^3$ cm^2/V s. If $p = 10\%$, the mobility is $\mu = 2 \times 10^4$ cm^2/V s and if $p = 1\%$, μ is as low as 2×10^5 cm^2/V s which shows the significant influence of imperfections. However, the above considerations must be taken with caution. Up to now all the donors have been assumed ionized. This is true for donors outside the wire. If there are only a few donors inside the wire (and no donors outside) these donors will keep their electrons and will be neutral. In the case of interest here (say $p = 1\%$), it is not obvious whether the few donors inside the wire will be ionized or not and in any case the possible ionization will be due to screening of electrons which originate from donors outside the wire which makes the cross section weaker. Under such conditions, donors inside the wire will have little influence on the mobility. It is beyond the scope of this paper to investigate the influence of such donors, but we must keep in mind that mobility limitation due to imperfections as described here may not be as drastic as one might think at first sight.

TABLE I. Ratio of the mobilities for a Fermi level just below ($\mu_<$) and just above ($\mu_>$) the bottom of the second conduction subband in a semiconductor of dimension d. (See discussion in the text). C_d the constant of proportionality ($\mu = C_d B_{00}^{-1}$) is taken equal to unity for clarity. E_0 and E_1 are, respectively, the energies of the bottom of the first ($n = 0$) and of the second ($n = 1$) conduction subband. E_F is the Fermi level. $B_{00} = B_{00}(E_F)$ gives the intraband ($0 \to 0$) scattering rate while $D_{01} = D_{01}(E_F)$ gives the interband ($0 \to 1$) scattering rate, both of them for $E_F = E_1$.

d	$\mu_{E_F \le E_1} = \mu_<$	$\mu_{E_F \ge E_1} = \mu_>$	$\mu_>/\mu_<$
3	B_{00}^{-1}	$\left[B_{00} + \left[\frac{E_F - E_1}{E_F - E_0} \right]^{1/2} D_{01} \right]^{-1}$	$\left[1 + 0 \frac{D_{01}}{B_{00}} \right]^{-1}$
2	B_{00}^{-1}	$\left[B_{00} + D_{01} \right]^{-1}$	$\left[1 + 1 \frac{D_{01}}{B_{00}} \right]^{-1}$
1	B_{00}^{-1}	$\left[B_{00} + \left[\frac{E_F - E_1}{E_F - E_0} \right]^{-1/2} D_{01} \right]^{-1}$	$\left[1 + \infty \frac{D_{01}}{B_{00}} \right]^{-1}$

TABLE II. Order of magnitude of the mobility of the mean free path l and of the quantity $k_F l$ for a linear electronic density $N \approx 10^6$ cm^{-1}, which corresponds to a Fermi wave $k_F \approx 10^6$ cm^{-1} for a wire of radius $R = 100$ Å. If the mobility is of the order of the theoretical maximum mobility (see Fig. 6) or even 10^3 weaker, the localization is negligible ($k_F l \gg 1$). This remains true as long as the mobility is larger than 10^3 cm^2/V s.

μ (cm^2/V s)	l	$k_F l$
10^9	1 cm	10^6
10^6	10 μm	10^3
10^3	100 Å	1

Although it is not the goal of our paper, another problem we have to look at is the question of localization. The simplest criterion for knowing whether localization plays a part or not is found by calculating $k_F l$ where l is the mean free path and comparing it to unity.[14] If $k_F l \gg 1$ then localization is negligible and within our model our results are correct, otherwise our calculation has to be modified. Results are given in Table II for $N \approx k_F \approx 10^6$ cm^{-1}. It is clear in a "perfect" 1D SC ($\mu_{\text{expt}} \approx \mu_{\text{theor}} \approx 10^9$ cm^2/V s, $k_F l \gg 1$) that there will be a huge mobility and in the contrary case ($\mu_{\text{expt}} \ll \mu_{\text{theor}}$, $k_F l \approx 1$) there will be a strong localization.

Another point of interest is to evaluate how good the approximation is which we have used for the dielectric function. This has no obvious answer. Fortunately a sophisticated calculation was recently carried out by Das Sarma and Wu-yan Lai[7] who took into account the influence of both finite temperature and collisional broadening on the dielectric function. It is easy to convince oneself that the quantity of interest is $\pi(q,0)/\pi_0$, where $\pi(q,0)$ is the static polarizability in the presence of collisional broadening Γ and at finite temperature T and π_0 the same quantity for $q = 0$ with $T = \Gamma = 0$ in the notation of Ref. 7 or

$$\frac{\pi(q,0)}{\pi_0} = \frac{\sqrt{E_F}}{\left[\frac{\hbar^2 q^2}{2m^*} \right]^{1/2}} \int_0^\infty dE \ln \left| \frac{\left[\frac{\hbar^2 q^2}{2m^*} \right]^{1/2} + 2\sqrt{E}}{\left[\frac{\hbar^2 q^2}{2m^*} \right]^{1/2} - 2\sqrt{E}} \right| \times \left[4 k_B T \cosh^2 \left[\frac{E - E_F}{2 k_B T} \right] \right]^{-1} \tag{36}$$

in our approximation where only finite-temperature effects are taken into account. The net result is given in Fig. 7. It is foreseeable that the static polarizability of Ref. 7 is smaller than that used in the present paper; however, the difference is too small to change the order of magnitude of the mobility. This is particularly true for $N \approx 10^6$ cm^{-1} where the dielectric function is near the background dielectric constant.

Throughout this paper we have assume that the temperature is equal to zero except for the dielectric constant for which we have assumed $T=4.2$ K. It is clear that for $k_B T \ll E_F$ the mobility will not change if we take into account the slight broadening around the Fermi level due to the finite temperature. This will be the case for $N \approx 10^6$ cm^{-1}, $E_F \approx 14$ meV, and $k_B T = 4.2$ K ≈ 0.3 meV. However, for $N \approx 10^5$ cm^{-1}, $E_F \approx 0.14$ meV and in this case our approximation becomes poor and can only give an order of magnitude. In other words in Figs. 5 and 6 the results are much better for high ($N=10^6$ cm^{-1}) than for low ($N=10^5$ cm^{-1}) density. (Anyhow in this last case the mobility is not very high and, due to localization, the calculation becomes questionable as discussed above.) Nevertheless we note that $N=10^5$ cm^{-1} corresponds to a very low electron density and the cases of interest will probably be around $N=10^6$ cm^{-1} (as in 2D SC, where the range of interest is around $N \approx 10^{12}$ cm^{-2}). This means that in pratical cases our calculation need not be altered.

Lastly, it is necessary to know the range of validity of our one-band approximation: this range depends on the energy difference E_{01} between the first ($n=0$) and the second ($n=1$) conduction subband. For an infinite well, $E_{01}=42$ meV, if the wire section is 200×200 Å^2 (as in Ref. 2) and if there is only one electron in the well. ($E_{01}=50$ meV for a cylindrical wire of radius $R=100$ Å). For $N=10^6$ cm^{-1}, which is the maximum density considered in this paper. E_F is equal to only 14 meV, well below 42 meV: the one-band approximation seems fully justified. However, the reality is more complex. The Fermi level E_F and the linear density N are related by $E_F = 14(N/10^6)^2$, where E_F is in meV and N is in cm^{-1}. The critical density N^* for which $E_F=42$ meV is $N^* = 1.7 \times 10^6$ cm^{-1} which is larger but not very much larger than 10^6 cm^{-1}. Furthermore, due to both the finite depth of the well and to the influence of the electronic density the energy levels will be closer than in the case considered just above [as in 2D SC (Ref. 1)]. In these conditions the critical density will be lower than 1.7×10^6 cm^{-1} and could be of the order of $N \approx 10^6$ cm^{-1}. Only detailed calculations taking into account the exact shape of the well can give accurate results. For a wire of square section 100×100 Å^2 (or a cylindrical wire of radius $R=50$ Å), all the energies must be multiplied by four and the critical density will be always much larger than 10^6 cm^{-1}.

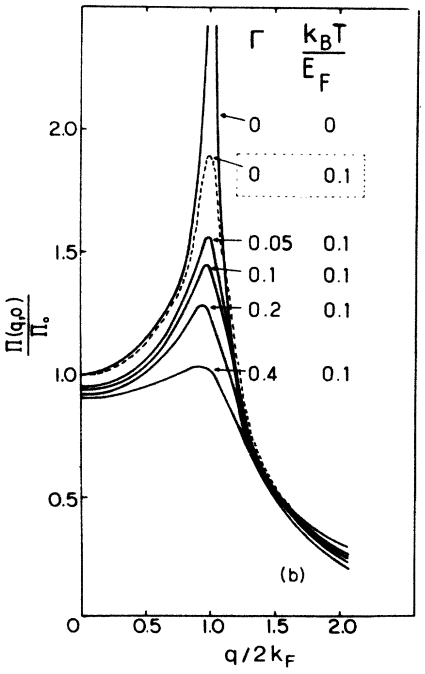

FIG. 7. Solid lines, which give the static polarizability function of the one-dimensional electron gas, are extracted from Fig. 1(b) of Ref. 7. Γ is the broadening, in units of E_F. The dotted line gives the same quantity in our approximation where the broadening is not taken into account.

Before concluding, we wish to discuss briefly the paper by Lee and Vassel:[15] In this paper the authors treat impurity scattering as well as phonon scattering from low to room temperature. To make a useful comparison with our paper we will concentrate on impurity scattering at low temperature. The main difference are the following: (i) the square of the wave function is approximated by a δ function [more precisely, $\delta(|\rho|-R/2)$; see Eq. (25) of Ref. 15] and (ii) they use a Thomas-Fermi approximation for the dielectric function. As pointed out by the authors themselves, "this approximation is incorrect particularly at very low temperature." This last point is the main difference between their approach and ours where the Lindhard function was used.[5]

VI. CONCLUDING REMARKS

We have shown that reasonable approximations lead to an analytical formulation of the mobility in 1D SC. The main approximation is the use of cylindrical symmetry which cannot alter the order of magnitude of the results for samples such described in Ref. 2. It is likely that in the future the main problem will not be to improve our model but to take into account the almost unavoidable imperfections of real samples to obtain a more realistic mobility. However, there is a problem which has not been tackled above and cannot be set aside, the limitation due to scattering by phonons. If the mobility is not overly high, say 10^7 cm^2/V s, phonons play no part and at low temperature scattering is only due to ionized impurities as in 2D SC.[16] But for very high mobility it is clear that it is not possible to neglect scattering by phonons. This problem will be investigated in a forthcoming publication.

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APPENDIX: SOME REMARKS ABOUT THE DIELECTRIC FUNCTION

First we wish to justify the approximation given in Eqs. (30) and (31) for $S(E_F/k_BT)$ defined in Eqs. (17) and (18), which gives the nonzero temperature dielectric function. For $E_F \gg k_BT$ the approximation defined by Eq. (30) is obtained by a simple calculation. For $E_F \ll k_BT$, we obtain

$$S(x)=\left[\frac{x}{2}\right]^{1/2} \int_0^\infty du[\sqrt{u}\cosh^2 u]^{-1}, \quad x \ll 1 . \quad \text{(A1)}$$

Numerically, this last equation gives back Eq. (31).

The problem is now to look at what happens if we are not in one of the two above limits. For this we plot the two curves defined by Eqs. (30) and (31). These two curves intersect for $E_F/k_BT=0.656$ as shown in Fig. 8, where $S(E_F/k_BT)$, numerically calculated, is also plotted. Even at the intersection point the ratio between the true value of $S(E_F/k_BT)$ and the approximations of Eq. (30) or (31) is only equal to 1.058. Indeed the approximations are the worst not at the intersection point but at $x=1.5$ for which the same ratio is equal to 0.941. This shows that these approximations are well justified.

We are now able to give the limits of the mobility when the electronic density is very small or very large (for $N=N_d$ and neglecting interband scattering). For $E_F \gg k_BT$ we obtain

$$\mu \sim \begin{cases} \epsilon_{BG}^2\alpha^4, & \text{if } f=f_U \\ \epsilon_{BG}^2(\delta_M^2-\delta_m^2)\alpha^3 e^{2(\delta_m-\alpha)}, & \text{if } f=f_M \end{cases}$$

and for $E_F \ll k_BT$:

$$\mu \sim \begin{cases} \alpha^{-6}, & \text{if } f=f_U \\ \alpha^{-3}(\ln\alpha)^2, & \text{if } f=f_M. \end{cases}$$

We thus find the results for the two limiting cases given without demonstration in Sec. V.

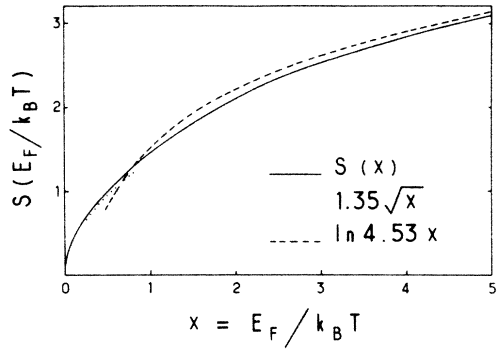

FIG. 8. Solid line give numerical values of $S(x)$, $x=E_F/k_BT$. [See Eq (18)]. The dotted line and dashed line give, respectively, the values of the functions defined by Eqs. (30) and (31), which intersect for $E_F/k_BT=0.656$.

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