PHYSICAL REVIEW B VOLUME 34, NUMBER 1 1 JULY 1986

New method for studying disordered quantum systems

H. G. Schuster* and V. R. Vieira
Centro de Fisica da Matéria Condensada, Instituto Nacional de Investigaçao Cientifica,
Avenida Prof. Gama Pinto 2, 1699 Lisboa Codex, Portugal

(Received 20 November 1985)

The effect of an electron heat bath on the motion of an electron system is studied using a quantum Langevin equation and deriving the influence functional of the Feynman-Vernon approach. The generating functional for the time-dependent, finite-temperature correlation functions of a disordered electron system with interactions is written down, and some of its symmetry properties are analyzed. Since the disorder only affects the time evolution of the system our formalism avoids the replica trick and provides a natural physical framework for studying a large variety of open problems in the field of disordered systems. As an example, we give an explicit expression for the averaged generating functional for a disordered electron system in the presence of a Coulomb interaction in terms of two complex matrix fields.

I. INTRODUCTION

One of the main difficulties arising in the theory of disordered systems is the so-called "denominator problem" which occurs when one tries to calculate the average value of a measurable quantity described by an operator A because, in addition to thermal averages, one has to take the average over the disorder. One then has

$$\langle\langle A\rangle\rangle_D=\left\langle\frac{\mathrm{Tr}(e^{-\beta H}A)}{\mathrm{Tr}e^{-\beta H}}\right\rangle_D , \tag{1.1}$$

where the thermal average is denoted by $\langle\cdots\rangle$ and the disorder average by $\langle\cdots\rangle_D$, and β is the inverse temperature.

Since the Hamiltonian H which contains the disorder appears also in the denominator, it becomes difficult to perform the disorder average. Up to now, essentially three methods have been found to circumvent this problem: The first two methods (the so-called replica trick[1] and the superfield method[2]) are formal manipulations to get rid of the denominator, whereas the third method[3] is more physical. Instead of the static problem, one considers in Ref. 3 a dynamical nonequilibrium system which evolves for large times to the proper equilibrium limit. The random average is then performed over the corresponding generating functional. The first two methods have been applied successfully, e.g., to describe the localization of noninteracting electrons,[4] whereas the last method was crucial in formulating the spin-glass problem for classical spins.[5]

It is our purpose to extend to quantum systems the last method, which also offers the possibility of studying dynamical properties of the system and which, up to now, has been applied only to classical systems. However, we do not start from the Langevin equations (as is the case of classical systems[3,5]) but instead use the Feynman-Vernon approach,[6] which studies the dynamics of the quantum system coupled to a heat bath. In this way we have the advantage both of studying physically interesting dynamical quantities and of being able to avoid the denominator problem at the same time.

The basic idea is as follows. We couple our quantum system described by a Hamiltonian H_s to a quantum heat bath described by a Hamiltonian H_b. The combined Hamiltonian is

$$H=H_s+H_b+H_{s\text{-}b} , \tag{1.2}$$

where $H_{s\text{-}b}$ describes the interaction of our system with the heat bath. Next, we consider the time evolution of the density matrix ρ of the total system from a time t_i to a time t_f:

$$\rho(t_f)=e^{-iH(t_f-t_i)}\rho(t_i)e^{iH(t_f-t_i)} . \tag{1.3}$$

If we assume that the system (s) and the bath (b) are initially uncorrelated at $t=t_i$ and that the heat bath is in thermal equilibrium, we can write

$$\rho(t_i)=\rho_s(t_i)\rho_b(t_i) , \tag{1.4}$$

with

$$\rho_b(t_i)=\frac{e^{-\beta H_b}}{\mathrm{Tr}_b e^{-\beta H_b}} . \tag{1.5}$$

This ensures the normalization of the trace:

$$\mathrm{Tr}\rho(t_f)=\mathrm{Tr}_s\rho_s(t_i)\,\mathrm{Tr}_b\rho_b(t_i)=1 . \tag{1.6}$$

Taking the trace over the heat-bath variables, we obtain the reduced density matrix $\rho_s(t_f)$ of our system. For $t_f\to\infty$, it will approach (in the zero-coupling limit) the proper equilibrium density matrix of our system:

$$\lim_{t_f\to\infty}\mathrm{Tr}_b\rho(t_f)=\lim_{t_f\to\infty}\mathrm{Tr}_b[e^{-iH(t_f-t_i)}\rho_s(t_i)\rho_b(t_i)\times e^{iH(t_f-t_i)}]=\rho_s^{\mathrm{eq}} . \tag{1.7}$$

After performing the random average, the equilibrium density matrix ρ_s^{eq} of our system becomes

$$\langle \rho_s^{\mathrm{eq}} \rangle_D = \lim_{t_f \to \infty} \langle \mathrm{Tr}_b[e^{-iH(t_f-t_i)}\rho_s(t_i)\rho_b(t_i)e^{iH(t_f-t_i)}] \rangle_D \tag{1.8}$$

Since $\rho_s(t_i)$ can be chosen to be independent of the randomness, one only has to average over the random interactions occurring in H, which appears in the exponentials, i.e., one has circumvented the denominator problem by studying the time evolution of the system in a heat bath.

The outline of this paper is as follows. In Sec. II we study the equations of motion for one free electron in a Fermi sea coupled to a heat bath which is also made up of electrons. The density of states of the heat bath and the coupling between the system and the heat bath are chosen appropriately to ensure that our system approaches thermal equilibrium in the long-time limit. In Sec. III we study the time development of the density matrix of a many-electron system coupled to a heat bath, using path integrals. We generalize the Feynman-Vernon theory[6,7] which has been established for oscillators using real variables, to fermions using Grassmann variables.[8] Eliminating the heat-bath variables, we calculate explicitly the influence functional for the many-electron system. This allows us to present a closed expression (in the form of a path integral) for the generating functional for an arbitrary many-electron system with interactions in a heat bath. This formalism can be immediately applied to bosons having a finite chemical potential simply by replacing Grassmann variables by complex numbers and Fermi functions by Bose functions. As an application, we calculate in Sec. IV the generating functional of a random electron system with interactions and study its symmetry properties. Performing the random average we obtain, after a Hubbard-Stratonovich[9] transformation, a closed expression for the generating functional in terms of complex matrix fields. Finally, in Sec. V we draw our conclusions.

II. ONE ELECTRON IN A HEAT BATH

In this section we study two coupled fermion systems described by the model Hamiltonian:

$$H_M = Ec^\dagger c + \sum_k (V_k^* c a_k + V_k a_k^\dagger c) + \sum_k \varepsilon_k a_k^\dagger a_k \ . \tag{2.1}$$

The creation and annihilation operators $c^\dagger$ and c describe an electron of energy E in a Fermi sea. The second fermion system described by $a_k^\dagger, a_k$ acts as a heat bath. Note that the energies E and ε_k are measured relative to the common chemical potential μ of both systems which are coupled via the coupling constants V_k. We will now study the equations of motion for both systems and choose the couplings V_k and the density of states for the ε_k's in such a way that the system described by $c^\dagger, c$ tends for $t \to \infty$ to thermal equilibrium. The coupling between the system and the heat bath in Eq. (2.1) is somewhat different from the one used by Caldeira and Leggett,[7] who couple a quantum particle to a set of quantum harmonic oscillators simulating the heat bath. In fact, our approach is similar to the method used for bosons by Haken.[10] Using Eq. (2.1), the coupled equations of motion for c and a_k become

$$i\dot{c} = [c,H] = Ec + \sum_k V_k^* a_k \ , \tag{2.2a}$$

$$i\dot{a}_k = [a_k,H] = \varepsilon_k a_k + V_k c \ . \tag{2.2b}$$

Equation (2.2b) can be solved formally,

$$a_k(t) = e^{-i\varepsilon_k t} a_k(0) - i \int_0^t dt' e^{-i\varepsilon_k(t-t')} V_k c(t') \ , \tag{2.3}$$

and yields for Eq. (2.2a)

$$\dot{c} = -iEc - \sum_k |V_k|^2 \int_0^t dt' e^{-i\varepsilon_k(t-t')} c(t') - i \sum_k V_k^* e^{-i\varepsilon_k t} a_k(0) \ . \tag{2.4}$$

This equation becomes a Langevin equation for the c operator if we choose the couplings V_k and the density of states $\rho(\varepsilon_k)$ for the ε_k's such that

$$\rho(\varepsilon_k) |V_k|^2 = \frac{\Gamma}{\pi} = \mathrm{const} \ , \tag{2.5}$$

which implies

$$\sum_k |V_k|^2 e^{-i\varepsilon_k \tau} = \frac{\Gamma}{\pi} \int_{-\mu}^{\infty} d\varepsilon\, e^{-i\varepsilon\tau} \simeq 2\Gamma\delta(\tau) \ , \tag{2.6}$$

where the last equation (which makes the integral approximately equal to twice a δ function in time) holds only for times τ that are much larger than typical time scales $1/\mu$ of the reservoir. Since we are interested only in the low-frequency behavior of the system, $1/\tau \ll \mu$, we can safely replace the integral by $2\Gamma\delta(\tau)$. Using Eq. (2.6), Eq. (2.4) becomes

$$\dot{c} = -(iE + \Gamma)c + F(t) \ , \tag{2.7}$$

i.e., we obtain indeed a Langevin equation with a damping constant Γ and a random force $F(t)$:

$$F(t) = -i \sum_k V_k^* e^{-i\varepsilon_k t} a_k(0) \ . \tag{2.8}$$

If we assume that our heat bath is initially in thermal equilibrium, i.e.,

$$\langle a_k^\dagger a_k \rangle \equiv \frac{\mathrm{Tr} e^{-\beta H_b} a_k^\dagger a_k}{\mathrm{Tr} e^{-\beta H_b}} = f(\varepsilon_k) \equiv \frac{1}{e^{\beta\varepsilon_k} + 1} \ , \tag{2.9}$$

with

$$H_b = \sum_k \varepsilon_k a_k^\dagger a_k \ , \tag{2.10}$$

the random forces $F(t)$ are correlated as follows: the anticommutator for different times is

$$\{F(t), F^\dagger(t')\} = 2\Gamma\delta(t - t') \tag{2.11}$$

and the expectation value of the antisymmetrized product is

$$\langle \tfrac{1}{2}[F(t)F^\dagger(t') - F^\dagger(t')F(t)] \rangle = \Gamma \int \frac{d\varepsilon}{2\pi} e^{-i\varepsilon(t-t')} \tanh\left[\frac{\beta\varepsilon}{2}\right] . \tag{2.12}$$

These are the fermion counterparts of the well-known noise-noise correlation functions for boson Brownian motion.[7,11] The choice of Eq. (2.5) ensures that the anticommutation relation between the c's is preserved, i.e.,

$$\{c(t),c^\dagger(t)\}=1 , \tag{2.13}$$

and that in the long-time limit $t\rightarrow\infty$ the thermal occupation number of the c system tends, for weak coupling, to the proper equilibrium limit

$$\begin{aligned}\lim_{\Gamma\rightarrow 0}\lim_{t\rightarrow\infty}\langle c^\dagger(t)c(t)\rangle &= \lim_{\Gamma\rightarrow 0}\frac{1}{\pi}\int d\varepsilon\frac{\Gamma}{\Gamma^2+(\varepsilon-E)^2}f(\varepsilon)\\ &=f(E) .\end{aligned} \tag{2.14}$$

Let us add two clarifying comments. First, the fermionic bath yields a grand canonical ensemble which conserves as usual the average particle number of the system. Equation (2.14) shows that our formalism yields the correct and well-known Fermi distribution for the electrons of the system. It is therefore physically sensible to use a fermionic heat bath. An explicit example for an electronic system coupled to a fermionic heat bath would be, e.g., d electrons in a transition metal which are coupled to s electrons which provide a heat bath. But we would like to stress that one can always use a fermionic heat bath for equilibrating an electron system, in the same way as one could simulate a heat bath using harmonic oscillators. Second, Eq. (2.14) shows that the limit $t\rightarrow\infty$ has to be taken before $\Gamma\rightarrow 0$. This means physically that one first has to wait until the system has reached thermal equilibrium and one can then make the coupling arbitrarily small. If one first switches off the coupling ($\Gamma\rightarrow 0$), the system could never reach thermal equilibrium even if one would wait an arbitrarily long time. Thus the sequence of limits is dictated by physics and their performance does not generate problems.

III. PATH INTEGRAL APPROACH FOR THE GENERATING FUNCTIONAL

We will now use the path integral approach to present an explicit expression for the generating functional for the correlation functions of a many-electron system in a heat bath. This method has been introduced originally by Feynman and Vernon[6] and was applied successfully by Caldeira and Leggett[7] to describe quantum Brownian motion. The main difference with our approach is that we are using path integrals with Grassmann variables,[8] instead of real or complex variables, because they are more appropriate for electron systems. The heat bath is made up of electrons and the coupling between it and our system is also different, as outlined in the preceding section.

Our starting point is the evolution equation [(1.3)–(1.7)] for ρ_s, the reduced density matrix of our system

$$\rho_s(t_f)=\mathrm{Tr}_b e^{-iH(t_f-t_i)}\rho_s(t_i)\rho_b(t_i)e^{iH(t_f-t_i)} . \tag{3.1}$$

The total Hamiltonian of the system (s) interacting with the heat bath (b) is

$$H=H_s+H_b+H_{s\text{-}b} , \tag{3.2}$$

where

$$H_s=H_s\{c_l^\dagger,c_l\} \tag{3.3}$$

describes a many-electron system (with arbitrary interactions) with creation and annihilation operators $c_l^\dagger,c_l$. The heat bath has, as in Sec. II, the Hamiltonian

$$H_b=\sum_k \varepsilon_k a_k^\dagger a_k , \tag{3.4}$$

and the coupling Hamiltonian is, in analogy to Eq. (2.1),

$$H_{s\text{-}b}=\sum_{l,k}(V_{l,k}^* c_l^\dagger a_k+\mathrm{H.c.}) , \tag{3.5}$$

assuming

$$\sum_k \rho(\varepsilon_k)V_{l,k}V_{l',k}^*\ldots=\delta_{l,l'}\frac{\Gamma}{\Pi}\int d\varepsilon_k\cdots , \tag{3.6}$$

in analogy to Eq. (2.5). We are again interested in time scales of our system much larger than those corresponding to the characteristic frequencies of the heat bath, as was discussed following Eq. (2.6).

Next we calculate $\rho_s(t_f)$ in the so-called holomorphic representation; i.e., we consider the matrix elements of the density operator $\rho_s(t_f)$ in a basis of unnormalized states

$$||\alpha\rangle\equiv e^{a^\dagger\alpha}|0\rangle , \tag{3.7}$$

where the α's are Grassmann variables (see Appendix A). These states are the fermion equivalent of the (unnormalized) coherent states used for bosons. For the convenience of the reader we list some properties.

(i) The unit element can be written as

$$\int d^2\alpha||\alpha\rangle e^{-\alpha^*\alpha}\langle\alpha||=1 . \tag{3.8}$$

(ii) The states are not orthogonal and their overlap is given by

$$(\langle\beta||)(||\alpha\rangle)=e^{\beta^*\alpha} . \tag{3.9}$$

(iii) Matrix elements can be easily calculated using the normal ordered form A_N of the operator A (expressed with creation and annihilation operators):

$$A(\beta^*,\alpha)=\langle\beta||A(a^\dagger,a)||\alpha\rangle=e^{\beta^*\alpha}A_N(\beta^*,\alpha) . \tag{3.10}$$

(iv) The trace of an operator is given by

$$\mathrm{Tr}A=-\int d^2\alpha e^{\alpha^*\alpha}A(\alpha^*,\alpha) \tag{3.11}$$

if $A(\alpha^*,\alpha)$ is an even function of the Grassmann variables.

By using these formulas in Eq. (3.1) the matrix element $\langle\gamma_f'||\rho_s(t_f)||\gamma_f\rangle$ can be written as

$$\begin{aligned}\langle \gamma'_f \| \rho_s(t_f) \| \gamma_f \rangle &= \mathrm{Tr}_b \langle \gamma'_f \| e^{-iH(t_f-t_i)} \rho_s(t_i) \rho_b(t_i) e^{iH(t_f-t_i)} \| \gamma_f \rangle \\ &= -\int \exp(\alpha_f^* \cdot \alpha_f - \alpha_i^* \cdot \alpha_i - \alpha_i'^* \cdot \alpha_i' - \gamma_i^* \cdot \gamma_i^* - \gamma_i'^* \cdot \gamma_i') d^2\alpha_f d^2\alpha_i d^2\alpha_i' d^2\gamma_i d^2\gamma_i' \\ &\quad \times \langle \gamma'_f, \alpha_f \| e^{-iH(t_f-t_i)} \| \alpha_i, \gamma_i \rangle \langle \gamma_i \| \rho_s(t_i) \| \gamma_i' \rangle \langle \alpha_i \| \rho_b(t_i) \| \alpha_i' \rangle \langle \gamma_i', \alpha_i' \| e^{iH(t_f-t_i)} \| \alpha_f, \gamma_f \rangle ,\end{aligned} \tag{3.12}$$

where

$$\alpha = (\alpha_1, \ldots, \alpha_k, \ldots), \quad \int d^2\alpha = \prod_k \int d^2\alpha_k$$

refers to the bath and

$$\gamma = (\gamma_1, \ldots, \gamma_l, \ldots), \quad \int d^2\gamma = \prod_l \int d^2\gamma_l$$

refers to the system. Representing the time-evolution operators by bath integrals, i.e.,

$$\langle \gamma'_f, \alpha_f \| e^{-iH(t_f-t_i)} \| \alpha_i, \gamma_i \rangle = \int_{\gamma_i}^{\gamma_f'^*} D^2\gamma \int_{\alpha_i}^{\alpha_f^*} D^2\alpha e^{i(S_S+S_R)} , \tag{3.13}$$

where

$$\begin{aligned} iS_S &= \tfrac{1}{2}(\gamma_f'^* \cdot \gamma_f + \gamma_i^* \cdot \gamma_i) \\ &\quad + \int_{t_i}^{t_f} dt [\tfrac{1}{2}(\dot{\gamma}^* \cdot \gamma - \gamma^* \cdot \dot{\gamma}) - iH_S(\gamma^*, \gamma)] \end{aligned}$$

and

$$\begin{aligned} iS_R &= \tfrac{1}{2}[\dot{\alpha}_f^* \cdot \alpha_f + \alpha_i^* \cdot \dot{\alpha}_i] \\ &\quad + \int_{t_i}^{t_f} dt [\tfrac{1}{2}(\dot{\alpha}^* \cdot \alpha - \alpha^* \cdot \dot{\alpha}) - iH_b(\alpha^*, \alpha) \\ &\qquad - iH_{s\text{-}b}(\gamma^*, \gamma; \alpha^*, \alpha)] , \end{aligned} \tag{3.14}$$

and introducing

$$K\{\alpha_f^*, t_f; \alpha_i, t_i; \gamma(t)\} = \int_{\alpha_i}^{\alpha_f^*} D^2\alpha e^{iS_R\{\alpha,\gamma\}} , \tag{3.15}$$

the matrix element of $\rho_s(t_f)$ in Eq. (3.12) can be written in the convenient form

$$\langle \gamma'_f \| \rho_s(t_f) \| \gamma_f \rangle = \int d^2\gamma_i d^2\gamma_i' e^{-\gamma_i^* \cdot \gamma_i - \gamma_i'^* \cdot \gamma_i'} \int_{\gamma_i}^{\gamma_f'^*} D^2\gamma \int_{\gamma_f}^{\gamma_i'^*} D^2\gamma' e^{iS_S\{\gamma\} - iS_S\{\gamma'\}} F\{\gamma, \gamma'\} \langle \gamma_i \| \rho_s(t_i) \| \gamma_i' \rangle , \tag{3.16}$$

where

$$\begin{aligned} F\{\gamma, \gamma'\} &= -\int e^{\alpha_f^* \cdot \alpha_f - \alpha_i^* \cdot \alpha_i - \alpha_i'^* \cdot \alpha_i'} d^2\alpha_f d^2\alpha_i d^2\alpha_i' \\ &\quad \times K\{\alpha_f^*, t_f; \alpha_i, t_i; \gamma\} K\{\alpha_i'^*, -i\beta; \alpha_i', 0; 0\} \\ &\quad \times K\{\alpha_i'^*, t_i; \alpha_f, t_f; \gamma'\} \end{aligned} \tag{3.17}$$

is the influence functional. Here we used

$$\begin{aligned} \langle \alpha \| \rho_b(t_i) \| \alpha' \rangle &= \frac{1}{Z} \langle \alpha \| e^{-\beta H_b} \| \alpha' \rangle \\ &= \prod_k \frac{1}{Z_k} \exp\left[\sum_k e^{-\beta \varepsilon_k} \alpha_k^* \alpha_k' \right] \\ &= K\{\alpha^*, -i\beta; \alpha', 0; 0\} \end{aligned} \tag{3.18}$$

and

$$Z_k = 1 + e^{-\beta \varepsilon_k} .$$

The influence functional $F\{\gamma, \gamma'\}$ is equal to 1 if the coupling between the system and the heat bath is switched off ($H_{s\text{-}b} = 0$). Then Eq. (3.16) describes the time evolution of the density matrix ρ_s as given by the time-evolution operators of the system H_s alone:

$$\int_{\gamma_i}^{\gamma_f'^*} D^2\gamma e^{iS_S\{\gamma\}}$$

and

$$\int_{\gamma_f}^{\gamma_i'^*} D^2\gamma' e^{-iS_S\{\gamma'\}} . \tag{3.19}$$

In the presence of coupling, the influence of the heat bath on the time evolution of ρ_s is contained in $F\{\gamma, \gamma'\}$.

Since $F\{\gamma, \gamma'\}$ contains only Gaussian integrals it can be calculated in explicit form. Using the expression for $K\{\alpha_f^*, t_f; \alpha_i, t_i; \gamma\}$ for a free system derived in Appendix B, one arrives at

$$\begin{aligned} &F\{\gamma(t), \gamma'(t)\} \\ &\quad = \exp\left[-i \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt' \sum_{l,k} | V_{lk} |^2 \tilde{\gamma}_l^\dagger(t) G_k(t,t') \tilde{\gamma}_l(t') \right] , \end{aligned} \tag{3.20}$$

where

$$\tilde{\gamma}_l^\dagger(t) = [\gamma_l^*(t), \gamma_l'^*(t)] \tag{3.21}$$

and

$$G_k(t,t') = i \begin{bmatrix} f(\varepsilon_k)\Theta(t'-t) - f(-\varepsilon_k)\Theta(t-t') & f(\varepsilon_k) \\ -f(-\varepsilon_k) & f(\varepsilon_k)\Theta(t-t') - f(-\varepsilon_k)\Theta(t'-t) \end{bmatrix} e^{-i\epsilon_k(t-t')} . \tag{3.22}$$

Its Fourier transform becomes, in the limit $t_f \to +\infty, t_i \to -\infty$

$$G_k(\omega)=\begin{bmatrix} \dfrac{f(-\varepsilon_k)}{\omega-\varepsilon_k+i\delta}+\dfrac{f(\varepsilon_k)}{\omega-\varepsilon_k i\delta} & 2\pi i f(\varepsilon_k)\delta(\omega\varepsilon_k) \\ -2\pi i f(-\varepsilon_k)\delta(\omega-\varepsilon_k) & -\dfrac{f(\varepsilon_k)}{\omega-\varepsilon_k+i\delta}-\dfrac{f(-\varepsilon_k)}{\omega-\varepsilon_k-i\delta} \end{bmatrix}, \tag{3.23}$$

where δ is an infinitesimal positive constant introduced to regularize the propagators when the limit $t_f \to +\infty, t_i \to -\infty$ is taken, and

$$\gamma_l(\omega)=\int_{-\infty}^{\infty} dt\, e^{i\omega t}\gamma_l(t) ,$$

with similar definitions for the other fields. Performing the sum over the states k of the heat bath and using Eq. (3.6), one obtains

$$F\{\gamma(\omega),\gamma'(\omega)\}=\exp\left[-i\int\frac{d\omega}{2\pi}\sum_l \tilde{\gamma}_l^{\dagger}(\omega)\overline{G}(\omega)\tilde{\gamma}_l(\omega)\right] , \tag{3.24}$$

with

$$\overline{G}(\omega)=-i\Gamma\begin{bmatrix}\tanh(\beta\omega/2) & -2f(\omega) \\ 2f(-\omega) & \tanh(\beta\omega/2)\end{bmatrix} . \tag{3.25}$$

In order to obtain the generating functional $Z\{\eta(t),\eta'(t)\}$ for the electron system from which one can derive all time-dependent correlation functions, we introduce in Eq. (3.16) source fields $\eta(t),\eta'(t)$, couple them to the $\gamma(t),\gamma'(t)$, use for ρ_s $(t_i=-\infty)=|0\rangle\langle 0|$ (for simplicity, since the initial value of ρ_s is not relevant), and take the trace. This yields

$$Z\{\eta(t),\eta'(t)\}=\int D^2\gamma\int D^2\gamma' \exp\left[\int_{-\infty}^{\infty} i[\tilde{\eta}^{\dagger}(t)\tilde{\gamma}(t)+\tilde{\gamma}^{\dagger}(t)\tilde{\eta}(t)]dt\right] e^{iS_S\{\gamma(t)\}}e^{-iS_S\{\gamma'(t)\}}F\{\gamma(t),\gamma'(t)\} , \tag{3.26}$$

with

$$\tilde{\eta}^{\dagger}(t)=[\eta^*(t),\eta'^*(t)] ,$$

where the functional integral over the variables $\gamma(t)$ and $\gamma'(t)$ extends now over all times $-\infty < t < \infty$.

The normalization of ρ_s is automatically assured since one has

$$Z\{0,0\}=\mathrm{Tr}\rho_s(\infty)=\mathrm{Tr}\rho_s(-\infty)=1 . \tag{3.27}$$

As a matter of fact, it is not necessary to set the sources $\eta(t),\eta'(t)$ equal to zero. It suffices to take them equal, i.e., $\eta'(t)=\eta(t)$ to find

$$Z\{\eta(t),\eta(t)\}=1 .$$

Equation (3.26) for the generating functional $Z\{\eta(t),\eta'(t)\}$ is, together with the explicit expression for $F\{\gamma(t),\gamma'(t)\}$ in Eqs. (3.24) and (3.25), one of our main results. By taking derivatives with respect to the source fields $\eta(t),\eta'(t)$, one can generate directly from $Z\{\eta(t),\eta'(t)\}$ all relevant correlation functions and $Z\{0,0\}=1$ ensures the absence of the denominator problem (the denominator is 1). Random averages over $Z\{\eta,\eta'\}$ can be easily performed since they involve only averages over $e^{iS_S\{\gamma\}}e^{-iS_S\{\gamma'\}}$.

In order to establish a connection to the results of Sec. II, we finally calculate $Z\{\eta,\eta'\}$ explicitly for a free electron in a heat bath which is described by H_M in Eq. (2.1). We can then drop the index l and obtain for the action of a free particle (without the boundary terms)

$$iS\{\gamma\}=\int dt[\tfrac{1}{2}(\dot{\gamma}^*\gamma-\gamma^*\dot{\gamma})-iE\gamma^*\gamma]$$
$$=i\int\frac{d\omega}{2\pi}(\omega-E)\gamma^*(\omega)\gamma(\omega) . \tag{3.28}$$

Using Eqs. (3.24)–(3.26), $Z\{\eta,\eta'\}$ can be calculated by performing Gaussian integrals and one obtains

$$Z\{\eta(\omega),\eta(\omega)\}=\exp\left[-i\int\frac{d\omega}{2\pi}\tilde{\eta}^{\dagger}(\omega)G_0(\omega)\tilde{\eta}(\omega)\right] , \tag{3.29}$$

where

$$\tilde{\eta}^{\dagger}(\omega)=[\eta^*(\omega),\eta'^*(\omega)]$$

and

$$G_0(\omega)=\frac{1}{(\omega-E)^2+\Gamma^2}\begin{bmatrix}\omega-E-i\tanh(\beta\omega/2) & -2i\Gamma f(\omega) \\ 2i\Gamma f(-\omega) & (\omega-E)-i\Gamma\tanh(\beta\omega/2)\end{bmatrix} . \tag{3.30}$$

This yields for the thermal occupation number

$$\langle c^\dagger c\rangle = -\int \frac{d\omega}{2\pi}\,\frac{\vec{\partial}}{\partial\eta^*(\omega)}\,\frac{\vec{\partial}}{\partial\eta'(\omega)}Z\Bigg|_{\eta=\eta'=0}$$
$$= \frac{1}{\pi}\int d\omega\frac{\Gamma}{(\omega-E)^2+\Gamma^2}f(\omega)=f(E)\quad\text{for } \Gamma\to 0\ , \tag{3.31}$$

i.e., the same result as Eq. (2.14). Therefore, the path integral formalism of this section is, for the case of one free electron in a Fermi sea, equivalent to the Langevin-equation approach of Sec. II. For an interacting system the result of Eq. (3.30) is the free propagator to be used in perturbation theory.

For $\Gamma\to 0$, Eq. (3.30) yields just Eq. (3.23) (with the sign of the off-diagonal elements changed). This is not accidental, as one can see, comparing this propagator, which refers to a system coupled to a heat bath and where the temperature dependence comes from this heat bath, to the propagator (3.23) for a system in thermal equilibrium at temperature T.[12,13] In this case, we do not have the heat bath and $\rho_s(t_i)$ is the equilibrium density matrix. To derive the propagator we could again add a source term to the action similarly to what we did in Eq. (3.26). Due to the similarity between this source term and the Hamiltonian of Eq. (3.5) for the coupling between the system and the heat bath, the calculation of the new propagator is similar to the calculation of the influence functional which we have performed [see Eq. (3.17)]. One then arrives again at Eqs. (3.22) and (3.23), the only difference being that the sign of the off-diagonal elements of those matrices must be changed, since in Eq. (3.26) the source terms in η and η' have the same sign. One concludes then that the propagator for the system coupled to a heat bath can be obtained from the propagator for the system in thermodynamic equilibrium replacing the infinitesimal positive constant δ by the finite damping Γ and using necessarily, for the argument of the Fermi functions, the frequency (and not the energy of the system). As a result, the physical quantities are expressed by Lorentzians of finite width Γ convoluted in frequency with the Fermi functions, as found in Eq. (3.31). In the weak-coupling limit, $\Gamma\to 0$, one recovers the equilibrium results.

Finally, Eq. (3.30) can be interpreted as being the zero-temperature propagator to which the heat bath gave a self-energy correction given by Eq. (3.25). The damping and the noise-noise correlation functions can be read off from Eq. (3.26), similarly to what happens in the case of harmonic oscillators.[7,11] The independence of the initial $\rho_s(t_i)$ is easily understood, since it only affects the infinitesimal δ regularization, and in the presence of a finite damping Γ, one can take $\delta\to 0$, washing out that dependence.

IV. DISORDERED ELECTRON SYSTEMS WITH INTERACTIONS

In this section we calculate the generating functional for a disordered electronic system[14–16] with interactions and discuss some of its symmetry properties. Our model Hamiltonian reads

$$H=\sum_{\mu,\mu'}(t_{\mu\mu'}c_\mu^\dagger c_{\mu'}+V_{\mu\mu'}c_\mu^\dagger c_{\mu'}^\dagger c_{\mu'}c_\mu)\ , \tag{4.1}$$

where $\mu=(i,\sigma)$ denotes the site i and the spin index σ.

The hopping matrix elements $t_{\mu\mu'}=t_{ij}\delta_{\sigma,\sigma'}$ with $t_{ji}^*=t_{ij}$ are chosen to be a random Gaussian distribution having a width $\Delta_{ij}=\langle\,|t_{ij}|^2\rangle_D$. The interaction term $V_{\mu\mu'}=V(i-j)(1-\delta_{ij})$ describes, e.g., the unscreened Coulomb interaction between the electrons. But we would like to point out that our formalism works as well if we take for $V_{\mu\mu'}$, the interaction term in the Hubbard model, i.e., $V_{\mu\mu}=U\delta_{ij}(1-\delta_{\sigma,\sigma'})$ or, with a proper change of spin indices

$$\sum_{\mu,\mu'}V_{\mu\mu'}c_\mu^\dagger c_{\mu'}^\dagger c_{\mu'}c_\mu\to V\sum_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger c_{i\downarrow}c_{i\uparrow},\quad V<0\ ,$$

the attractive electron-electron interaction of a model superconductor. Using Eq. (3.26) the generating functional which corresponds to the Hamiltonian of Eq. (4.1) becomes

$$Z\{h\}=\int D^2\gamma\int D^2\gamma' e^{L_0\{\gamma,\gamma'\}}\ , \tag{4.2}$$

where

$$L_0\{\gamma,\gamma'\}=L_n\{\gamma,\gamma'\}+L_r\{\gamma,\gamma'\} \tag{4.3}$$

and L_r, L_n denote the random and nonrandom part of the Lagrangian L_0, respectively. The nonrandom part is given by

$$\begin{aligned}L_n\{\gamma,\gamma'\}=\tfrac{1}{2}\sum_\mu\int dt\{[\dot\gamma_\mu^*(t)\gamma_\mu(t)-\gamma_\mu^*(t)\dot\gamma_\mu(t)]-[\dot\gamma_\mu'^*(t)\gamma_\mu'(t)-\gamma_\mu'^*(t)\dot\gamma_\mu'(t)]\}\\ -i\sum_{\mu,\mu'}V_{\mu\mu'}\int dt[\gamma_\mu^*(t)\gamma_{\mu'}^*(t)\gamma_{\mu'}'(t)\gamma_\mu(t)-\gamma_\mu'^*(t)\gamma_{\mu'}'^*(t)\gamma_{\mu'}'(t)\gamma_\mu'(t)]\\ +\sum_{\mu,\mu'}\int dt\int dt'\gamma_\mu^\dagger(t)[G(t-t')\delta_{\mu\mu'}+h_{\mu\mu'}(t,t')]\gamma_{\mu'}'(t')\ .\end{aligned} \tag{4.4}$$

Here, $\gamma_\mu^\dagger(t)=[\gamma_\mu^*(t),\gamma_\mu'^*(t)]$; the matrix $G(t-t')$ is given by the Fourier transform of Eq. (3.25),

$$G(t)=\int\frac{d\omega}{2\pi}e^{-i\omega t}G(\omega)\ , \tag{4.5}$$

and we introduced a matrix of source fields $h_{\mu\mu'}(t,t')$. The random part of L_0 reads

$$L_r=-i\sum_{\mu,\mu'}t_{\mu\mu'}\int dt[\gamma_\mu^*(t)\gamma_{\mu'}'(t)-\gamma_\mu'^*(t)\gamma_{\mu'}'(t)]\ . \tag{4.6}$$

Let us now discuss some symmetry properties of L. We consider three questions: (i) What corresponds in our formalism to the replica symmetry? (ii) What happens to the hyperbolic symmetry of the localization problem in the presence of interactions? (iii) How does time-reversal symmetry show up in L_0?

To answer the first question we consider the noninteracting case, i.e., $V_{\mu\mu'}=0$, and put $h_{\mu\mu'}(t,t')=0$ and $\Gamma=0$. Then Eq. (4.3) becomes, after Fourier transformation in time,

$$L_n=i\sum_{\mu}\int\frac{d\omega}{2\pi}\omega[\gamma_\mu^*(\omega)\gamma_\mu(\omega)-\gamma_\mu'^*(\omega)\gamma_\mu'(\omega)]\ , \tag{4.7}$$

$$L_r=i\sum_{\mu,\mu'}t_{\mu\mu'}\int\frac{d\omega}{2\pi}[\gamma_\mu^*(\omega)\gamma_{\mu'}(\omega)-\gamma_\mu'^*(\omega)\gamma_{\mu'}'(\omega)]\ . \tag{4.8}$$

By comparing L_r to the corresponding expression in the formulation which uses replicas [Eqs. (2.7) and (2.8) in Ref. 16], one can see that the frequency ω plays a role which is somewhat similar to the replica index n and that the static limit $\omega\rightarrow 0$ corresponds to $n\rightarrow 0$. But although L_r is invariant with respect to rotations of $\gamma(\omega)$ or $\gamma'(\omega)$, the Lagrangian L_0 does not have this invariance because of the factor ω in L_n. Therefore, we conclude that a symmetry which is equivalent to the rotation symmetry in replica space does not exist in our formulation.

However, the hyperbolic symmetry[15,16] which exchanges the branches γ and γ' and whose important role for localization was first observed by Wegner is preserved by L_0 [see Eqs. (4.4)–(4.6)]. This symmetry is weakly broken for $\Gamma\neq 0$. But, if we include the interaction part $V_{\mu\mu'}\neq 0$, then the hyperbolic symmetry is broken massively. This means that, in the presence of interactions, a delocalization transition in two-dimensional systems is no longer forbidden by massless Goldstone modes.

We observe finally that time-reversal invariance shows up as a Z_2 symmetry in L_0,[13] i.e., L_0 is invariant against taking the Hermitian conjugate and interchanging primed and unprimed variables:

$$L_0^\dagger\{\gamma',\gamma\}=L\{\gamma,\gamma'\}\ . \tag{4.9}$$

This symmetry is always present in the full expression for L_0 [i.e., $\Gamma\neq 0$, $V_{\mu\mu'}\neq 0$, but $h_{\mu\mu'}(t,t')=0$] and does not change if one takes the random average of $Z\{h\}$.

In order to obtain a field-theoretic formulation of the localization problem in the presence of interactions, which is amenable to a renormalization-group analysis, we next calculate the averaged generating functional $\langle Z\{h\}\rangle_D$ and perform a Hubbard-Stratonovich[9] transformation to express $\langle Z\{h\}\rangle_D$ in terms of complex (not Grassmann) matrices.

Taking the random average of Eq. (4.2) yields

$$\langle Z\{h\}\rangle_D=\int D^2\gamma\int D^2\gamma' e^{L}\ , \tag{4.10}$$

where $L=L_n+\widetilde{L}_r$ and

$$\widetilde{L}_r=-\tfrac{1}{2}\sum_{i,j,\sigma,\sigma'}\Delta_{ij}\int dt\int dt'[\gamma_{i,\sigma}^*(t)\gamma_{i,\sigma'}(t')\gamma_{j,\sigma'}^*(t')\gamma_{j,\sigma}(t)-\gamma_{i,\sigma}^*(t)\gamma_{i,\sigma}'(t')\gamma_{j,\sigma'}^*(t')\gamma_{j,\sigma}(t)$$
$$-\gamma_{i,\sigma}'(t)\gamma_{i,\sigma}^*(t')\gamma_{j,\sigma'}(t')\gamma_{j,\sigma'}'^*(t)+\gamma_{i,\sigma}'^*(t)\gamma_{i,\sigma'}'(t')\gamma_{j,\sigma'}'^*(t')\gamma_{j,\sigma}'(t)]\ . \tag{4.11}$$

As a next step, variables which are coupled by Δ_{ij} and $V_{\mu\mu'}$ are decoupled by a Hubbard-Stratonovich transformation and $\langle Z\{h\}\rangle_D$ can be written in terms of complex matrices:

$$\langle Z\{h\}\rangle_D\sim\int DQ\int DP e^{\hat{L}\{Q,P\}}\ , \tag{4.12}$$

where

$$\hat{L}\{Q,P\}=-\sum_{i,j}[(\Delta^{-1})_{ij}\,\mathrm{Tr}Q_iQ_j+i(V^{-1})_{ij}\,\mathrm{Tr}(P_iMP_j)]$$
$$-\mathrm{Tr}\ln(E-P-Q+h) \tag{4.13}$$

and the matrices Q, P, E, h, and M are given by

$$Q_i\{a,\omega,\sigma;a',\omega',\sigma'\};\ \ h_i\{a,\omega,\sigma;a'\omega'\sigma'\}\ ,$$

$$P_i\{a,\omega,\sigma;a',\omega',\sigma'\}=\delta_{a,a'}\delta_{\sigma,\sigma'}P_i(a,\omega+\omega')\ , \tag{4.14}$$

$$E=\delta_{\sigma,\sigma'}\delta(\omega-\omega')\left[\begin{bmatrix} i\omega & 0\\ 0 & -i\omega\end{bmatrix}-i\overline{G}(\omega)\right],M=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}.$$

Here, the index a labels the different branches γ and γ' and we have specialized the source field to be diagonal in the spatial variables. Equation (4.13) yields, to the best of our knowledge, for the first time, a closed expression for the generating functional of a disordered electronic system with interaction, in terms of complex matrices. One recovers for $P=0$, $\beta\rightarrow\infty$, the noninteracting case in a form which looks strikingly similar to previous results[14–16] but which avoids the replica trick. The interaction introduces an additional matrix field $P\neq 0$. As mentioned in the introduction to this section this formulation is appropriate to study the competition between localization represented by the Q fields and ferromagnetism represented by the P fields, or superconducting represented by some other P fields associated with the Cooper-pair densities.

V. SUMMARY AND CONCLUSIONS

We started by studying the motion of an electron system in the presence of an electron heat bath, i.e., the Brownian motion for electrons. First, we considered a system of free electrons. Choosing in an appropriate way the density of states for the heat bath and the coupling between the system and the heat bath, we derived a Langevin equation for the electron system, with damping and noise terms. The noise-noise correlation functions were obtained, providing us with the fermion analog of the noise-noise correlation functions for the usual harmonic-oscillator Brownian motion. In order to be able to consider interacting electron systems, we used the Feynmann-Vernon approach. Using path integral techniques with Grassmann variables and making use of the density of states and coupling chosen before, we obtained the influence functional reflecting the presence of the heat bath on the action for our system. The unperturbed propagator for a perturbation theory was derived. It is independent of the precise expression of the initial density matrix for our system, and it gives immediately the results obtained before for the free system. Comparing it with the propagator for an electron system in thermal equilibrium, one observes the broadening and the frequency convolution effect due to the finite damping. In the weak-coupling limit we recover the equilibrium results. We wrote then the generating functional for the correlation functions of a many-electron system, with interactions, in the presence of a heat bath.

This formalism is particularly important for disordered systems where one has a thermal and a random average to perform. Choosing initial conditions independent of the disorder, all disorder will be in the time evolution of our system and the so-called denominator problem is avoided. The heat bath ensures that the system approaches equilibrium.

Finally, we considered the electron localization problem due to a disordered hopping matrix, in the presence of an interaction. Our method provides a natural formulation of this problem, on physical grounds, avoiding formal manipulations like the replica trick or the superfield method. Some symmetry properties of the effective action of the electron localization problem in the presence of a Coulomb interaction were discussed and we gave an explicit expression of the corresponding generating functional in terms of two sets of complex matrices (one referring to the disordered noninteracting electron system and the other taking into account the interaction).

We would finally like to point out that our method has a number of important physical applications. It could be used to study in electronic systems the interplay between disorder and Coulomb interaction or disorder and superconductivity. Furthermore, one could also study the influence of disorder on the electron-phonon interaction by using for the electrons our method and for the phonon field, e.g., the Caldeira-Leggett approach. Finally, our formalism could be immediately applied to bosons with a finite chemical potential, just using complex variables instead of Grassmann variables and replacing Fermi functions by Bose functions. This would, e.g., be important for the study of superfluidity of helium on a random substrate.

ACKNOWLEDGMENT

One of us (H.G.S.) would like to acknowledge the financial support of Sonderforschungsbereich 65 Frankfurt/Darmstadt.

APPENDIX A: GRASSMANN VARIABLES AND HOLOMORPHIC REPRESENTATION

Grassmann variables are the classical fields associated with electrons and are therefore characterized by their anticommutation relations,[8] given by

$$\{\alpha_i,\alpha_j\}=\{\alpha_i^*,\alpha_j^*\}=\{\alpha_i,\alpha_j^*\}=0 \ . \tag{A1}$$

As a result, the most general function of a finite number of them is a polynomial, with the product of all the variables in the highest monomial. When working with them one tries to keep, as much as possible, to the usual rules. As an example, differentiation still follows the usual rule

$$d(\alpha^*\beta)=(d\alpha^*)\beta+\alpha^* d\beta \ , \tag{A2}$$

but since the variables anticommute, we must make the distinction between left $(\overleftarrow{\partial}/\partial)$ and right $(\overrightarrow{\partial}/\partial)$ derivatives, according to out of which side we take the differential $d\alpha^*$ (or $d\beta$).

The requirement of translational invariance for integration, i.e., of

$$\int f(\alpha+\beta)d\alpha=\int f(\alpha)d\alpha \ , \tag{A3}$$

implies that

$$\int d\alpha=\int d\alpha^*=0 \ . \tag{A4}$$

As a convention, one then defines that

$$\int \alpha d\alpha=\int d\alpha^*\alpha^*=1 \tag{A5}$$

and uses the notation

$$d^2\alpha=d\alpha\, d\alpha^* \ . \tag{A6}$$

Complex conjugation is the analog of Hermitian conjugation and so it reverses the order of multiplication. As a result of the rules for integration, the change of variables is similar to the usual one, with the only difference being that one must divide by the Jacobian of the transformation instead of multiplying by it [see also Eq. (B6)].

Finally, by definition, the electron creation and annihilation operators anticommute with the Grassmann variables, and if one chooses that

$$\alpha\,|0\rangle=|0\rangle \ , \tag{A7}$$

one finds that

$$\alpha\,|1\rangle=-|1\rangle\alpha \ , \tag{A8}$$

where $|0\rangle$ and $|1\rangle$ are the usual electron states.

Electron coherent states $|\alpha\rangle$ are, by definition, (right) eigenvectors of the annihilation operator a, with eigenvalue α, i.e., one has

$$a\,|\alpha\rangle=\alpha\,|\alpha\rangle \ . \tag{A9}$$

Their properties are formally similar to the boson coherent states. They are given by

$$|\alpha\rangle=D(\alpha)\,|0\rangle \ , \tag{A10}$$

where

$$D(\alpha)=e^{a^\dagger\alpha-\alpha^* a} \tag{A11}$$

$$=e^{-(1/2)\alpha^*\alpha}e^{a^\dagger\alpha}e^{-\alpha^* a} \ , \tag{A12}$$

as can be seen from the property

$$D^{-1}(\alpha)aD(\alpha)=a+\alpha \ . \tag{A13}$$

Their inner product is given by

$$\langle\beta\,|\,\alpha\rangle=\exp(-\tfrac{1}{2}\beta^*\beta-\tfrac{1}{2}\alpha^*\alpha+\beta^*\alpha) \tag{A14}$$

$$=\exp[\tfrac{1}{2}(\beta^*-\alpha^*)\alpha-\tfrac{1}{2}\beta^*(\beta-\alpha)] \tag{A15}$$

and they can be used to write down a decomposition of the identity

$$\int |\,\alpha\rangle d^2\alpha\langle\alpha\,| =1 \ . \tag{A16}$$

One could use these coherent states $|\,\alpha\rangle$ and consider the matrix elements of the different operators between these states. In the path integral formalism, however, where the variables α and α^* are treated as independent, it is preferable to use the unnormalized states $||\alpha\rangle$ given by

$$||\alpha\rangle=e^{(1/2)\alpha^*\alpha}\,|\,\alpha\rangle=e^{a^{\dagger}\alpha}\,|\,0\rangle \ , \tag{A17}$$

which depend on the single variable α (and not on α^*) and are still (right) eigenvectors of the annihilation operator.

The decomposition of the identity, the inner product of these states, and the holomorphic representation of operators is given in Eqs. (3.8)–(3.11).

APPENDIX B: PATH INTEGRAL FOR ELECTRONS

The path integral expression for the evolution operator,

$$K\{\alpha_f,t_f;\alpha_i,t_i;0\}\equiv\langle\alpha_f||e^{-iH(t_f-t_i)}||\alpha_i\rangle \ , \tag{B1}$$

can be obtained by splitting t_f-t_i into N subintervals of length $(t_f-t_i)/N$, using the decomposition of the identity given by Eq. (3.8), defining $\alpha_0=\alpha_i$, $\alpha_N^*=\alpha_f^*$ and keeping only terms of order $1/N$ for the action,

$$K\{\alpha_f^*,t_f;\alpha_i,t_i;0\}=\lim_{N\to\infty}\int\prod_{k=1}^{N-1}d^2\alpha_k\exp\left[\tfrac{1}{2}(\alpha_N^*\alpha_{N-1}+\alpha_1^*\alpha_0)+\tfrac{1}{2}\sum_{k=1}^{N-1}(\alpha_{k+1}^*-\alpha_k^*)\alpha_k\right.$$

$$\left.-\tfrac{1}{2}\sum_{k=0}^{N-2}\alpha_{k+1}^*(\alpha_{k+1}-\alpha_k)-\sum_{k=0}^{N-1}H_N(\alpha_{k+1}^*,\alpha_k)\varepsilon\right] \tag{B2}$$

$$=\int D^2\alpha\exp\left\{\tfrac{1}{2}[\alpha_f^*\alpha(t_f)+\alpha^*(t_i)\alpha_i]+\int_{t_i}^{t_f}\left[\frac{1}{2}\left[\frac{d\alpha^*}{dt}\alpha-\alpha^*\frac{d\alpha}{dt}\right]-iH_N(\alpha^*,\alpha)\right]dt\right\} , \tag{B3}$$

where

$$D^2\alpha=\lim_{N\to\infty}\prod_{k=1}^{N-1}d^2\alpha_k \ . \tag{B4}$$

The appropriate boundary conditions for the path integral trajectories are $\alpha(t_i)=\alpha_i$ and $\alpha^*(t_f)=\alpha_f^*$, as one should expect from the functional dependence of $K\{\alpha_f^*,t_f;\alpha_i,t_f;0\}$ on α_i and α_f^* or from the derivation of the path integral. For the case of a free-electron system with time-dependent sources $\eta(t),\eta^*(t)$ described by a Hamiltonian

$$H=\varepsilon a^{\dagger}a-\eta^*a-a^{\dagger}\eta \ , \tag{B5}$$

the path integral for the matrix element (B2) can be solved exactly using the basic Gaussian integral with Grassmann variables:

$$\int d^2\alpha e^{i(\alpha^*G_0^{-1}\alpha+\eta^*\alpha+\alpha^*\eta)}=\det(-iG_0^{-1})e^{-i\eta^*G_0} \ . \tag{B6}$$

As for the case of Gaussian path integrals with real or complex variables the exponential factor of $K[\alpha_f^*,t_f;\alpha_i,t_i;0]$ is in fact equal to $\exp(iS)$, where S is the action evaluated at the trajectory which satisfies the equations of motion

$$\frac{d\alpha^*}{dt}=i\frac{\overleftarrow{\partial}H}{\partial\alpha},\quad \frac{d\alpha}{dt}=\frac{\vec{\partial}H}{\partial\alpha^*} \ . \tag{B7}$$

Their solutions, with the appropriate boundary conditions,

$$\alpha(t)=e^{-i\varepsilon(t-t_i)}\alpha_i+i\int_{t_i}^{t}e^{-i\varepsilon(t-t')}\eta(t')dt' \ , \tag{B8a}$$

$$\alpha^*(t)=\alpha_f^*e^{-i\varepsilon(t_f-t)}+i\int_{t}^{t_f}\eta^*(t')e^{+i\varepsilon(t-t')}dt' \ , \tag{B8b}$$

yield

$$K\{\alpha_f^*,t_f;\alpha_i,t_i;\eta\}=\exp(\alpha_f^*e^{-i\varepsilon(t_f-t_i)}\alpha_i+\alpha_f^*\overline{\eta}+\overline{\eta}^*\alpha_i+A) \tag{B9}$$

with

$$\overline{\eta}=i\int_{t_i}^{t_f}dt\,e^{-i\varepsilon(t_f-t)}\eta(t) \ , \tag{B10a}$$

$$\overline{\eta}^*=i\int_{t_i}^{t_f}dt\,\eta^*(t)e^{-i\varepsilon(t-t_i)} \ , \tag{B10b}$$

and

$$A=-\int_{t_i}^{t_f}dt\int_{t_i}^{t}dt'\eta^*(t)e^{-i\varepsilon(t-t')}\eta(t') \ . \tag{B10c}$$

As can be seen from Eq. (B2) the determinant for the prefactor in Eq. (B9) is simply 1.

[*] Permanent address: Institut für Theoretische Physik der Universität Frankfurt/M, Robert-Mayer-Str. 8, 6000 Frankfurt, Federal Republic of Germany.

[1] S. F. Edwards and P. W. Anderson, J. Phys. F **5**, 965 (1975).

[2] K. B. Efetov, Adv. Phys. **32**, 53 (1983).

[3] H. Sompolinsky and A. Zippelins, Phys. Rev. Lett. **47**, 359 (1981); P. C. Martin, E. D. Siggia, and H. A. Rose, Phys. Rev. A **8**, 423 (1973); C. de Dominicis, in *Dynamical Critical Phenomena and Related Topics* Vol. 104 of *Lecture Notes in Physics,* edited by C. P. Enz (Springer-Verlag, Berlin, 1979).

[4] A. J. McKane and M. Stone, Ann. Phys. **131**, 36 (1981).

[5] H. Sompolinsky, Phys. Rev. Lett. **47**, 935 (1981).

[6] R. P. Feynman and F. L. Vernon, Ann. Phys. **24**, 118 (1963); R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).

[7] A. O. Caldeira and A. J. Leggett, Physica **121A**, 587 (1983).

[8] F. A. Berezin, *The Method of Second Quantization* (Academic, New York, 1966).

[9] J. Hubbard, Phys. Rev. Lett. **3**, 77 (1959); R. L. Stratonovich, Dokl. Akad. Nauk SSSR **2**, 1097 (1957) [Sov. Phys.—Doklady **2**, 416 (1957)].

[10] H. Haken, Rev. Mod. Phys. **47**, 67 (1975); *Synergetics. An Introduction. Nonequilibrium Phase Transitions and Self-Organization in Physics, Chemistry and Biology* (Springer-Verlag, Berlin, 1977).

[11] G. W. Ford, M. Kac, and P. Mazur, J. Math. Phys. **6**, 504 (1965); R. Benguria and M. Kac, Phys. Rev. Lett. **46**, 1 (1981); A. Schmid, J. Low Temp. Phys. **49**, 609 (1982).

[12] J. Schwinger, J. Math. Phys. **2**, 407 (1961); L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys.—JETP **20**, 1018 (1965)]; H. Umezawa, H. Matsumoto, and M. Tachiki, *Thermo Field Dynamics and Condensed States* (North-Holland, Amsterdam, 1982), and references therein.

[13] A. J. Niemi and G. W. Semenoff, Ann. Phys. **152**, 105 (1984).

[14] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985), and references therein.

[15] F. J. Wegner, Z. Phys. B **35**, 207 (1979).

[16] L. Schäfer and F. J. Wegner, Z. Phys. B **38**, 113 (1980).