Effect of an arbitrary dissipative circuit on the quantum energy levels and tunneling of a Josephson junction

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The complex energy shifts of the energy levels of a macroscopic system subject to dissipation are calculated as a function of the phenomenological damping parameters describing the classical motion of the system. These results are applied to the energy levels of the zero-voltage state of a current-biased Josephson junction in parallel with an arbitrary dissipative circuit. Following the approach of Leggett, the influence of the same dissipative circuit on the tunneling rate out of the zero-voltage state is also calculated. The dependences of both phenomena, quantization of energy levels, and quantum tunneling, on the admittance of the circuit are compared.

I. INTRODUCTION

It has been recently observed that a macroscopic system—a current-biased Josephson junction—exhibits well-defined atomlike absorption resonances corresponding to transitions between quantized energy levels.[1] The Josephson junction is governed by a single macroscopic variable, which is the phase difference across the junction and which has already been shown experimentally[2–7] to display macroscopic quantum tunneling, in agreement with theoretical predictions.[8–11]

In contrast with quantum tunneling, which has no classical analog and for which the effect of friction was not really understood before the work of Caldeira and Leggett,[8] the phenomenon of absorption can be transposed in the correspondence limit. One thus expects dissipation to give a finite lifetime to the quantized levels and eventually to induce a shift in their energies. One purpose of this paper is to calculate this lifetime and shift for the experimental situation of Ref. 1.

Whereas dissipation in an atom is fixed once and for all by the interaction between an electron and the radiation field and cannot be predicted from measurements in a classical regime, dissipation in a macroscopic system such as a Josephson junction can be varied externally and measured classically.[12] Thus, as already stressed by Leggett in the case of tunneling,[13] one is not faced with a first-principles microscopic calculation as in the Lamb-shift problem, but rather with the establishment of a correspondence between the classical and quantum absorption of power by the junction in the presence of an arbitrary dissipative circuit.

Because it is possible to measure the modification of tunneling rates and, simultaneously, the absorption line position and width in the presence of dissipation, another problem arises: How would the two results be related? In particular, is it possible that the same circuit could strongly affect absorption lines and weakly affect tunneling or vice versa? The second purpose of this paper is to answer this question.

The paper is organized as follows. We first calculate, in Sec. II, the complex energy shifts of the discrete quantum levels of a general macroscopic variable coupled to its environment in terms of the damping response function appearing in the phenomenological equation which this variable obeys classically. We then apply these results, in Sec. III, to a current-biased Josephson junction in parallel with a linear dissipating circuit of admittance $Y(\omega)$ that can be treated as a perturbation. We evaluate numerically the importance of the effect in the particular example of Ref. 1. In Sec. IV, we apply Leggett's theory of the influence of a frequency-dependent dissipation mechanism[13] on quantum tunneling to the circuit treated in Sec. III. We investigate in detail the simple cases where the influence of the admittance on tunneling can be exactly calculated. Finally, in Sec. IV, we discuss the differences in the dependence of both quantum tunneling rates and absorption widths and positions on the temporal response of the admittance.

II. CALCULATION OF ENERGY SHIFTS OF THE QUANTIZED LEVELS OF A MACROSCOPIC VARIABLE

We consider a single degree of freedom X with potential energy $V(x)$ and kinetic energy $\frac{1}{2}\dot{X}^2$ subject to friction through its coupling to the many degrees of freedom of a large energy reservoir. In the classical regime, the coordinate X obeys a phenomenological equation of the type

$$\ddot{X}+\hat{K}\{X(t)\}=-\frac{\partial}{\partial X}V(X)\ , \qquad (2.1)$$

where $\hat{K}$ is a linear operator subject to causality requirements. Its Fourier transform $K(\omega)$ is analytical in the lower half of the complex plane. Another requirement on $\hat{K}$ is that $K(0)=0$, which simply states that the equilibrium value of X is not affected by friction. In this article, X is a macroscopic degree of freedom and $\hat{K}$ can therefore be obtained experimentally by direct observation of the classical behavior of X.

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A. The Hamiltonian description of friction

Following Leggett,[13] the Hamiltonian H which describes this system can be taken as the sum of a bare Hamiltonian H_0 and the Hamiltonian H_c of a set of oscillators of coordinates x_j coupled to the macroscopic degree of freedom X,

$$H_0=\tfrac{1}{2}P^2+V(X)\ , \tag{2.2}$$

$$H_c=\sum_j \tfrac{1}{2}\left[p_j^2+\omega_j^2\left[x_j-\frac{c_j}{\omega_j^2}X\right]^2\right]\ . \tag{2.3}$$

The coordinates P and p_j are the conjugate momenta of X and x_j, respectively. The symbols ω_j and c_j refer, respectively, to the frequency and coupling strength of the jth oscillator.

The spectral density $J(\omega)$ of the set of oscillators is defined by

$$J(\omega)=\frac{\pi}{2}\sum_j \frac{c_j^2}{\omega_j}\delta(\omega-\omega_j)\ , \tag{2.4}$$

and is chosen according to the following relation:

$$K(\omega)=\lim_{\mathrm{Im}z\to 0-}K(z)\ , \tag{2.5}$$

where $K(z)$, given by

$$K(z)=-\frac{2z^2}{\pi}\int_0^\infty \frac{J(\omega')d\omega'}{\omega'[(\omega')^2-z^2]}\ , \tag{2.6}$$

is such that $K(z)/z$ is the analytical continuation of $iJ(\omega)/\omega$ in the lower half of the complex plane.

B. Calculation of energy levels

We now begin an explicitly quantum-mechanical treatment of Eqs. (2.2) and (2.3) and introduce the eigenenergies E_n and eigenstates $|n\rangle$ of H_0, which are assumed to be known. We compute the energy shift ΔE_n of the nth eigenstate due to the coupling to the set of harmonic oscillators. The Hamiltonian H can be developed as

$$H=H_0+\sum_j \tfrac{1}{2}(p_j^2+\omega_j^2x_j^2)-X\sum_j c_jx_j+\tfrac{1}{2}X^2\sum_j\frac{c_j^2}{\omega_j^2}\ . \tag{2.7}$$

This Hamiltonian is similar to the Hamiltonian of an atom coupled to the radiation field except that the coupling here involves the position coordinate X and an extra potential term which is quadratic in X.

Using standard perturbation theory up to second order in the coupling constants c_j, one obtains

$$\Delta E_n=\sum_m M_{nm}\omega_{nm}\sum_j\frac{c_j^2}{2\omega_j^2(\omega_{nm}-\omega_j)}\ , \tag{2.8}$$

where

$$M_{nm}=|\langle n|X|m\rangle|^2\ , \tag{2.9}$$

and where $\omega_{nm}=E_n-E_m$. We take, in this section, rad s^{-1} as the energy unit, which is equivalent to setting $\hbar=1$.

Using the definition (2.4) of $J(\omega)$ and causality requirements on $\exp(-iHt)$, we can write

$$\Delta E_n=\sum_m M_{nm}\frac{\omega_{nm}}{\pi}\lim_{\epsilon\to 0+}\int_0^\infty\frac{J(\omega)d\omega}{\omega(\omega_{nm}-\omega+i\epsilon)}\ . \tag{2.10}$$

Because $J(\omega)$ is a continuous function of ω in the limit of an infinite number of reservoir oscillators, ΔE_n is, in general, complex. We can write

$$\Delta E_n=\Delta E_n'+i\Delta E_n''\ . \tag{2.11}$$

The real part $\Delta E_n'$ is the energy displacement proper of level n. The imaginary part $\Delta E_n''$ has to be interpreted as $-\frac{1}{2}\tau_n^{-1}$, where τ_n is the lifetime of level n.

We can remove the poles appearing in (2.10) for $\omega=\omega_{nm}$ by treating separately the contributions due to states with energies respectively higher and lower than E_n, and by use of relations (2.5) and (2.6). We obtain, after performing some algebra, a well behaved expression for ΔE_n, apart from an eventual divergence when $\omega\to\infty$,

$$\Delta E_n=L_n+G_n\ , \tag{2.12}$$

$$L_n=\sum_{m<n}M_{nm}K^*(\omega_{nm})\ , \tag{2.13}$$

$$G_n=-\sum_m M_{nm}\frac{\omega_{nm}}{\pi}\int_0^\infty\frac{d\omega\,\mathrm{Im}K(\omega)}{\omega(|\omega_{nm}|+\omega)}\ . \tag{2.14}$$

It follows that

$$\Delta E_n'=\mathrm{Re}(L_n)+G_n\ , \tag{2.15}$$

$$\Delta E_n''=\mathrm{Im}(L_n)\ . \tag{2.16}$$

The term L_n is "local" in frequency in the sense that it involves only the value of $K(\omega)$ at the transition frequencies towards lower-energy states. Its real part contributes to a pure energy shift and vanishes when $K(\omega)$ is purely dissipative $[\mathrm{Re}K(\omega)=0]$. On the other hand, the term G_n is "global" in frequency in the sense that it involves the values of $\mathrm{Im}K(\omega)$ over *all* frequencies. This term is real, vanishes when $K(\omega)$ is purely dispersive $[\mathrm{Im}K(\omega)=0]$, and only produces a shift in the energy levels. We note that this term is divergent for $\omega\to\infty$ if $\mathrm{Im}K(\omega)$ behaves as ω^α with $\alpha\geq 1$.

We now consider two illustrative examples.

1. The harmonic oscillator

When $V(x)=\frac{1}{2}\omega_0^2X^2$, formulas (2.13) and (2.14) give

$$\Delta E_n=\frac{n}{2}\frac{K^*(\omega_0)}{\omega_0}+\frac{1}{2\pi}\int_0^\infty\frac{d\omega\,\mathrm{Im}K(\omega)}{\omega(\omega_0+\omega)}\ . \tag{2.17}$$

The second term, the global term, can be divergent but is independent of the level number n. As the physically measurable quantities are energy level *differences,* this divergence has no physical significance. The change in the energy level spacing is

$$\Delta E_n-\Delta E_{n-1}=\frac{K^*(\omega_0)}{2\omega_0}\ , \tag{2.18}$$

which naturally coincides with the expression of the complex frequency shift of the classical oscillator.

2. The case of friction proportional to velocity

We consider the case when $\hat{K}$ is of the form $\lambda d/dt$ which yields $K(\omega)=i\lambda\omega$. The quantity L_n is then purely imaginary and G_n is now

$$G_n=-\frac{\lambda}{\pi}\sum_m M_{nm}\omega_{nm}\int_0^\infty \frac{d\omega}{|\omega_{nm}|+\omega}\ . \tag{2.19}$$

This term is divergent at $\omega\to\infty$ and to proceed, we introduce a cutoff frequency ω_c to the upper bound of the integral. We then find

$$G_n=-\frac{\lambda}{\pi}\sum_m M_{nm}\omega_{nm}(\ln\omega_c-\ln|\omega_{nm}|)\ . \tag{2.20}$$

Using the fact that $|n\rangle$ is a nondegenerate bound state, one easily shows that

$$\sum_m M_{nm}\omega_{nm}=\langle n|[H_0,X]X|n\rangle=\frac{i}{2}\langle n|[P,X]|n\rangle=\tfrac{1}{2}\ , \tag{2.21}$$

and hence,

$$G_n=-\frac{\lambda}{2\pi}\ln\omega_c+\frac{\lambda}{\pi}\sum_m M_{nm}\omega_{nm}\ln|\omega_{nm}|\ . \tag{2.22}$$

The term containing the cutoff frequency in this expression is independent of n and thus does not contribute to a shift in the energy-level differences.

III. THE QUANTIZED ENERGY LEVELS OF THE CURRENT-BIASED JOSEPHSON JUNCTION

At low temperature, a Josephson junction of small dimensions has only one degree of freedom, the phase difference δ across the junction. This phase difference is a macroscopic variable involving the correlated motion of a large number of Cooper pairs. The junction can be represented as a particle of coordinate $X=C(\Phi_0/2\pi)^2\delta$ ($\Phi_0=h/2e$ and C is the junction capacitance) moving in a tilted potential, the tilt being proportional to the steady current flowing through the junction. The mass of the particle can be taken as unity and the coupling to the environment is due to the circuit of admittance $Y(\omega)$ in parallel with the junction (see Fig. 1). In the classical regime, X obeys an equation of type (2.1) with[13]

$$K(\omega)=\frac{i\omega}{C}Y(\omega) \tag{3.1}$$

and

$$V(X)=-I_0\frac{\Phi_0}{2\pi}\left[\cos\left[\frac{X}{C(\Phi_0/2\pi)^2}\right]+\frac{I}{I_0}\frac{X}{C(\Phi_0/2\pi)^2}\right]\ . \tag{3.2}$$

We consider the case where the particle is trapped in one of the metastable minima of $V(X)$ (zero-voltage state).

For I slightly under the critical current I_0, the potential well is very well approximated in the vicinity of each one of its minima by a cubic potential $U(X)$ (see Fig. 2). One can choose as independent parameters of the well, the barrier height ΔU and the frequency ω_P of oscillations at the bottom of the well when there is no dissipation, i.e., when $Y(\omega)=0$. The ratio $\Delta U/\hbar\omega_p$ roughly indicates the number of levels in the well.

The existence of these quantum levels has been demonstrated by applying an external microwave excitation to the junction. When the microwave frequency coincides with a transition frequency between levels, the population in the upper level increases, thereby enhancing the escape rate of the particle out of the metastable well. This corresponds to the switching of the junction to the nonzero-

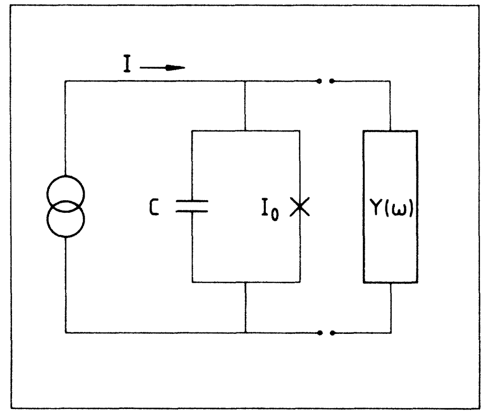

FIG. 1. A Josephson junction can be represented as an ideal junction of critical current I_0 parallel with a capacitance C. We consider a junction biased with a current source I. A linear dissipative circuit of admittance $Y(\omega)$ can be added in parallel with the junction.

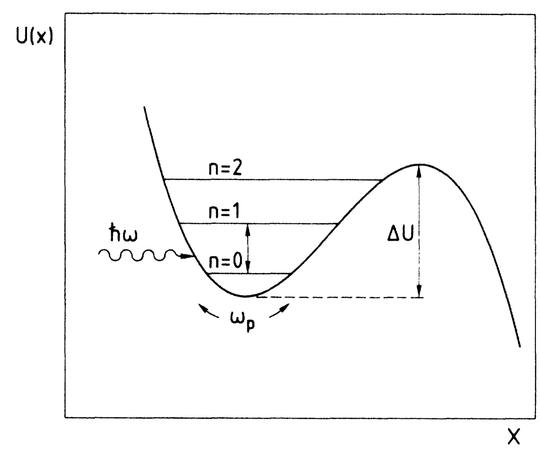

FIG. 2. Cubic potential $U(x)$ involved in the motion of the particle representing the junction in its zero voltage state when it is biased slightly below the critical current I_0. Here, ΔU is the barrier height and ω_p is the frequency of the small oscillations of the particle at the bottom of the well. Transitions between quantized energy levels in this metastable well can be induced by applying a microwave excitation at the transition frequencies.

voltage state which can be detected easily.

One therefore obtains resonances whose positions give energy differences between levels and whose widths give the lifetimes of the excited states involved in the transition. These lifetimes are dominated by the influence of friction which forces energy states to decay towards lower-energy states; the influence of tunneling through the barrier can be neglected.

We have evaluated the magnitude of the energy-level shifts G_0 and G_1 of the ground and first excited states for an experimentally relevant value of the well parameter $\Delta U/\hbar\omega_p$, where there are two bound states in the well. This corresponds to Fig. 3 of Ref. 1. The dissipative circuit in parallel with the junction is taken as a pure resistor, giving a quality factor $Q=RC\omega_p$ to the classical oscillations in the well.

The matrix elements and energies needed in Eqs. (2.13) and (2.22) were calculated by solving numerically the eigenstates of the cubic potential. A basis consisting of the first 30 eigenfunctions of the harmonic oscillator was used. In this basis, the matrix elements of the Hamiltonian for the cubic potential are known exactly and the eigenstates can be found using a simple diagonalization routine. This procedure gives a good precision for the quasi bound states in the well and their energies. For example, the sum rule of Eq. (2.21) is verified with 2% accuracy. The continuum of states outside the well, however, is sampled by a relatively small number of discrete states. This approximation is sufficient to compute the order of magnitude of G_n, which turns out to be only a small correction.

We thus obtained the following estimate for the complex shift $\Delta\omega_{10}=\Delta\omega'_{10}+i\Delta\omega''_{10}$ of the transition frequency between the ground and first excited states $\omega_{10}=0.84\omega_P$:

$$\Delta\omega'_{10}=(-0.09\pm0.05)\frac{\omega_p}{Q} , \tag{3.3}$$

$$\Delta\omega''_{10}=-\tfrac{1}{2}\tau_1^{-1}=-\tfrac{1}{2}(1.08\pm0.05)\frac{\omega_p}{Q} . \tag{3.4}$$

The results are valid as long as a perturbative approach can be used, which is when $Q \gg 1$. They show that (i) the linewidth of the transition between ground and first excited states is approximately given by $1/Q$, and (ii) the global term G_n induces a shift of the transition frequency which is much smaller than the linewidth. These results support the interpretation of the resonance experiment of Ref. 1 which neglected any frequency shift due to the resistive part of the admittance.

Thus, resonant escape experiments in a current-biased Josephson junction essentially probe the admittance of the circuit in parallel with the junction at the transition frequencies between levels. In the next section, we will see that quantum tunneling is affected in a very different way by the admittance.

IV. THE INFLUENCE OF AN ARBITRARY ADMITTANCE ON THE QUANTUM TUNNELING OF A JOSEPHSON JUNCTION

We begin with a result derived by Leggett[13] which expresses the effect of an arbitrary linear dissipating mechanism on the tunneling of a quantum variable. The tunneling rate Γ can be written as

$$\Gamma=A\exp\left[-\frac{B_0+\Delta B}{\hbar}\right] , \tag{4.1}$$

where B_0 is the part of the tunneling exponent which does not depend on the dissipation ($B_0=\frac{36}{5}\Delta U/\omega_p$ for the particle in the cubic potential of Sec. III) and where ΔB is the modification due to friction. Following the perturbative approach of Caldeira and Leggett, one finds for the tunneling of the variable X of the preceding section

$$\Delta B=\frac{1}{2\pi C}\int_0^\infty Y(-i\omega)\omega\,|\tilde{X}_B(\omega)|^2 d\omega , \tag{4.2}$$

where $X_B(\omega)$ is the Fourier transform of the "bounce" trajectory $X_B(t)$, that is the zero-energy solution of

$$\ddot{X}_B-\frac{\partial}{\partial x}U(X_B)=0 . \tag{4.3}$$

For the case of the cubic potential, this trajectory is given by

$$X_B(t)=\frac{(\frac{27}{2}\Delta U)^{1/2}}{\omega_p}\frac{1}{\cosh^2(\omega_p t/2)} . \tag{4.4}$$

Taking the Fourier transform of $X_B(t)$, we arrive at

$$\Delta B=\frac{108}{\pi^3}\frac{\Delta U}{C\omega_p^2}\int_0^\infty Y\left[\frac{-i\omega_p u}{\pi}\right]\frac{u^3 du}{\sinh^2 u} . \tag{4.5}$$

Therefore, ΔB is affected by the value of the admittance at *imaginary negative frequencies* $-i\omega_p u/\pi$ with u extending over a broad range given by the kernel $u^3/\sinh^2 u$. It is, in fact, more convenient to express ΔB in terms of the temporal response $y(t)$ of the admittance defined as

$$y(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}Y(\omega)e^{i\omega t}d\omega . \tag{4.6}$$

The response $y(t)$ can either be measured directly or calculated from the measured $Y(\omega)$. One thus obtains

$$\Delta B=\frac{162}{\pi^3}\frac{\Delta U}{C\omega_p^2}\int_0^\infty y(t)w\left[\frac{\omega_p t}{2\pi}\right]dt , \tag{4.7}$$

with

$$w(x)=\sum_{n=1}^{\infty}\frac{n}{(n+x)^4}=\zeta(3,x)-x\zeta(4,x) . \tag{4.8}$$

The function $\zeta(n,x)$ is the Rieman function.[14] Expression (4.7) is more useful than (4.5) when one is trying to grasp intuitively what influence a given admittance will have on tunneling.

The weight function $w(\omega_p t/2\pi)$ is a monotonic function which decreases for long times as $(\omega_p t/2\pi)^{-2}/6$ (see Fig. 3). The time scale of this weight function is $\tau_T=(\omega_p/2\pi)^{-1}$ and can be interpreted as the time scale of the tunneling events. There are two cases when expression (4.7) gives simple results: this is when $y(t)$ has a characteristic time τ either much shorter or much greater than τ_T.

(i) $\tau \ll \tau_T$ (admittance with short response time). In that case, the integral in Eq. (4.7) will be dominated by the first two moments of $y(t)$,

$$\Delta B \approx \frac{162}{\pi^3}\frac{\Delta U}{C\omega_p^2}\left[w(0)\int_0^\infty y(t)dt + w'(0)\int_0^\infty t y(t)dt\right] . \quad (4.9)$$

Using

$$\int_0^\infty t^n y(t)dt = i^n \left.\frac{d^n Y(\omega)}{d\omega^n}\right|_{\omega=0} , \quad (4.10)$$

one can rewrite (4.9) as

$$\Delta B \approx \frac{162}{\pi^3}\zeta(3)\frac{\Delta U}{C\omega_p^2}Y(0) + \frac{18}{5}\frac{\Delta U}{C\omega_p}\frac{1}{i}Y'(0) . \quad (4.11)$$

One can interpret $Y(0)$ and $Y'(0)$ in terms of an effective resistance and capacitance

$$Y(0) = \frac{1}{R_Y} = \frac{C\omega_p}{Q_Y} , \quad (4.12)$$

$$Y'(0) = iC_Y . \quad (4.13)$$

The first term of the right-hand side of Eq. (4.11) can easily be recognized as the result of Caldeira and Leggett for a pure resistor,[8] while the second term is just a renormalization of the capacitance of the junction by the effective capacitance of the admittance

$$\frac{\Delta B}{B_0} \approx \frac{45}{2\pi^3}\zeta(3)\frac{1}{Q_Y} + \frac{1}{2}\frac{C_Y}{C} . \quad (4.14)$$

(ii) $\tau \gg \tau_T$ (admittance with long response time). In this case, the integral in Eq. (4.7) will be dominated by the $t=0$ value of the function $y(t)$,

$$\Delta B \approx \frac{162}{\pi^3}\frac{\Delta U}{C\omega_p^2}Y(0)\int_0^\infty w\left[\frac{\omega_p}{2\pi}t\right]dt . \quad (4.15)$$

Using

$$y(0) = \lim_{\omega\to\infty} i\omega Y(\omega) , \quad (4.16)$$

and interpreting the limit of $i\omega Y(\omega)$ as the inverse of an effective inductance L_Y, one can rewrite (4.15) as

$$\frac{\Delta B}{B_0} \approx \frac{5}{2}\frac{L}{L_Y} , \quad (4.17)$$

where L is the effective inductance of the junction given by $L=(C\omega_p^2)^{-1}$.

This last formula shows that a bias circuitry with resonances well below the plasma frequency can still drastically affect quantum tunneling rates.

Finally, there is another case which lends itself to a simple analysis: it is the case of an ideal transmission line of length l large compared to its wavelength at the plasma frequency, terminated on an impedance Z_0.

When Z_0 is a pure resistor, $Y(\omega)$ is a periodic function of ω. The response function thus consists of δ functions,

$$y(t) = \sum_0^\infty a_n \delta(t - n\tau_l) , \quad (4.18)$$

where $\tau_l = 2l/v$, v being the line wave velocity and where a_n is given by

$$a_0 = \frac{1}{Z_c} \quad (n=0) , \quad (4.19)$$

$$a_n = \frac{2}{Z_c}\left[\frac{Z_c - Z_0}{Z_c + Z_0}\right]^n \quad (n \geq 1) , \quad (4.20)$$

where Z_c is the characteristic impedance of the transmission line.

We find

$$\Delta B = \frac{162}{\pi^3}\frac{\Delta U}{C\omega_p^2}\sum_n a_n w\left[n\frac{\omega_p}{2\pi}\tau_l\right] , \quad (4.21)$$

which, in the case $\tau_l \gg \tau_T$, reduces to

$$\Delta B = \frac{162}{\pi^3}\zeta(3)\frac{\Delta U}{C\omega_p^2}\frac{1}{Z_c}\left[1 + O\left[\frac{1}{\omega_p\tau_l}\right]^2\right] . \quad (4.22)$$

Thus, for long transmission lines, ΔB is independent of l and of Z_0, and is the same as obtained for a pure resistor of admittance $Y = Z_c^{-1}$.[15] One easily shows that this result still holds when Z_0 depends on frequency, since the response function $y(t)$, is, in that case, the sum of $\delta(t)/Z_c$ and of a function which is equal to zero for $t < \tau_l$.

We see that by biasing the junction through a transmission line of characteristic impedance Z_c much greater than the junction impedance $(C\omega_p)^{-1}$, the influence of the biasing circuitry can be neglected as far as tunneling is concerned. In contrast, the effect of the line on the energy levels would be a periodic function of l^{-1} and would depend on Z_0. This difference in behavior is a remarkable property of friction in the quantum regime.

V. DISCUSSION

We can now discuss the relationship between the modification of the tunneling exponent and the width and shift of an absorption resonance. In Fig. 3, we show for comparison the weight functions entering in the integration of the temporal response of the admittance to get either the tunneling exponent [Fig. 3(a)] or the width of the first excited state [Fig. 3(b)]. The tunneling weight function decreases monotonically towards zero as $t \to \infty$, whereas the level-width weight function is a simple cosine that keeps a constant amplitude as $t \to \infty$. The qualitative differences between the two functions is maintained if one considers higher excited states. For these higher states, the level-width weight function would be a sum of cosines with different frequencies. Note that the weight function for the shift of a level is also an oscillatory function which is, for the most part, a sum of sines instead of cosines.

The time scale of the variations of the weight functions for tunneling and level broadening is in all cases the same and of the order of $2\pi/\omega_p$. If the temporal response of the admittance decays rapidly on this time scale, then only the value at $t=0$ of these two weight functions will affect the physical phenomena and the time evolution of the functions will be not revealed. Neglecting the contribution of the small global term to the shift of the transition frequency, one obtains, using expressions (2.13) and

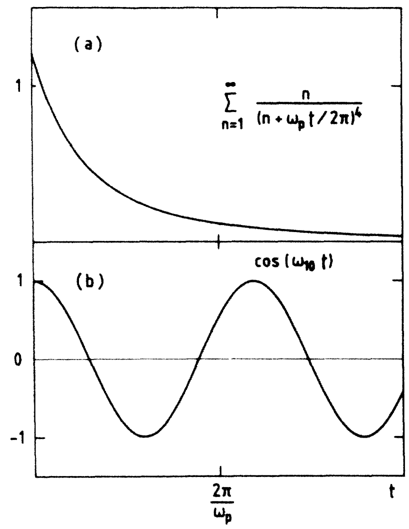

FIG. 3. The modification ΔB of the exponent of the tunneling rate and the energy level width $\mathrm{Im}\Delta E$ of a junction are obtained by integrating the temporal response $y(t)$ of the admittance with a weighting function shown, respectively, in (a) and (b). Panel (b) pertains, in particular, to the level width of the first excited state when its transition frequency to the ground state is $\omega_{10}=0.84\omega_p$.

(4.14), a simple relation between the modification of tunneling and the frequency shift $\Delta\omega'_{10}$ and linewidth $\Delta\omega''_{10}$,

$$\frac{\Delta B}{B_0} \approx -\alpha\frac{\Delta\omega'_{10}}{\omega_{10}} + \beta\frac{|\Delta\omega''_{10}|}{\omega_{10}} , \tag{5.1}$$

where α and β are coefficients of the order of unity. Their values for the particular example $\Delta U/(\hbar\omega_p)=1.4$ are $\alpha=1.1$ and $\beta=1.36$. If, on the other hand, the temporal response of the admittance decays only slowly on the time scale $2\pi/\omega_p$, then the weight functions will select different features of that temporal response. We will have

$$\Delta B \propto \lim_{\omega\to\infty} i\omega Y(\omega) , \tag{5.2}$$

$$\Delta\omega_{10} \propto Y(\omega_{10}) . \tag{5.3}$$

We see that in this case, there is no longer a simple relationship between tunneling and level shift and broadening, which are affected in two independent ways by the admittance.

A solvable case that belongs to neither the short- nor the long-response-time case is when the admittance consists of a simple transmission line with characteristic impedance Z_c terminated by a resistor Z_0. When the length l of the line is long compared with the wavelength λ at ω_p, one gets the simple result

$$\frac{\Delta B}{B_0} = \beta\frac{Z_0}{Z_c}\frac{|\Delta\omega''_{10}|}{\omega_{10}} \quad \text{when } l=n\lambda \ (n \text{ is an integer}) , \tag{5.4}$$

$$\frac{\Delta B}{B_0} = \beta\frac{Z_c}{Z_0}\frac{|\Delta\omega''_{10}|}{\omega_{10}} \quad \text{when } l=(n+\tfrac{1}{2})\lambda . \tag{5.5}$$

By choosing properly the values of l, Z_c, and Z_0, one can have the transmission line affecting predominantly tunneling rather than the absorption resonance or vice versa. The observation of these phenomena would constitute an interesting demonstration of the somewhat nonintuitive aspects of friction in the quantum regime.

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[15]As this work was completed, we received a copy of work by W. Bialek, S. Kivelson, and S. Chakravarty prior to its publication and a paper by S. Chakravarty and A. Schmid, Phys. Rev. B **33**, 2000 (1986). These authors approach the problems presented in this paper differently but get similar results concerning the smallness of the global term G_L and the effect of a transmission line on tunneling.