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Impact ionization mechanism for self-generated chaos in semiconductors

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A novel physical mechanism is proposed as an explanation of intrinsic self-generated chaotic oscillations in semiconductors under static external conditions. It is based upon impact ionization from at least two impurity levels, and includes trapping and dielectric relaxation. Conditions for an oscillatory instability are derived, singling out high-purity relaxation semiconductors with low differential mobility. A period-doubling route to chaos and a strange attractor of spiral type are found.

Chaotic oscillations in high-purity Ge (Refs. 1–4), GaAs (Refs. 5 and 6), and InSb (Ref. 7) have recently been discovered under a wide variety of experimental conditions, ranging from low temperatures[1–5,7] to room temperature,[6] and including weak infrared[1,7] or visible[5] irradiation as well as complete shielding against external irradiation,[3] and, in some cases, parallel[2] or transverse[3,7] magnetic fields.

Upon variation of the applied bias, taken as a control parameter, different routes to chaos were observed: the period-doubling (Feigenbaum) scenario,[1–3,5,7] quasiperiodic (Ruelle-Takens-Newhouse) breakdown[2,3,5] and intermittent switching (Pomeau-Manneville) between two oscillatory states.[3,6] The oscillation frequencies were typically quite low, between a few Hz and several kHz.

The physical mechanism of these chaotic oscillations is—with the exception of the helical instability[2]—not well understood, although there is strong evidence that impact ionization from impurity levels is involved in a majority of these experiments.[1,3,5–7] The onset of chaos occurred either just below[1,5,6] or above[3] the threshold field for impurity breakdown. The similarity of the mechanism in various materials is indicated by the observation of an empirical scaling law[6] between the impurity level energy and the breakdown field, which ranged from a few V/cm for shallow donor or acceptor states at helium temperatures to several kV/cm for deep levels in semi-insulating GaAs at room temperature.

The experiments can be divided into two classes: (i) driven chaos,[5,7] which is induced by periodically chopped external radiation or ac-modulated external currents or pulsed voltage, (ii) self-generated chaos,[1,3,4,6,7] which is observed under static applied electric fields and time-independent, if any, irradiation, and is widely independent of external circuit conditions. A theoretical model for the first class of phenomena was recently proposed by Teitsworth and Westervelt (TW),[8] using an ac-modulated driving current in combination with impact ionization from a single acceptor level, but the second, larger class of experiments has so far not been explained theoretically.

In this Rapid Communication I present a novel model which can account for self-generated chaotic oscillations in semiconductors, as in class (ii). It is based upon impact ionization and trapping at impurities, coupled with dielectric relaxation. The essential innovation is that it explicitly includes the excited state(s) of the trapped carriers in the generation-recombination kinetics, in addition to the ground state.

Analytical conditions for the occurrence of self-oscillatory instabilities in a general model with two impurity levels are derived for what is believed to be the first time, thus restricting the range of material parameters for which chaotic oscillations can be expected. This could be of potential importance for the design of far-infrared photodetectors and integrated electronic circuits where oscillatory instabilities and broad-band noise are detrimental. For the simplest two-level model the route to chaos is investigated numerically with typical parameters. This is to my knowledge the first consistent theoretical explanation of impact-ionization-induced self-generated chaos in semiconductors, as observed by several groups in different materials and under widely varied experimental conditions.[1,3,6,7]

For convenience, all formulas below will be given for an n-type semiconductor, although the model can easily be applied to p-type material by making the appropriate replacements. The temperature is assumed to be sufficiently low that the impurities are not thermally ionized. This implies helium temperatures and breakdown fields of only a few V/cm in the case of shallow donors or acceptors, e.g., in n-type InSb (Ref. 7), n-type GaAs (Ref. 5), and p-type Ge (Refs. 1–3), or room temperatures and breakdown fields $\gtrsim 1$ kV/cm in the case of deep traps, e.g., in semi-insulating GaAs (Ref. 6). In the following we normalize all concentrations by the effective doping density $N_D^* \equiv N_D - N_A$, all lengths by the effective Debye length $L_D \equiv (D_0 \tau_M)^{1/2}$, the time by the effective dielectric relaxation time $\tau_M \equiv \epsilon_s/(4\pi e \mu_0 N_D^*)$, and the electric field by kT/eL_D, where N_D, N_A are the donor and compensating acceptor concentrations, respectively, μ_0 and D_0 are the low-field mobility and the diffusion constant, and ϵ_s is the static dielectric constant.

The basic model assumption is that the electrons can be bound at an impurity center in M different energy levels, corresponding to ground and excited states. The nonlinear generation-recombination (GR) and transport processes are governed by the continuity equations for the electrons in the conduction band (of density ν) and in the M impurity levels (densities ν_i, $i = 1, \ldots, M$), and by Poisson's equation:

$$\dot{\nu} - \nabla \cdot \mathbf{j} = \phi_0(\nu, \nu_1, \ldots, \nu_M, \boldsymbol{\varepsilon}) ,$$
$$\dot{\nu}_i = \phi_i(\nu, \nu_1, \ldots, \nu_M, \boldsymbol{\varepsilon}) ,$$
$$\nabla \cdot \boldsymbol{\varepsilon} = 1 - \nu - \sum_i \nu_i . \tag{1}$$

Here $\boldsymbol{\varepsilon}$ is the electric field, $\mathbf{j} = \nu \mathbf{v}(\boldsymbol{\varepsilon}) + D(\boldsymbol{\varepsilon}) \nabla \nu$ is the current density with dimensionless field-dependent drift velocity $\mathbf{v}$ and diffusion constant D, and

$$\phi_i \equiv \sum_j B_{ij}(\nu,\varepsilon)\nu_j + c_i(\varepsilon)\nu, \quad \phi_0 \equiv -\sum_i \phi_i$$

are the GR rates. The functions B_{ij} and c_i are specified by the GR mechanism which is effective in a particular material; they depend upon the magnitude of the electric field through the GR coefficients, in particular the impact ionization coefficients. The B term, which depends upon ν through trapping and impact ionization, provides the crucial nonlinearities. The simplest GR mechanism of this type, which employs two donor levels and impact ionization from both levels, has been advanced previously[9] to explain S-type negative differential conductivity (SNDC) and current filamentation at low temperatures.

Self-generated chaos is generally preceded by an oscillatory instability of the steady state. Conditions for this can be derived from the response of the system (1) to small longitudinal fluctuations proportional to $\exp(ikx+\lambda t)$ of the field ε and the carrier concentrations $\nu, \nu_1, \ldots, \nu_M$. The resulting dispersion relation $\lambda(k)$ contains a complexity of possible diffusion-driven, mobility-driven, and GR-induced instabilities, and has been solved for special cases elsewhere.[9,10] The simplest oscillatory instability, a Hopf bifurcation[11] of a spatially homogeneous limit cycle oscillation, occurs if two complex conjugate eigenvalues $\lambda(k=0)$ cross the imaginary axis, from Re $\lambda<0$ to Re $\lambda>0$, as the control parameter (for instance, the external static current density J or field ε_0) is varied. For a two-level model, the condition for a Hopf bifurcation at ε_0 becomes

$$v(\varepsilon_0)[(\tilde{\nu}-A_{11}-A_{21})\partial\phi_1/\partial\varepsilon + (\tilde{\nu}-A_{22}-A_{12})\partial\phi_2/\partial\varepsilon] + (\lambda_1+\lambda_2)(\tilde{\nu}-\lambda_1)(\tilde{\nu}-\lambda_2)=0 \ , \quad (2)$$

where $\tilde{\nu}\equiv \nu\, dv/d\varepsilon$, and λ_1, λ_2 are the eigenvalues of the 2×2 GR matrix

$$A_{ij} \equiv B_{ij} - \sum_k (\partial B_{ik}/\partial \nu)\nu_k - c_i \ ,$$

all evaluated at the steady state. For positive differential mobility and standard GR kinetics, impact ionization coefficients monotonically increasing with field, and trapping cross sections nonincreasing with field, a necessary condition following from (2) is $\tilde{\nu}<\lambda_1$. This requires GR induced SNDC (whence $\lambda_1>0>\lambda_2$),[9] a low electron concentration ν on the NDC branch of the static current-field characteristics, and a small differential mobility $dv/d\varepsilon$. Thus, self-sustained oscillations and chaos are to be expected in high-purity relaxation semiconductors, and the oscillations occur on a very slow time scale, which is in agreement with the experiments.[1,3-7] The angular frequency of oscillation near the Hopf bifurcation is given by

$$\mathrm{Im}\lambda = [\det A - \tilde{\nu}(A_{11}+A_{22}) + v(\varepsilon_0)\partial\phi_0/\partial\varepsilon]^{1/2} \ .$$

For the simplest two-level GR mechanism[9] and a choice of typical numerical parameters, the evolution of the limit-cycle oscillations in the nonlinear regime beyond the Hopf bifurcation point ε_0^H, given by (2), has been investigated. Assuming spatial homogeneity and a time-independent external current density J, Eqs. (1) can be reduced to a set of three nonlinear ordinary differential equations:

$$\dot{\varepsilon} = J - \nu v(\varepsilon) \ ,$$

$$\dot{\nu} = X_1\nu\nu_1 + (X_1^S + X_1^*\nu)\nu_2 - T_1^S(N_A/N_D^* + \nu)\nu \ ,$$

$$\dot{\nu}_1 = -(X^* + X_1\nu)\nu_1 + T^*\nu_2 \ ,$$

with $\nu_2 = 1-\nu-\nu_1$. These have been solved numerically for a series of increasing fields $\varepsilon_0 > \varepsilon_0^H = 98$ (Fig. 1). The drift velocity was modeled by the empirical saturable form[12] $v(\varepsilon)=\arctan(0.3\varepsilon)/0.3$. The (dimensionless) impact ionization coefficients for ground- and excited-state ionization were approximated by the Shockley formula[13] $X_1 = 5\times 10^{-4}\exp(-6E_t/\varepsilon)$ and $X_1^* = 10^{-2}\exp(-1.5E_t/\varepsilon)$, with a normalized impurity ground-state energy $E_t=1$. The trapping coefficients were $T_1^S=10^{-2}$, $T^*=10^{-5}$, the generation coefficients $X_1^S = X^* = 10^{-7}$, and the compensation was $N_A/N_D^* = 0.3$.

The time series $\nu(t)$ and the phase portraits of ν versus ε in Fig. 1 exhibit a period-doubling route to chaos. As ε_0 is increased, the amplitude of the limit cycle grows, and oscillations of period two [Fig. 1(b)], four [Fig. 1(c)], eight [Fig.

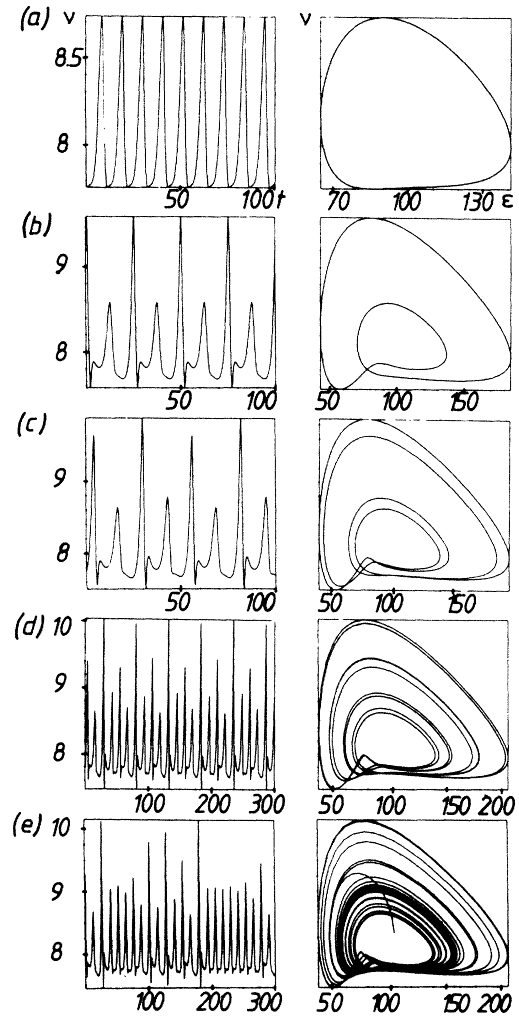

FIG. 1. Carrier concentration ν in units of $10^{-3}N_D^*$ vs time in units of $10^4\,\tau_M$ (left column) and phase portraits of ν vs field ε in units $(kTE_t)/(eL_D)$ (right column) for the following steady-state fields ε_0: (a) 102, (b) 105, (c) 105.3, (d) 105.42, and (e) 105.5.

1(d)], and chaotic oscillations [Fig. 1(e)] are successively displayed. Corresponding power spectra of $\varepsilon(t)$ are shown in Fig. 2. For a limit cycle of period one, the spectrum is sharply peaked at the intrinsic oscillation frequency $\omega_0 = 2\pi f_0 = 0.57\times 10^{-4}/\tau_M$ [Fig. 2(a)], while in the chaotic regime a high level of broad-band noise is present [Fig. 2(b)]. In Fig. 3 the local field maxima ε_n are plotted versus the control parameter ε_0. The Feigenbaum period-doubling cascade, as well as chaotic bands at $\varepsilon_0 > 105.43$ and noise-free windows of period six at $\varepsilon_0 = 105.475$, and of period five at $\varepsilon_0 = 105.575$ can be seen. The inset shows the Poincare return map of ε_{n+1} vs ε_n reconstructed from successive maxima of $\varepsilon(t)$ at $\varepsilon_0 = 105.5$. It is strongly reminiscent of the one-dimensional iterated maps studied in the theory of discrete dynamic systems.[14,15]

The physical origin of the obtained chaotic dielectric relaxation oscillations can be understood as follows: Injected charge is trapped, which increases the electric field. This enhances impact ionization of the trapped charge, which creates more free carriers and leads to a reduced field due to increased dielectric relaxation. Hereby the trapping rate becomes dominant over the ionization rate, which completes the cycle. The fact that the present model gives chaos for dc conditions, as opposed to the TW model,[8] which gives chaos under ac drive only, is due to an essential difference in the underlying physical mechanisms. In the absence of an ac drive the TW model does not exhibit limit cycles or chaos, but only *damped* dielectric relaxation oscillations. Chaos is generated by the coupling of this intrinsic frequency with an appropriate driving frequency, similar to a driven Van der Pol oscillator. In the present model the mixing of orbits which is necessary on a chaotic attractor is provided by the redistribution of trapped carriers between the ground and the excited state due to the competing impact ionization of these two levels. This causes an *intrinsic instability* of the dielectric relaxation oscillations, which is absent in the TW model. The additional internal degree of freedom of the trap ground-state occupancy furnishes the third dynamic variable which is required in autonomous chaotic systems. The strange attractor is characterized by the *displaced reinjection* of phase trajectories in three-dimensional phase space (ε, ν, ν_1) onto the bistable ("rippled") slow submanifold of dielectric relaxation oscillations. This represents a novel physical example of "spiral-type" chaos.[16] As a further difference to the TW model, it suffices to use the simple standard Shockley formula for the impact ionization coefficients, while the TW mechanism requires a more elaborate, nonmonotonic dependence of the coefficient upon the field, which has not yet been corroborated.[17]

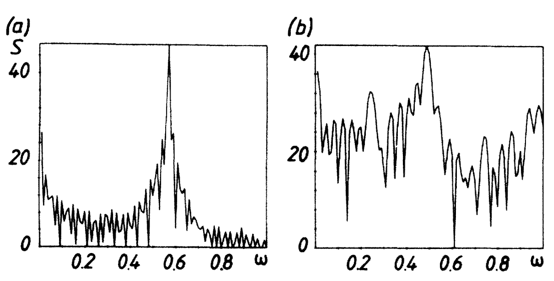

FIG. 2. Power spectra $S(\omega)$ of $\varepsilon(t)$ for (a) $\varepsilon_0 = 103$ (limit cycle of period one) and (b) $\varepsilon_0 = 105.5$ (chaos). The power S is in *dB*, and the angular frequency is in units of $10^{-4}\tau_M^{-1}$.

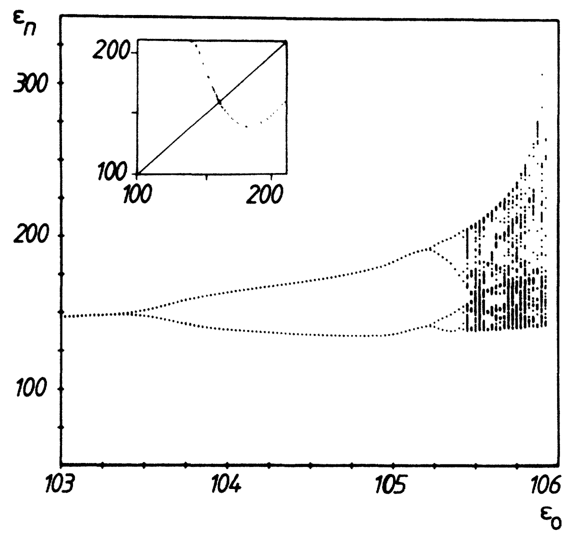

FIG. 3. Bifurcation diagram of the field maxima ε_n vs the control parameter ε_0. The inset shows the return map ε_{n+1} vs ε_n, where $\{\varepsilon_n\}_{n\in \mathrm{IN}}$ is the set of successive field maxima, for $\varepsilon_0 = 105.5$.

The numerical parameters chosen are representative of high-purity materials at low temperatures. Inspection of Figs. 1–3 reveals a close similarity with experimental diagrams.[1–7] In the figures presented, all times, fields, and concentrations can be scaled by varying the temperature, the impurity energy, and the doping density. For *p*-type Ge at 4.2 K, with $\mu_0 = 10^6$ cm^2/Vs, $\epsilon_s = 16$, effective doping $\sim 10^{11}$ cm^{-3}, and an acceptor level at 10 meV, for example, the physical units in Figs. 1–3 are $\tau_M \sim 10^{-4}$ μs, $(kTE_t)/(eL_D) \sim 0.2$ V/cm, while for semi-insulating GaAs at 300 K with $\mu_0 = 10^4$ cm^2/Vs, $\epsilon_s = 12.5$, $N_D^* = 10^{10}$ cm^{-3}, and a trap level at 700 meV, $\tau_M \sim 10^{-4}$ ms, $(kTE_t)/(eL_D) \sim 10$ V/cm.

The condition for an oscillatory instability (2) requires a delicate balance between the differential mobility, the GR time constants, and the carrier density, which can be sensitively controlled by the temperature, by optical radiation, or by magnetic freeze-out due to a small transverse magnetic field. This elucidates the role of such additional control parameters in some of the experiments.[1,3,5–7]

The longitudinal oscillatory instability may be coupled with a transverse filamentary instability[9] if the transverse dimension of the SNDC element is sufficiently large. Since the dielectric relaxation occurs on a very slow timescale, as inferred from condition (2), whereas the current filamentation is governed by the faster generation-recombination processes coupled with transverse diffusion,[9] a quasistationary transverse filamentary profile may form for each instantaneous value of the electric field, being slaved by the slow dielectric relaxation oscillations of the field. This will result in an oscillatory "breathing" of the current filaments which shows up as slow periodic or chaotic current oscillations.

In conclusion, a simple rate-equation model with impact ionization from two impurity levels has been advanced as a novel explanation of self-generated chaos in semiconductors. Its validity could be experimentally tested by slightly increasing the temperature such that the excited level, but not the ground level, is thermally ionized. The understanding of these phenomena might be useful in guiding the tailoring of low-noise electronic devices.

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