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Magnetoplasmon-phonon coupling in a two-dimensional electron gas

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We have studied the coupling of longitudinal-optical (LO) phonons in a polar semiconductor to the collective density oscillations of a two-dimensional electron gas (2DEG) in a strong magnetic field. Both random-phase and time-dependent Hartree-Fock approximations for the electronic contribution to the dielectric function have been considered. The strong field plays an essential role since the magnetoplasmon resonance of the 2DEG occurs near the cyclotron frequency which, for GaAs, becomes comparable to the LO-phonon frequency at available magnetic fields. The coupling of magnetoroton modes [S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. Lett. **54**, 581 (1985)], which are relevant to the fractional quantum Hall effect, to LO phonons is also briefly discussed.

Electrons in the conduction band of polar semiconductors interact strongly with longitudinal-optical (LO) phonons. This interaction leads to a strong coupling between plasmons and phonons when their frequencies are comparable. For degenerate bulk polar semiconductors, where the phonon and long-wavelength plasmon frequencies are of the same order, this effect has been predicted theoretically[1] and confirmed experimentally.[2,3] In a recent Rapid Communication, Xiaoguang, Peeters, and Devreese[4] have calculated the plasmon-phonon coupling for a two-dimensional electron gas (2DEG) occurring at a GaAs/$Al_xGa_{1-x}As$ heterojunction using the usual random-phase approximation (RPA) for the electronic contribution to the dielectric function.[5] Unlike the three-dimensional case where the plasmon frequency at $q=0$ is finite, the 2D plasmon frequency[6] varies as $q^{1/2}$ so that in the long-wavelength limit, which is most accessible to experimental investigation,[7] the phonon and plasmon frequencies are never comparable. In this article we discuss the plasmon-phonon coupling when the 2DEG is placed in a strong perpendicular magnetic field. At long wavelengths the magnetoplasmon resonance frequency approaches the cyclotron frequency so that the magnetic field can be used to sweep the electronic collective-excitation frequencies through the LO-phonon frequency and achieve a strong coupling between the modes. We also discuss coupling of the electronic intra-Landau-level magnetoroton[8,9] modes of the 2DEG, which are relevant to the fractional quantum Hall effect,[10] to the LO phonons although it seems that the magnetoroton frequencies will always lie well below those of the LO phonon modes.

The collective-excitation frequencies of the coupled electron-phonon system are determined by the equation,

$$\mathrm{Re}\,\epsilon(q,\omega(q))=0\ , \tag{1}$$

where $\epsilon(q,\omega)$ is the total dielectric function:[5,11]

$$\epsilon(q,\omega)=1+(\epsilon_\infty-1)+\frac{\epsilon_0-\epsilon_\infty}{1-\omega^2/\omega_{\mathrm{TO}}^2}-\frac{2\pi e^2}{q}\Pi(q,\omega). \tag{2}$$

In Eq. (2) $\Pi(q,\omega)$ is the proper polarizability of the 2DEG and ϵ_∞ and ϵ_0 are the high- ($\omega\gg\omega_{\mathrm{TO}}$) and low- ($\omega\ll\omega_{\mathrm{TO}}$) frequency dielectric constants in the absence of electrons. The second, third, and fourth terms in Eq. (2) represent, respectively, the contributions to the screening from interband transitions in the semiconductor, from optical phonons, and from the mobile electrons of the 2DEG. [Note that in the absence of the electronic contribution $\epsilon(q,\omega_{LO})=0$, where $\omega_{\mathrm{LO}}^2=\omega_{\mathrm{TO}}^2(\epsilon_0/\epsilon_\infty)$.] We begin by considering the random-phase approximation[12] for which $\Pi(q,\omega)$ is given by[13]

$$\Pi(q,\omega)=\frac{2}{2\pi l^2\hbar}\sum_{n,n'}[1-n_F(\varepsilon_{n'})]n_F(\varepsilon_n)\,|F_{n',n}(q)|^2\left(\frac{1}{\omega-\omega_c(n'-n)+i\delta}-\frac{1}{\omega+\omega_c(n'-n)+i\delta}\right)\ , \tag{3}$$

where $l\equiv(\hbar c/eB)^{1/2}$ is the magnetic length, $\varepsilon_n=\hbar\omega_c(n+\frac{1}{2})$ ($n=0,1,2,\dots$) are the Landau-level energies and

$$F_{n',n}(q)=\left(\frac{n!}{(n')!}\right)^{1/2}\left[\frac{(iq_x-q_y)l}{\sqrt{2}}\right]^{n'-n}\exp\left(\frac{-q^2l^2}{4}\right)L_n^{n'-n}\left(\frac{q^2l^2}{2}\right)\ . \tag{4}$$

[In Eq. (4) $L_n^m(x)$ is the generalized Laguerre polynomial.[14]] To discuss the collective excitations qualitatively, we restrict our attention to the case where only the lowest Landau level is full, and include only the contribution to the polarizability from transitions between the $n=0$ and $n=1$ Landau levels. Then,

$$\Pi(q,\omega)\simeq\frac{2}{\pi l^2\hbar}\frac{(q^2l^2/2)\exp(-q^2l^2/2)\omega_c}{\omega^2-\omega_c^2}\ , \tag{5}$$

which, when inserted into Eqs. (1) and (2) yields,

$$\omega_\pm^2(q)=\left(\frac{\omega_{LO}^2+\omega_{mp}^2}{2}\right)\pm\left[\left(\frac{\omega_{LO}^2-\omega_{mp}^2}{2}\right)^2+(\omega_{LO}^2-\omega_{TO}^2)(\omega_{mp}^2-\omega_c^2)\right]^{1/2}, \tag{5a}$$

where ω_{mp} is the RPA magnetoplasmon frequency[15] calculated for a nonpolar lattice (i.e., $\omega_{LO}=\omega_{TO}$),

$$\omega_{mp}^2=\omega_c^2(1+A(q))=\omega_c^2+\omega_c\left(\frac{e^2}{\epsilon_\infty l\hbar}\right)2ql\exp(-q^2l^2/2) \tag{5b}$$

In the long-wavelength limit, Eq. (5a) has a very simple relation to the corresponding expression[4,16] for the resonant frequencies at $B=0$,

$$\omega_\pm^2(q)=\left(\frac{\omega_{LO}^2+\omega_p^2}{2}\right)\pm\left[\left(\frac{\omega_{LO}^2-\omega_p^2}{2}\right)+(\omega_{LO}^2-\omega_{TO}^2)\omega_p^2\right]^{1/2}, \tag{6a}$$

where ω_p is the plasmon frequency in a nonpolar lattice at $B=0$

$$\omega_p^2=2\pi ne^2q/\epsilon_\infty m\ . \tag{6b}$$

Since for $q\rightarrow 0$, as we can verify from Eq. (5b), the classical magnetoplasmon dispersion relation[17] $(\omega_{mp}^2-\omega_c^2=\omega_p^2)$ applies, it follows that the terms proportional to $\omega_{LO}^2-\omega_{TO}^2$ in Eqs. (5a) and (6a) which produce the coupling between phonon and electron modes become identical. The only effect of the magnetic field then is to increase the uncoupled electronic resonance frequency from ω_p to $\omega_{mp}=(\omega_c^2+\omega_p^2)^{1/2}$. Since for GaAs, $\omega_{LO}=36.7$ meV, $\omega_{TO}=33.8$ meV, and ω_c (in meV) $=1.70B$ (in T), the electronic and lattice resonance frequencies become comparable at available magnetic fields. Note that for $\omega\ll\omega_{TO}$ Eq. (5a) reduces to

$$\omega_-^2(q)=\omega_{mp}^2-(\omega_{mp}^2-\omega_c^2)\left(\frac{\omega_{LO}^2-\omega_{TO}^2}{\omega_{LO}^2}\right)=\omega_c^2+\omega_c\left(\frac{e^2}{\epsilon_0 l\hbar}\right)2ql\exp\left(\frac{-q^2l^2}{2}\right), \tag{7}$$

i.e., the interaction contribution to the lower-frequency magnetoplasmonlike resonance is identical to that in a nonpolar lattice except that it is to be calculated with the low-frequency dielectric constant ϵ_0 rather than ϵ_∞. The modes are most strongly coupled when $\omega_c=\omega_{LO}$ so that, at least as $q\rightarrow 0$, $\omega_{mp}^2=\omega_{LO}^2+\omega_p^2$. Using this in Eq. (5a) gives

$$\omega_\pm^2(q)=\omega_{LO}^2+\frac{\omega_p^2}{2}\pm\left[(\omega_{LO}^2-\omega_{TO}^2)\omega_p^2+\left(\frac{\omega_p^2}{2}\right)^2\right]^{1/2}\simeq\omega_{LO}^2\pm\omega_p(\omega_{LO}^2-\omega_{TO}^2)^{1/2}, \tag{8}$$

i.e., for both modes the dispersion at long wavelength shows a $q^{1/2}$ behavior which should be easy to observe.

In order to obtain accurate results for the coupled resonance frequencies at all wave vectors it is necessary to solve Eq. (1) numerically including all Landau levels in Eq. (3). The results of such a calculation are shown in Fig. 1. (For $q\rightarrow 0$ analytic results discussed above are

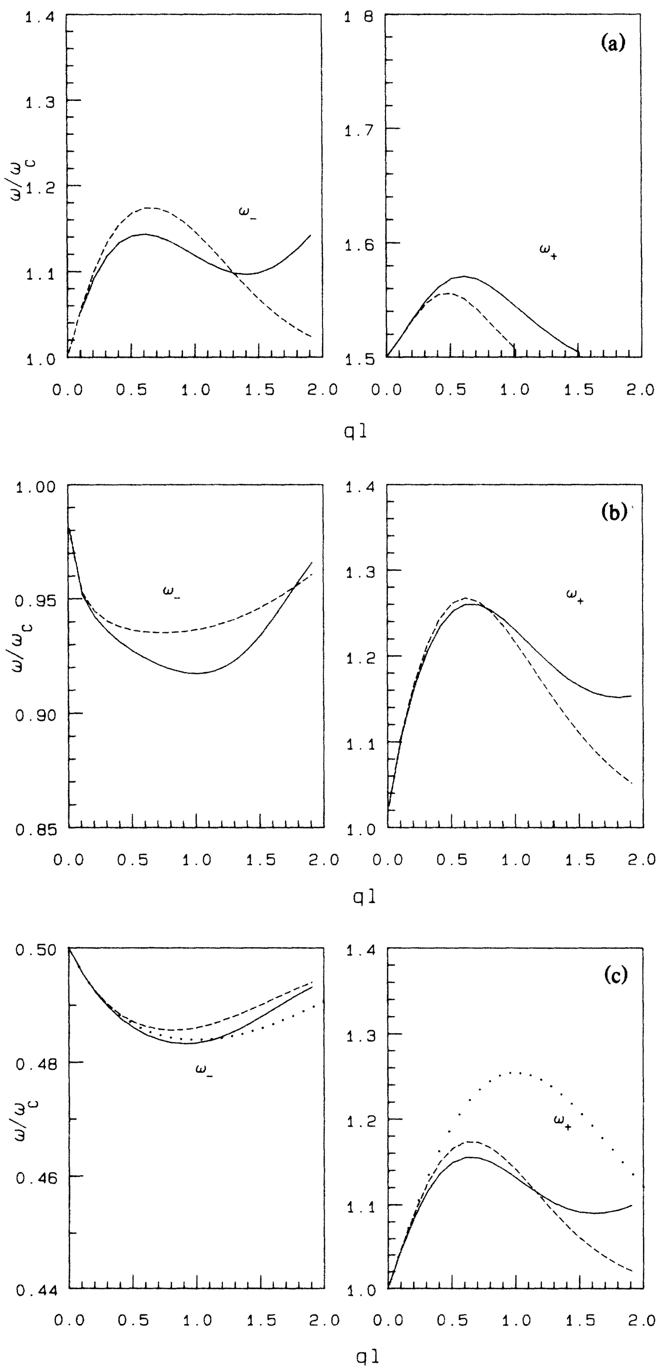

FIG. 1. Magnetoplasmon-phonon modes in the RPA (dashed curves) and in the HFA (full curves). (a) $\omega_c=\frac{2}{3}\omega_{LO}$, (b) $\omega_c=\omega_{LO}$, and (c) $\omega_c=2\omega_{LO}$. The dotted curves in (c) are the result obtained when transition between only $n=0$ and $n=1$ Landau levels are included [Eq. (5)].

recovered since $|F_{n,0}(q)|^2 \propto q^{2n}$.) Figures 1(a)–1(c) illustrate the cases where $\omega_c = \frac{2}{3}\omega_{LO}$ ($B \simeq 14.4$ T), $\omega_c = \omega_{LO}$ ($B \simeq 21.6$ T), and $\omega_c = 2\omega_{LO}$ ($B \simeq 43.2$ T), respectively. All calculations are for a full lowest Landau level[18] and so correspond to densities of 3.6×10^{11} cm^{-2}, 5×10^{11} cm^{-2}, and 1×10^{12} cm^{-2}, respectively.[19] For $\omega_c < \omega_{TO}$ the lower resonance is magnetoplasmonlike while for $\omega_c > \omega_{LO}$ the lower frequency is phononlike. For $\omega_{TO} < \omega_c < \omega_{LO}$ the modes are very strongly coupled with the $q^{1/2}$ behavior mentioned previously evident in Fig. 1(b). The solid lines in Figs. 1(a)–1(c) result from the time-dependent Hartree-Fock approximation[20] to $\Pi(q,\omega)$ which introduces substantial corrections to the RPA away from the long-wavelength limit. In particular, the uncoupled magnetoplasmon resonance frequencies are larger in the time-dependent Hartree-Fock approximation so that the mode coupling is increased for $\omega_c < \omega_{LO}$.

We conclude by discussing the coupling of the magnetoroton modes[8,9] relevant to the fractional quantum Hall effect to LO phonons. To do so we combine Eqs. (1) and (2) with the single mode approximation for the magnetoroton contribution to the density response function[9]

$$\chi(q,\omega) \simeq \frac{\nu\bar{s}(q)\Delta(q)}{\pi l^2[\omega^2-\Delta^2(q)]} \ . \tag{9}$$

In Eq. (9) ν is the Landau-level filling factor, $\bar{s}(q)$ is the projected static structure factor, and $\Delta(q)$ is the magnetoroton energy. Since $\Delta(q) \ll \omega_{LO}$ $\chi(q,\omega)$ should be evaluated with ϵ_0 and

$$\Pi(q,\omega) \simeq \frac{\chi(q,\omega)}{1+(2\pi e^2/q\epsilon_0)\chi(q,\omega)} \ . \tag{10}$$

Inserting Eq. (10) into Eq. (2) gives

$$\omega_\pm^2 = \frac{\omega_{LO}^2+\Delta^2[1+B(q)]}{2} \pm \left[\left(\frac{\omega_{LO}^2-\Delta^2}{2}\right)+\frac{\Delta^2(\omega_{LO}^2+\Delta^2)B(q)}{2}+\frac{\Delta^4B^2}{4}\right]^{1/2} , \tag{11}$$

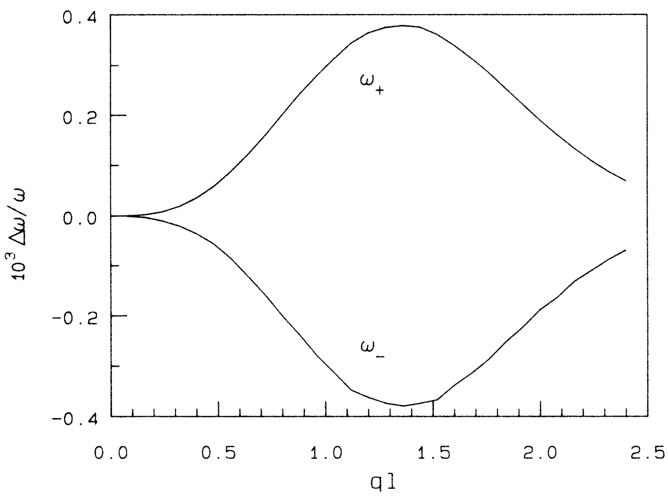

FIG. 2. Relative changes in the magnetoroton and phonon frequencies at $\nu = \frac{1}{3}$ and $B = 20$ T.

where

$$B(q) = \frac{2\nu\bar{s}(q)e^2}{l^2q\epsilon_0\Delta(q)}\left(\frac{\omega_{LO}^2}{\omega_{TO}^2}-1\right) \ . \tag{12}$$

We have displayed in Fig. 2 the relative changes in $\omega_\pm$, i.e., $(\omega_+ - \omega_{LO})/\omega_{LO}$ and $(\omega_- - \Delta)/\Delta$ at $B = 20$ T. Since $\Delta \sim 0.1e^2/\epsilon_0 l \ll \omega_{LO}$ we can expand in Δ/ω_{LO} to obtain $\omega_+^2 \simeq \omega_{LO}^2[1+(\Delta/\omega_{LO})^2B(q)]$ and $\omega_-^2 \simeq \Delta^2[1-(\Delta/\omega_{LO})^2B(q)]$. The coupling between the two modes is very weak; however, the result shows that one can in *principle* observe the magnetoroton mode through careful analysis of the experimental results for the LO phonon frequency in a very strong magnetic field.

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[2]A. Mooradian and G. B. Wright, Phys. Rev. Lett. **16**, 999 (1966); A. Mooradian and A. L. McWhorter, *ibid.* **19**, 849 (1967).
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[4]Wu Xiaoguang, F. M. Peeters, and J. T. Devreese, Phys. Rev. B **32**, 6982 (1985).
[5]For a derivation of the total dielectric function of the coupled electron-phonon system see G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1981), Chap. 6.
[6]At extremely long wavelengths the plasmon dispersion is modified by retardation effects which we do not discuss here.
[7]The plasmon and phonon resonance frequencies can be seen by Raman spectroscopy, and far-infrared (FIR) emission and FIR transmission spectroscopy. A recent review has been presented by D. Heitmann, in *Proceedings of Yamada Conference XIII (EP2DS VI)* [Surf. Sci. (to be published)].
[8]S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. Lett. **54**, 581 (1985).
[9]S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. B **33**, 2481 (1986). For $q \to 0$, $\bar{s}(q) \propto q^4$ so that inter-Landau-level contributions which give $\chi \propto q^2$ always dominate at sufficiently small q. The discussion here is appropriate to the extremely strong field limit where it will be valid except for very small q.
[10]For example, *The Quantum Hall Effect,* edited by S. M. Girvin and R. Prange (Springer, New York, 1986).
[11]We assume that the 2DEG is many lattice constants wide and, for the case where the 2DEG occurs at semiconductor heterojunction ignore the penetration of the electrons into the barrier material and any distortion of the well material lattice dynamics near the interface.
[12]This approximation was briefly considered by R. Lassnig, in *Proceedings of Yamada Conference XIII (EP2DS VI)* [Surf. Sci. (to be published)], in discussing 2D magnetopolarons.
[13]Equation (3) gives the polarizability of a noninteracting 2DEG

in a magnetic field and can easily be derived using standard many-body techniques. See, e.g., A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971). Note that we have set the spin splitting to zero.

[14]For example, I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products* (Academic, New York, 1965).

[15]H. Chiu and J. J. Quinn, Phys. Rev. B **9**, 4724 (1974); N. Horing and M. Yildiz, Ann. Phys. (N.Y.) **97**, 216 (1976); T. N. Theis, Surf. Sci. **98**, 515 (1980), and references therein.

[16]This is equivalent to Eqs. (5) of Ref. 4 for $q \rightarrow 0$.

[17]A. V. Chaplik, Zh. Eksp. Teor. Fiz. **62**, 746 (1972) [Sov. Phys. JETP **35**, 395 (1972)].

[18]Results can be extended to the case of partially full Landau levels using the techniques discussed by A. H. MacDonald, H. C. A. Oji, and S. M. Girvin, Phys. Rev. Lett. **55**, 2208 (1985).

[19]The last case corresponds to an unrealistically high electron density and field strengths and is included for purposes of illustration.

[20]See A. H. MacDonald, J. Phys. C **18**, 1003 (1985). The time-dependent Hartree-Fock approximation includes corrections to the RPA from the Coulombic electron-electron interactions and approaches the RPA when $q \rightarrow 0$ or when $(e^2/\epsilon_\infty l) \ll \hbar\omega_c$. To be consistent corrections due to phonon-mediated electron-electron interactions should also be included. This is difficult except for $\omega \ll \omega_{\rm LO}$ where we can simply replace $2\pi e^2/\epsilon_\infty q$ by $2\pi e^2/\epsilon_0 q$, and to our knowledge has not been accomplished even for the $B=0$ case.