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Magnetoconductance oscillations of a quasi-one-dimensional electron gas in a parabolic transverse potential

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We have studied theoretically the magnetoconductance oscillations in a quasi-one-dimensional electron gas with a parabolic transverse confining potential. The solution to Schrödinger's equation is that of a hybrid harmonic oscillator with a frequency ω that depends on both the parabolic potential and the magnetic field B. At $B=0$, ω equals the classical oscillation frequency of the parabolic potential. In the high-field limit, ω approaches the cyclotron frequency. The result is a nonlinear fan plot for the magnetoconductance minima, which should help to clarify the origin of conductance oscillations in narrow-channel metal-oxide-semiconductor field-effect transistors.

There has been much activity recently in the study of the quantum-mechanical properties of ultranarrow conducting systems. Much effort has been spent in fabricating narrow-channel metal-oxide-semiconductor field-effect transistor (MOSFET) structures in which an electron gas, which is typically confined to (but free in) the $\hat{\mathbf{x}}$-$\hat{\mathbf{y}}$ plane [two-dimensional electron gas (2DEG)], is confined in the $\hat{\mathbf{x}}$ direction as well.[1-4] The aim of much of this work has been to observe the effects of the quasi-one-dimensional (Q1D) density of states on the conductance of these devices. However, the existence of universal conductance fluctuations[5] and sample inhomogeneities has resulted in marginal demonstrated success until recently,[6] when structure was observed in the conductance of many Q1D lines measured in parallel.

There has been substantial controversy concerning the interpretation of the available data in devices producing single Q1D lines. We suggest that magnetoconductance measurements be made on such devices in order to clarify the relationship between the observed conductance oscillations and the Q1D density of states. In this Rapid Communication, we propose a simple theory of the magnetoconductance of a narrow Q1D line. We demonstrate that a mixing of the Q1D states and the two-dimensional Landau levels occurs. By correctly taking into account the Q1D density of states, we obtain a nonlinear fan plot for the magnetoconductance minima that should be observable.

We model the transverse confining potential $V(x)$ with an harmonic approximation. Although this model is not applicable to confinement due to a physical boundary,[2] simulations[6,7] and analytic calculations[8] have shown it to be a reasonable approximation for many realizable structures. Here we demonstrate that a parabolic transverse potential also has the advantage of allowing an exact solution to Schrödinger's equation. We assume that only the lowest electric subband of the electron gas in the $\hat{\mathbf{z}}$ direction is occupied, and we neglect spin and valley splitting. We also do not explicitly include the effects of scattering, which primarily serve to broaden the magnetostructure, as in the usual case with Shubnikov–de Haas oscillations.[9]

The quantum-mechanical solution follows the treatment given for the two-dimensional electron gas in a magnetic field.[9] The Hamiltonian for an electron with effective mass m^* is given by $\mathcal{H}=\mathcal{H}_1+\mathcal{H}_2$, with

$$\mathcal{H}_1=\frac{1}{2m^*}\left(p+\frac{e}{c}A\right)^2$$

and

$$\mathcal{H}_2=V(x)=\tfrac{1}{2}m^*\omega_0^2x^2 ,$$

where ω_0 is the classical oscillation frequency in the transverse parabolic potential. The symmetry of the problem suggests that the vector potential A be represented in the Landau gauge, $A=A_y=Bx$. Schrödinger's equation may then be expressed

$$-\frac{\hbar^2}{2m^*}\left[\frac{\partial^2\Psi}{\partial x^2}+\left(\frac{\partial}{\partial y}-\frac{ieB}{\hbar}x\right)^2\Psi\right]+\tfrac{1}{2}m^*\omega_0^2x^2\Psi=E\Psi . \tag{1}$$

With the ansatz

$$\Psi(x,y)=U(x)\exp(i\beta y) ,$$

Eq. (1) becomes separable in x and y. Here β is the momentum variable for the $\hat{\mathbf{y}}$ direction, which is quantized in units of $2\pi/L$, where L is the sample length. We change the variable x to the dimensionless unit ξ, where $\xi^2=x^2/al_B^2$, $a=\omega_c/\omega$,

$$\omega=(\omega_0^2+\omega_c^2)^{1/2} , \tag{2}$$

$\omega_c=eB/m^*$ is the cyclotron frequency, and $l_B^2=\hbar/eB$. U is then foud to obey the one-dimensional equation

$$-\frac{\partial^2U(\xi)}{\partial\xi^2}+(\xi-\bar{\xi})^2U(\xi)=\frac{2E'}{\hbar\omega}U(\xi) ,$$

where

$$E'=E-\tfrac{1}{2}(\hbar^2\beta^2/m^*)(1-a^2) .$$

This is the equation of an harmonic oscillator with level spacings $\hbar\omega$ and center coordinate $\bar{\xi}=\beta l_B a^{3/2}$. The full eigenstates are given by

$$\Psi(r)=\frac{1}{\sqrt{L}}\exp(i\beta y)\chi_N\left(\frac{x}{\sqrt{a}l_B}-\bar{\xi}\right) ,$$

and

$$\chi_N(\xi)=\left[\frac{1}{\alpha 2^N N!\sqrt{\pi}l_B}\right]^{1/2}\exp(-\xi^2)H_N(\xi)\ ,$$

where $H_N(\xi)$ is the Hermite polynomial of integer order N. The eigenenergies of the system are given by

$$E_N(\beta)=\hbar\omega(N+\tfrac{1}{2})+\frac{1}{2}\frac{\hbar^2\beta^2}{m^*}(1-\alpha^2)\ , \quad (3)$$

where the hybrid oscillator frequency ω is a mixture of the bare harmonic oscillator and cyclotron frequencies. This has the proper asymptotic behavior since at $B=0$, ω is equal to ω_0, the classical oscillation frequency in the parabolic potential, while as B increases, ω smoothly approaches the cyclotron frequency ω_c. It is also clear that the density of states for each hybrid level must vary smoothly from the $\sqrt{E}$ behavior of a 1D system to the δ function of a Landau level. This is important since it is the variations in the density of states that lead to oscillations in the magnetoconductance. In addition to the hybrid oscillator contribution to the energy, we see that the kinetic energy contribution due to the Q1D motion in the $\hat{\mathbf{y}}$ direction scales toward zero as B increases ($\alpha\rightarrow 1$). It should also be noted here that a simple semiclassical calculation of electron motion in a magnetic field perpendicular to a parabolic confining potential results in elliptic trajectories with an oscillation frequency identical to the quantum-mechanical result in Eq. (2).

In order to connect such an energy spectrum to measured magnetoconductance oscillations, one must relate the level spacings to an electron density spacing through the density of states, and hence to the gate voltage V_G through the oxide capacitance C_{ox}. In a purely 2D system (with no transverse confining potential), the density of states per unit area per Landau level is just $m^*\omega_c/h$, so that V_G is proportional to B for each index N. A plot of the locations of the magnetoconductance extrema in V_G-B space is known as a fan plot. In Q1D system with no magnetic field, the density of states per unit length for each level is given by

$$\rho_{1D}(E)=\sum_N \rho_0\left[\frac{\hbar\omega_0}{E-E_N}\right]^{1/2}\ , \quad (4)$$

where

$$\rho_0=\frac{g_s g_v}{2\pi}\left[\frac{2m^*}{\hbar^3\omega_0}\right]^{1/2}\ ,$$

and where g_s and g_v are the respective spin and valley degeneracies. We can define the change in number density per unit length to be $\Delta n_N^{Q1D}\equiv n(E_N)-n(E_{N-1})$. By integrating Eq. (4), it can be shown that for the equally spaced energies of an harmonic oscillator,

$$\Delta n_N^{Q1D}=2\rho_0\hbar\omega_0 N^{1/2}$$

(see Fig. 1). To convert this to an area density, we assert that the appropriate length is simply the oscillator amplitude x_N which also goes as $N^{1/2}$. We then have that $\Delta n=\Delta n_N^{Q1D}/x_N$, which is just $m^*\omega_0/h$, independent of N. In a similar fashion, for hybrid levels we find that the spacing in number density per unit area is simply $m^*\omega/h$, so that when the Nth level is completely occupied, the change in number density is

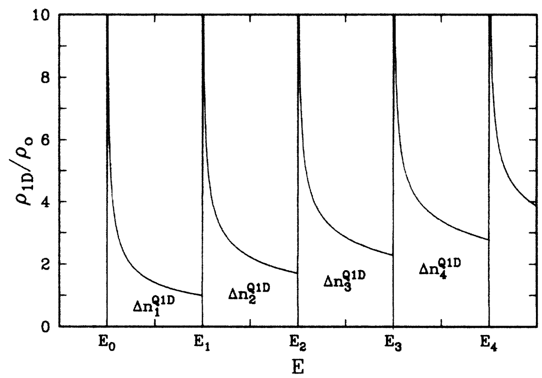

FIG. 1. The Q1D density of states ρ_{1D} normalized to ρ_0 (see text), plotted vs energy E. The changes in number density per unit length Δn_N^{Q1D} are the integrated areas between the eigenvalues E_N and E_{N-1}, and are labeled for $N=1$–4.

$$N\Delta n=N\frac{g_s g_v e}{h}(B^2+B_0^2)^{1/2}\ ,$$

where $B_0\equiv m^*\omega_0/e$. With this, we can use

$$\Delta V_G=\frac{Ne\Delta n}{C_{ox}}$$

to obtain a simple expression for the rays of the fan for the conductance minima in terms of the measured quantities V_G and B:

$$V_G=V_T+N\frac{g_s g_v e^2}{h}\frac{d}{\varepsilon\varepsilon_0}(B^2+B_0^2)^{1/2}\ , \quad (5)$$

where d is the gate oxide thickness, ε is the oxide dielectric constant, and V_T is the threshold voltage. The key point here is that V_G is no longer proportional to B (as in the 2D case), except in the limit of $B\gg B_0$.

Using this result, predictions for a realistic system can now be made: The frequency ω_0 is determined mainly by the available space charge through the Poisson equation. For a fixed space charge of 5×10^{15} electrons/cm^3, energy level spacings of order 2 meV are possible. This results in a value of B_0 of $\sim$3 T. A fan plot of Eq. (5), using $B_0=3$ T, $V_T=0.1$ V, and $d=30$ nm, is shown in Fig. 2, including the first 10 levels. The dotted lines illustrate the corresponding fan plot for a wide 2DEG ($B_0=0$). The intercepts of the solid lines at $B=0$ are the strictly Q1D states with no mixing with Landau levels. This result is similar to numerical calculations performed previously for a magnetic field applied parallel to a square-well heterostructure.[10] In that case the $B=0$ energy-level spacings were quadratic, as expected for electrons confined to a square well, but the density of states problem prevented a straightforward connection to gate voltage. It is also related to calculations by Mikeska and Schmidt[11] concerning electrons in a 2DEG with localized-state potential wells modeled as harmonic oscillators. They obtained a different hybrid mixing (because their wells were circular), and related the contribution of a statistical distribution of

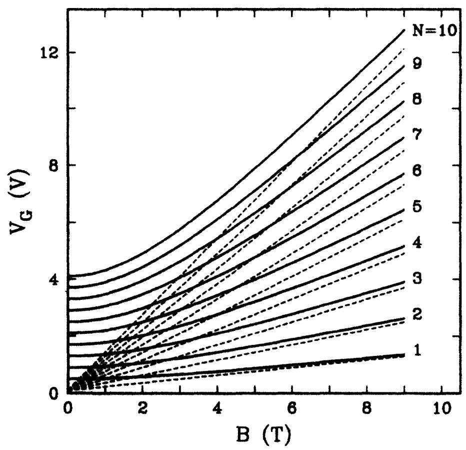

FIG. 2. The solid lines represent a fan plot for the magnetoconductance minima from the equation

$$V_G = V_T + (4Ne^2/h)(d/\varepsilon\varepsilon_0 h)(B^2+B_0^2)^{1/2} ,$$

for a Si MOSFET, where $B_0 = 3$ T, the gate oxide thickness $d = 30$ nm, and for energy levels N from 1 to 10. The dotted lines represent the solution for the 2DEG ($B_0 = 0$).

such wells to an observed increase in the cyclotron mass at low electron densities.

The implication of Eq. (5) is striking in that it shows clearly how the application of a magnetic field may be used to clarify the origin of conductance oscillations in Q1D systems at $B = 0$. In multiple-line systems the relatively clear oscillations observed at $B = 0$ should shift visibly in position and spacing for easily accessible fields (0–10 T). For single-line systems where the presence of strong universal conductance fluctuations has obscured the Q1D oscillations,[12] it may be possible to track levels from the strong-field regime toward low field and determine what structure at $B = 0$, if any, is actually due to the Q1D states.

In conclusion, we have considered the effect of a magnetic field on the electronic energy levels of a quasi-one-dimensional electron gas. The energy levels for the case of a parabolic transverse potential were found to be hybrids of the bare Landau levels and the Q1D states. This suggests that the Q1D energy levels at zero field may be identified by tracking the magnetoconductance maxima along a nonlinear fan plot. In the case of nonharmonic confining potentials, the only significant difference will lie in the level spacings at $B = 0$.

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