PHYSICAL REVIEW B VOLUME 34, NUMBER 2 15 JULY 1986

Influence of dissipation on the accuracy of the integral quantum Hall effect

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(Received 10 December 1985)

The dc conductivity σ_{xx} of a two-dimensional electron gas, in the presence of strong magnetic fields, is evaluated for elastic impurity scattering in the first Born approximation. Short-range, long-range, and Gaussian potentials are considered. The results depend explicitly on the scattering, the temperature (T), and the magnetic field. For low temperatures and high magnetic fields the conductivity shows an activated type of behavior as observed experimentally. For an integer filling factor the deviation $\Delta\sigma_{yx}(T)$ of the Hall conductivity from its zero-temperature quantized value, obtained previously, is equal to $\alpha\sigma_{xx}(T)$; the coefficient α depends on the scattering, on the magnetic field, and, for constant impurity concentration, on the temperature.

I. INTRODUCTION

Most of the theoretical studies of the integral quantum Hall effect[1] are concerned with the evaluation, at zero temperature, of the Hall conductivity σ_{yx} which shows plateaus as a function of the magnetic field, where the latter occur between Landau levels. The other conductivity component σ_{xx}, which measures the dissipation, is usually dismissed on account of the large gap between Landau levels and the existence of localized states between them; the Fermi level is pinned by these states in the gap.[2–4] However, experiments show that σ_{xx} is different from zero, although very small, for finite temperatures and it extrapolates to zero for zero temperature.[5] Moreover, for finite temperatures and strong magnetic fields σ_{xx} shows an activated type of behavior.[6,5,21]

We are not aware of explicit evaluations of the component σ_{xx} other than those of Refs. 7–9, valid at zero temperature. For short-range impurity scattering the peak values of σ_{xx}, corresponding to filled or half-filled Landau levels, are shown to be independent of the scattering and the magnetic field and are different from zero.[7] In Ref. 8, however, a slight dependence of σ_{xx} on the magnetic field is reported for magnetic fields that are not too strong. The conclusions of Ref. 9 are similar to those of Refs. 7 and 8. Early experiments are in good agreement with the theory[10,11] but later ones show dependence of the peak values on the magnetic field[5,11,12] or on the electron concentration.[13]

It is clear from the above that more work is needed in order to clarify the role of the dissipation on the accuracy of the effect, that is, the role of the scattering, the temperature, and the magnetic field. In this paper, we evaluate σ_{xx} explicitly for finite temperatures in the first Born approximation. We consider only elastic impurity scattering (short-range, long-range, or Gaussian-type potentials). The dependence of the conductivity on the scattering and the magnetic field is shown explicitly. For strong magnetic fields (for which the Born approximation is expected to apply) σ_{xx} has an activated type of behavior as observed experimentally. The result for σ_{xx} combined with the corresponding one for σ_{yx}, published previously,[14] helps explain the observed behavior of the resistivity peaks.

In Sec. II, we present the formalism and the results. In Sec. III, we present a simplified version of the results for strong magnetic fields and we make a comparison with the experiment.

II. THE MAGNETOCONDUCTIVITY σ_{xx}

A. Preliminaries

We consider a two-dimensional electron gas, such as the one realized in the inversion layer of a metal-oxide-semiconductor field-effect transistor (MOSFET), in a strong magnetic field $\mathbf{B}$ normal to the surface and parallel to the $\mathbf{z}$ axis. In the Landau gauge, the one-electron Hamiltonian, states, and eigenvalues read

$$h^0=(\mathbf{P}+e\mathbf{A})^2/2m^*,\quad \mathbf{A}=(0,Bx,0)\ , \tag{2.1}$$

$$|\zeta)=|N,k_y)=\phi_N(x+x_0)e^{ik_y y}/L_y^{1/2}\ , \tag{2.2}$$

$$\varepsilon_\zeta\equiv\varepsilon_N=(N+1/2)\hbar\omega_0,\quad N=0,1,2,\ldots\ , \tag{2.3}$$

where $\omega_0=eB/m^*$ is the cyclotron frequency, m^* is the effective mass, and $l^2=\hbar/m^*\omega_0$. ϕ_N represents harmonic-oscillator wave functions, N denotes the Landau levels, $\mathbf{A}$ is the vector potential, and $A_0=L_xL_y$ is the area. We set $x_0=-l^2k_y$. In the representation (2.2) the matrix elements necessary for the evaluation of σ_{xx} are

$$(\zeta|x|\zeta')=x_0\delta_{N,N'}\delta_{k_y,k_y'}+(l/\sqrt{2})\times(\sqrt{N+1}\,\delta_{N',N+1}-\sqrt{N}\,\delta_{N',N-1})\times\delta_{k_y,k_y'}\ , \tag{2.4}$$

$$(\zeta|e^{i\mathbf{q}\cdot\mathbf{r}}|\zeta')=J_{NN'}(q_x,k_y,k_y')\delta_{k_y,k_y'+q_y}\ , \tag{2.5}$$

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with

$$|J_{NN'}(\cdots)|^2 \equiv |J_{NN'}(u)|^2 = (N'!/N!)e^{-u}u^{N'-N}[L_{N'}^{N'-N}(u)]^2 \quad \text{with } N \le N' ; \qquad (2.6)$$

here $u = l^2 q_1^2/2$, $q_1^2 = q_x^2 + q_y^2$, and $L_N^M(u)$ is a Laguerre polynomial.

The conductivity tensor has been evaluated in Ref. 15. When an electric field is applied in the x direction, the two-dimensional version of the dc conductivity σ_{xx} (spin included) reads [cf. Ref. 15, Eq. (2.84)]

$$\sigma_{xx}^d = \frac{\beta e^2}{A_0} \sum_{\zeta,\zeta'} \langle n_\zeta \rangle_{\text{eq}} (1 - \langle n_{\zeta'} \rangle_{\text{eq}}) w_{\zeta\zeta'} (X_{\zeta'} - X_\zeta)^2 , \qquad (2.7)$$

where e is the electron charge, $\beta = 1/k_B T$, T is the temperature, k_B is Boltzmann's constant, and where $\langle n_\zeta \rangle_{\text{eq}}$ is the equilibrium Fermi-Dirac distribution function. Further, $X_\zeta = (\zeta | X | \zeta)$, $w_{\zeta\zeta'}$ is the transition rate given by the "golden rule," and the superscript d denotes that (2.7) comes from the solution of the diagonal master equation for the density operator $\rho(\rho = \rho_d + \rho_{nd})$ or from the corresponding diagonal *quantum* Boltzmann equation. In Ref. 14, it has been shown that the nondiagonal contribution σ_{xx}^{nd} vanishes.

It is worth noting that (2.7) is valid for both elastic and inelastic collisions. In what follows, we evaluate (2.7) for elastic scattering with impurities.

B. Impurity scattering

We assume that the electrons are scattered quasielastically by randomly distributed impurities. Writing the impurity potential $U(\mathbf{r} - \mathbf{R})$ as $\sum_{\mathbf{q}} U(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}}$ ($\mathbf{r}$ and $\mathbf{R}$ are the positions of the electron and the impurity, respectively), we find, with the help of (2.5), that the transition rate $w_{\zeta\zeta'}$ is given by

$$w_{\zeta\zeta'} \equiv w_{NN',k_y k_y'} = (2\pi/\hbar)(N_I/A_0) \sum_{\mathbf{q}} |U(\mathbf{q})|^2 |J_{NN'}(u)|^2 \times \delta(\varepsilon_N - \varepsilon_{N'}) \delta_{k_y, k_y' + q_y} , \qquad (2.8)$$

where N_I is the impurity concentration. Further, since the functions $\phi_N(x + x_0)$ oscillate around the point $-x_0$, we have

$$\sum_{k_y} \to \frac{L_y}{2\pi} \int_{-L_x/2l^2}^{L_x/2l^2} dk_y = \frac{A_0}{2\pi l^2} , \qquad (2.9)$$

and, using cylindrical coordinates,

$$\sum_{\mathbf{q}} \to \frac{A_0}{2\pi l^2} \int_0^\infty du . \qquad (2.10)$$

We can now evaluate (2.7).

1. *Screened interaction*

For

$$U(\mathbf{r}) = (e^2/k)\exp(-k_s r)/r ,$$

where k is the dielectric constant and k_s the inverse screening length, we have (in two dimensions) $U(\mathbf{q}) = (2\pi e^2/k)(q_1^2 + k_s^2)^{1/2}$. Using (2.4) and (2.7)–(2.10), we obtain

$$\sigma_{xx}^d = \frac{e^2}{h} \frac{\beta N_I}{\hbar\omega_0} \left[\frac{2\pi e^2}{k} \right]^2 \frac{1}{2\pi} \times \sum_N f_N (1 - f_N) \int_0^\infty \frac{q_y^2 |J_{NN}(u)|^2}{q_1^2 + k_s^2} du , \qquad (2.11)$$

where $f_N = \langle n_\zeta \rangle_{\text{eq}}$. Due to symmetry, σ_{yy}^d will be given by (2.11) with q_y^2 replaced by q_x^2. With $\sigma_{xx} = (\sigma_{xx} + \sigma_{yy})/2$ we obtain

$$\sigma_{xx}^d = \frac{e^2}{h} \frac{\beta N_I}{\hbar\omega_0} \left[\frac{\sqrt{2}\pi e^2}{k} \right]^2 \frac{1}{2\pi} \times \sum_N f_N (1 - f_N) \int_0^\infty \frac{u |J_{NN}(u)|^2}{u + b} du , \qquad (2.12)$$

where $b = k_s^2 l^2/2$. Since $|J_{NN}(u)|^2 \sim e^{-u}$ the major contribution to the integral, at least for small N, comes from small values of u. For $u \ll b$, $(u + b)^{-1}$ is expanded in powers of u/b and the result for the integral over u, $I(N,b)$, in (2.12), is

$$I(N,b) = \sum_m (-1)^{m+1} I_m / b^m , \quad m = 1,2,3,\ldots , \qquad (2.13)$$

where

$$I_m = \int u^m |J_{NN}(u)|^2 du , \quad m = 1,2,3,\ldots . \qquad (2.14)$$

The integral I_m has been evaluated explicitly in Ref. 16 for $m = 1,2,3$ and the same method is applied for $m > 3$. For $m = 1,\ldots,4$ the result is

$$I_1 = 2N + 1 ,$$
$$I_2 = 2(3N^2 + 3N + 1) ,$$
$$I_3 = 2(2N + 1)(5N^2 + 5N + 3) ,$$
$$I_4 = 4(N + 1)(2N + 1)(7N^2 + 7N + 6) + 2N^2(7N^2 + 5) . \qquad (2.15)$$

Alternatively, one can use the explicit expressions for $L_N^N(u)$ and express the integral in terms of exponential integrals or evaluate it numerically with b as a parameter.

2. *Gaussian potential*

The Fourier transform $U(\mathbf{q})$ of the potential $U(\mathbf{r}) = (V_0/\pi d^2) e^{-r^2}/d^2$, where d is the range of the potential, is $U(\mathbf{q}) = V_0 e^{-q^2 d^2}/2$. Repeating the steps in subsection B 1 we obtain ($b' = d^2/l^2$)

$$\sigma_{xx}^d = (e^2/h)(m^* \beta N_I V_0^2 / h\hbar) \times \sum_N f_N (1 - f_N) \int_0^\infty e^{-b'u} u |J_{NN}(u)|^2 du . \qquad (2.16)$$

For $d \to 0$, (2.16) reduces to the delta function result given below [cf. (2.18)]. The integral over u in (2.16) is equal to

$$-(\partial/\partial b')\int_0^\infty e^{-b'u}\,|J_{NN}(u)|^2 du$$

and the modified integral can be done exactly.[17] Alternatively, for b' small, one can expand the exponential in powers of b' and use (2.14) and (2.15). The result is following:

$$\sigma_{xx}^d = -\left[\frac{e^2}{h}\right]\left[\frac{m^*\beta N_I V_0^2}{h\hbar}\right] \times \sum_N f_N(1-f_N)\frac{\partial}{\partial b'}\left[\frac{(b'-1)^N}{(b'+1)^{N+1}} P_N\left[\frac{(b')^2+1}{(b')^2-1}\right]\right]$$

$$= (e^2/h)(m^*\beta N_I V_0^2/h\hbar) \times \sum_N f_N(1-f_N)\sum_m (-b')^{m-1} I_m/(m-1)!\ , \qquad (2.17)$$

where $P_N(z)$ is a Legendre polynomial.[17]

3. Short-range interaction

For $U(\mathbf{r}) = V\delta(\mathbf{r})$, $U(\mathbf{q})$ is independent of q. The integral over u now becomes $\int_0^\infty u\,|J_{NN}(u)|^2 du$ and is given by I_1, cf. (2.15). Moreover, the quantity $N_I\,|U(\mathbf{q})|^2$ is equal to $\hbar^3/m^*\tau_f$, where τ_f is the relaxation time in the absence of the magnetic field.[7] The result is

$$\sigma_{xx}^d = (e^2/h)(\beta\hbar/2\pi\tau_f)\sum_N f_N(1-f_N)(2N+1)\ . \qquad (2.18)$$

In contrast with previous results (valid for zero temperature[7–9]) this finite temperature result depends on the scattering through τ_f and on the magnetic field through the factor f_N. This is also the case with Eqs. (2.12) and (2.17) but with a different dependence on the scattering.

The above results have been obtained within linear-response theory. For corrections to these results, obtained by the two-parameter scaling theory of localization at zero temperature (and less explicitly at finite temperatures) see Ref. 18.

C. Collision broadening

Strictly speaking, the results (2.12), (2.17), and (2.18) give a series of isolated peaks (N is an integer). Broadening of the levels can be incorporated heuristically by replacing the delta function in (2.8) by a Lorentzian of zero shift (for simplicity) and of width Γ_N. (For more rigorous treatments, see Refs. 7–9.) The level width Γ_N is estimated from the relaxation time, $\Gamma_N \approx \hbar/\tau$. For elastic scattering we have $1/\tau = \sum_{\zeta'} w_{\zeta\zeta'}$, where $w_{\zeta\zeta'}$ is given by (2.8). Replacing the delta function in (2.8) by a Lorentzian, we obtain, in correspondence with subsections B 1, B 2, and B 3, the following level widths:

$$\Gamma_N^1 = \sqrt{N_1/2\pi}(2\pi e^2/k)\left[\int_0^\infty |J_{NN}(u)|^2 du/(u+b)\right]^{1/2},$$

$$\Gamma_N^2 = (N_I V_0^2/\pi l^2)^{1/2}\left[\frac{(b'-1)^N}{(b'+1)^{N+1}} P_N\left[\frac{(b')^2+1}{(b')^2-1}\right]\right]^{1/2},$$

$$\Gamma_N^3 = (N_I\,|U(\mathbf{q})|^2/\pi l^2)^{1/2} = (\hbar^2\omega_0/\pi\tau_f)^{1/2}\ , \qquad (2.19)$$

where we have used the fact that $\int_0^\infty |J_{NN}(u)|^2 du = 1$ for Γ_N^3. Notice that the results (2.19), obtained here in a simple way, differ only by a factor of $\sqrt{2}$ from the more rigorous self-consistent results of Refs. 7 and 19. The corresponding results for the conductivities are given by (2.12), (2.16), and (2.18) multiplied by a factor $\hbar\omega_0/\pi\Gamma_N$.

D. Zero-temperature limit

If the integrals over u in Sec. II B are known functions of N, we can also perform the sums over N at zero temperature. However, we obtain a simple result only for short-range potentials. This is also the result (apart from numerical factors) for the first term of (2.12) and (2.16) [cf. (2.15)] and is given below.

At zero temperature the factor $\beta f_N(1-f_N)$ is equal to $\delta(\varepsilon_N - \varepsilon_F)$, where ε_F is the Fermi level. To sum the product $(2N+1)\delta(\varepsilon_N-\varepsilon_F)$ we use Poisson's summation formula[20] and we replace $\delta(\varepsilon_N - \varepsilon_F)$ by a Lorentzian of width Γ_N. We can then show that

$$\sum_N (N+\tfrac{1}{2})\delta(\varepsilon_N-\varepsilon_F) = \frac{\epsilon_F}{(\hbar\omega_0)^2}\left[1+2\sum_{s=1}^\infty (-1)^s e^{-2\pi s(\pi\Gamma_N/\hbar\omega_0)} \times \cos[2\pi s(\varepsilon_F/\hbar\omega_0)]\right]. \qquad (2.20)$$

At zero temperature $\varepsilon_F \approx (N+\frac{1}{2})\hbar\omega_0$, $\cos[2\pi s(\varepsilon_F/\hbar\omega_0)] = (-1)^s$, and the quantity in the large parens is equal to $\coth(\pi\Gamma_N/\hbar\omega_0)$. Thus (2.18) becomes

$$\lim_{T\to 0}\sigma_{xx}^d = \frac{e^2}{h}(2N+1)\coth(\sqrt{\pi/\omega_0\tau_f})/2\pi\omega_0\tau_f\ , \qquad (2.21)$$

where we used (2.19). For $\sqrt{\pi/\omega_0\tau_f} \ll 1$, $\coth x \approx 1/x$ and (2.21) becomes simpler:

$$\lim_{T\to 0}\sigma_{xx}^d = \frac{e^2}{h}(2N+1)/2\pi\sqrt{\pi\omega_0\tau_f}\ . \qquad (2.22)$$

Thus the conductivity, which goes to zero for $\omega_0\tau_f \to \infty$, decreases with increasing magnetic field as observed at 50 mK (Ref. 5) and at 4.2 K (Ref. 12); in the latter case, however, $\beta f_N(1-f_N)$ is only approximately equal to $\delta(\varepsilon_N-\varepsilon_F)$.

The main difference of (2.22) from the results of Refs. 7 and 9 [equal to $(e^2/h)(2N+1)$ divided by π and 2, respectively] is its dependence on $\omega_0\tau_f$ absent in those references. However, a dependence on $\omega_0\tau_f$ is reported in Ref. 8 but it is opposite to that in (2.22).

III. COMPARISON WITH THE EXPERIMENT

A. Strong magnetic fields

For electrons interacting with randomly distributed impurities, the Born approximation applies for magnetic fields such that $l \ll R_I$, where R_I is the average impurity separation and $l=(\hbar/m^*\omega_0)^{1/2}$. This restriction however, as discussed in Ref. 14, can be relaxed.

In what follows we assume that the magnetic field is so strong that for low temperatures, only the Nth term contributes to the sum over N in the results of Sec. II B. Setting $C=\beta(\varepsilon_F-\varepsilon_N)$, assuming $e^{-C}\ll 1$, and expanding the Fermi factors, we obtain

$$f_N(1-f_N)=\sum_{m=1}(-1)^{m+1}me^{-mC}\approx e^{-C}\,. \tag{3.1}$$

With (3.1), the results (2.12), (2.17), and (2.18) take the approximate form ($\sigma_{xx}^d\equiv\sigma_{xx}$)

$$\sigma_{xx}\approx(e^2/h)(\beta N_I/\hbar\omega_0)(\sqrt{2}\pi e^2/k)^2e^{-C}\times\int_0^\infty u\,|J_{NN}(u)|^2du/(u+b)/2\pi\,, \tag{3.2}$$

$$\sigma_{xx}\approx(e^2/h)(m^*\beta N_I V_0^2/h\hbar)e^{-C}\times\left[-\frac{\partial}{\partial b'}\frac{(b'-1)^N}{(b'+1)^{N+1}}P_N\left[\frac{(b')^2+1}{(b')^2-1}\right]\right]\,, \tag{3.3}$$

$$\sigma_{xx}\approx(e^2/h)(2N+1)(\beta\hbar/2\pi\tau_f)e^{-C}\,. \tag{3.4}$$

These results depend on the magnetic field B through C. The first one, Eq. (3.2), however, has an additional B dependence through $\hbar\omega_0$ and b and Eq. (3.3) through b'.

B. Activated behavior

The temperature dependence of the above results is contained in the factor βe^{-C}. The conductivity behaves as $e^{-\varepsilon/T}/T$, i.e., it shows an activated type of behavior as observed experimentally.[5,6,21]

We now make a comparison of (3.2)—(3.4), with the results of Ref. 5 for the levels marked $n_{1\uparrow}$ at $(B=6.55T)$ and $n_{1\downarrow}$ at $B=4.98T$. For those fields, the Born approximation is expected to apply. The (constant) impurity concentration and zero-field mobility are $4.0\times10^{15}/\text{m}^2$ and $8.6\ \text{m}^2/\text{V s}$, respectively. The activation energy is taken from the data.

1. Screened interaction

We evaluate the integral over u, in Eq. (3.2), numerically with b as a parameter. The results are shown in Fig. 1. Curve I is obtained with $k_sl\approx80$ $(b=k_s^2l^2/2)$ and curve II with $k_sl\approx20$. These values of k_sl are in qualitative agreement with the self-consistent results for k_sl as a function of the magnetic field of Ref. 19. For a density of $10^{16}/\text{m}^2$ and the lowest Landau level these values are $k_sl\approx65$ and $k_sl\approx2$, respectively. Since l does not vary much for B between $4.98T$ and $6.65T$, this shows that k_s varies with the magnetic field B.

2. Gaussian potential

The quantity $\{\cdots\}$, in Eq. (3.3), is equal to

$$[(1+b')^2-4(1+b')+6]/(1+b')^4\,,$$

$b'=d^2/l^2$. The factor $N_IV_0^2$ is replaced by the corresponding value, $\hbar^3/\tau_fm^*$, for short-range interaction and τ_f is taken from the mobility. The results are again given by curves I and II, in Fig. 1, and have been obtained with $d/l\simeq0.6$ and $d/l\simeq0.0$, respectively. Again, since l does not vary much between the two curves this indicates, in analogy with subsection B 1, that the range d of the potential varies with the magnetic field.

3. Short-range interaction

The relaxation time τ_f is taken from the mobility. The results are shown in Fig. 1 by the curves III and IV, respectively.

As can be seen from Fig. 1 the agreement of the theory with the experiment is reasonable for the screened interaction and the Gaussian potential but rather poor for the short-range interaction, especially for the half-filled level (curve III). The discrepancies arise mainly from the fact that the Born approximation requires $l \ll R_I$, where R_I is the average impurity separation, while in the reported experiment ($l\sim100$ Å, $R_I\sim200$ Å) this condition is not well fulfilled. This is formally reflected in the values of e^{-C} which are not much smaller than 1 as we assumed, especially for low $1/T$ values. On the average, the Born approximation, as discussed in Ref. 14, is valid for $B\gtrsim10T$.

We also notice that the prefactor in $\sigma_{xx}(T)$ varies with the temperature, whereas the experimental points indicate

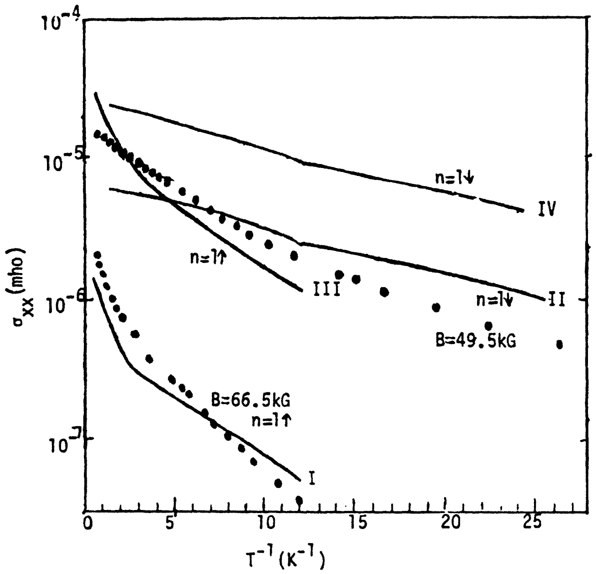

FIG. 1. Logarithmic plot of $\sigma_{xx}(T)$ vs $1/T$. The circles are experimental points from Ref. 5. The curves I and II are obtained from Eq. (3.2) or Eq. (3.3). The curves III and IV are obtained from Eq. (3.4).

the opposite, especially for the half-filled level. Had we assumed a constant $N_I\beta$ product in Eqs. (3.2) and (3.3) the fit to the data would be perfect (curves I and II). It is not clear what suppresses the temperature dependence of the prefactor. However, thermal activation can change the density of mobile carriers at the wings of the plateaus [where the results for $\sigma_{xx}(T)$ apply (Ref. 22)] or can lead to hopping conduction, not treated here. These reasons or corrections to the components $\sigma_{\mu\nu}(T)$, missed by linear-response theory,[18] could probably account for the discrepancies by suppressing or weakening the temperature dependence of the prefactor in $\sigma_{xx}(T)$.

If we use the collision broadening version of Eqs. (3.2)–(3.4) (multiplication by $\hbar\omega_0/\pi\Gamma_N$) together with (2.19) and the above-quoted values for $k_s l$ and d/l, the results get worse by a factor of 3 indicating that the replacement of the delta function by a Lorentzian, at the conductivity level, is a poor approximation.

Before closing this subsection, we note that an activated behavior of the conductivity or resistivity minima ($\sigma_{xx}=\rho_{xx}/\rho_{xx}^2+\rho_{xy}^2$, $\rho_{xx}\ll\rho_{xy}$) has been observed in Ref. 6 (in Si MOSFET's) at magnetic fields roughly twice as strong as those of Ref. 5 ($8.1T\le B\le 14T$). Assuming the same temperature dependence for both minima and maxima (results for the latter are not given in Ref. 5), one can describe the activated behavior of the minima with one or at most two terms in (3.1). [The term $2e^{-2C}$ in (3.1) is about 5 times smaller than the first one e^{-C} for $B\ge 10T$]. This reflects the fact that the Born approximation becomes better as the magnetic field increases.[23]

C. Relationship between $\Delta\sigma_{yx}(T)$ and $\sigma_{xx}(T)$

In a previous paper[14] the Hall conductivity $\sigma_{yx}(T)$ has been evaluated. The deviation

$$\Delta\sigma_{yx}(T)=\sigma_{yx}(0)-\sigma_{yx}(T)\ ,$$

where $\sigma_{yx}(0)$ is the zero-temperature quantized value, $(e^2/h)(N+1)$, when only the Nth level is occupied, is equal to $(e^2/h)(N+1)e^{-C}$, where $C=\beta(\varepsilon_F-\varepsilon_N)$. We see that both $\sigma_{xx}(T)$ and $\Delta\sigma_{yx}(T)$ show the same activated behavior (for $e^{-C}\ll 1$). Using Eqs. (3.2)–(3.4), we can write

$$\Delta\sigma_{yx}(T)=\frac{2\pi\hbar}{m^*N_I\beta}(N+1)\alpha_i\sigma_{xx}(T),\quad i=1,2,3\ , \tag{3.5}$$

where

$$\alpha_1=\frac{eB}{(\sqrt{2}\pi e^2/k)}\times\left[\int_0^\infty u\,|J_{NN}(u)|^2du\Big/(u+b)\right]^{-1},$$

$$\alpha_2=\frac{\hbar}{V_0^2}\left[-\frac{\partial}{\partial b}\frac{(b'-1)^N}{(b'+1)^{N+1}}P_N\left[\frac{(b')^2+1}{(b')^2-1}\right]\right]^{-1},$$

$$\alpha_3=\frac{\hbar}{V_0^2}(2N+1)^{-1}\ . \tag{3.6}$$

Equation (3.5) holds for the resistivity components $\Delta\rho_{yx}(T)$ and $\rho_{xx}(T)$ for $\sigma_{yx}(T)\gg\sigma_{xx}(T)$ and $\sigma_{yx}(T)/\sigma_{yx}(0)\approx 1$. This has been observed experimentally (cf. Refs. 21, 22, and 24–27) but with a temperature-independent proportionality coefficient. The temperature dependence of this coefficient, in Eq. (3.5), comes from the prefactor in $\sigma_{xx}(T)$, cf. Eqs. (3.2)–(3.4); again, thermal activation of the mobile carriers[22] or hopping could probably suppress or weaken this dependence by keeping $N_I\beta$ approximately constant.

From Eqs. (3.5) and (3.6) we notice that the proportionality coefficient depends on the scattering, the range of the potentials (b,b'), the impurity concentration N_I, and the Landau level index N. The dependence on the magnetic field B is explicit for the screened interaction or the Gaussian potential, but not for the short-range interaction (only through N_I and N).

The validity of Eqs. (3.5) and (3.6) could be easily checked experimentally by changing the impurity concentration (e.g., by illumination of the samples) or by considering various Landau levels and measuring $\sigma_{xx}(T)$ at the wings of the plateaus [maxima in $\sigma_{xx}(T)$].

Finally, the order of magnitude of the proportionality coefficient appears to be correct for the only pertinent data[28] that we are aware of, i.e., those of Ref. 22. The reported mobility is 1.6 m^2/V s. Assuming $m^*\approx 0.19m_0$, we find for short-range scattering and $T=0.6$ K, a slope 0.95 whereas the reported one is between 0.3 and 0.4. For T between 1.2 and 3.0 K this is also the order of magnitude[25,26] for the coefficient between $\Delta\rho_{yx}(T)$ and $\rho_{xx}^{\min}(T)$.

IV. CONCLUDING REMARKS

In this paper we have evaluated the conductivity σ_{xx} for finite temperatures. For strong magnetic fields, for which the Born approximation applies, an activated behavior of the conductivity is obtained. Deviations from this behavior have also been observed and are usually attributed to hopping conduction,[29] not treated here. Our results depend explicitly on the magnetic field, the scattering, and the impurity concentration in contrast with some of the earlier results.[7,9] Besides, they are in reasonable agreement with the experimental data[6,5] and the adjustable parameters used to describe the latter are in agreement with those of the literature when available [e.g., $k_s l$ in (3.2)]. For Gaussian potentials and in analogy with the results for $k_s l$ of Ref. 19, they also indicate that the range of the potential varies with magnetic field. Moreover, the dependence of the proportionality coefficient between $\Delta\rho_{yx}(T)$ and $\rho_{xx}(T)$ on the scattering, the magnetic field, etc., is made explicit.

In a previous paper,[14] it has been shown that, for the temperatures and the magnetic fields in which most of the quantum Hall experiments have been done, the conductivity component σ_{yx} remains quantized to an accuracy better than 10^{-5}. It can be shown that this result remains unaffected when electron-electron interaction is considered. Since the quantization of σ_{yx} (or ρ_{yx}) has been used in Ref. 5 in order to extract the σ_{xx} peak values, which compare relatively well with Eqs. (3.2)–(3.4), it can be said that the present theory gives a reasonable quantitative account of the dissipation for the integral quantum Hall effect. For a complete description of σ_{xx} however, other factors have to be considered as well, e.g., hopping, electron-electron interaction, etc.

ACKNOWLEDGMENT

This work was supported by the NSERC Grant No. A-9522.

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