Light scattering from a layered electron gas

Narkis Tzoar and Chao Zhang
Department of Physics, City College of New York, New York, New York 10031
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The Raman scattering cross section from a two-component layered electron gas (such as in InAs-GaSb, GaAs-$Al_xGa_{1-x}As$ superlattices, etc.) has been calculated. It is found that for a separation between the two components larger than a critical value, the scattered spectra have two resonant peaks in the high-frequency regime. For small separation and small mass ratio, there is a resonant peak due to an ion acoustic mode in the low-frequency regime and to a plasma mode in the high-frequency regime.

I. INTRODUCTION

Recently, many light scattering experiments have been done on the layered electron gas (LEG) in superlattice structures.[1] Olego *et al.*[2] observed the bulk plasmon of a LEG by inelastic light scattering from GaAs-$Al_xGa_{1-x}As$ heterostructures. This experiment confirmed the random-phase-approximation (RPA) prediction[3,4] of the bulk-plasmon dispersion relation. By imposing standard electromagnetic boundary conditions at the layers of a semi-infinite LEG, Giuliani and Quinn[5] predicted the existence and dispersion relation of surface plasmons if the dielectric media outside and inside the semi-infinite LEG are different. Jain and Allen[6] calculated the Raman intensities of the bulk and surface plasmons. In this work we give a microscopic theory for the Raman intensity of the bulk plasmons in a LEG where two different carriers are localized on alternating layers. Our method involves the exact construction of the density-density correlation functions in the RPA for such LEG. From these correlation functions, the Raman intensity is calculated.

The Raman scattering cross section is completely characterized by the wave-number transfer $\mathbf{q}$ and energy loss ω in the scattering event. Here, $\mathbf{q}=\mathbf{k}_{\rm in}-\mathbf{k}_{\rm out}$ and $\omega=\omega_{\rm in}-\omega_{\rm out}$ (subscripts in and out refer to incoming and outgoing photons). For most light scattering experiments from the semiconductor plasmas the wave-number transfer is smaller than the inverse screening length, i.e., $q\lambda_s<1$. For a one-component isotropic plasma the cross section is proportional to the density fluctuations of the electron gas. For the case $q\lambda_s<1$ the scattered intensity resides almost entirely in the plasma line.

II. CALCULATION OF THE SCATTERING CROSS-SECTION

We use the model of Visscher and Falicov[7] for a LEG, which has a δ-function-localized carrier density in a plane. The carriers are free to move in the plane and the carriers in different planes interact only via the Coulomb potential. The possibility of tunneling between two planes as well as of interband excitations within a plane is ignored under the assumption that both the carrier temperature and their Fermi energies are small compared to the subband splitting. Let us consider electrons of density n_e per unit area and mass m_e occupying layers which are positioned at $z=ja$ and holes of density n_h per unit area and mass m_h occupying layers which are positioned at $z=ja+b$, where j can be any integer, a is the length of the unit cell in the z direction, and b is the separation between electrons and holes on each cell. The Hamiltonian that describes such a system is given by

$$H=H_1+H_I\ , \qquad (1)$$

where the first term contains the kinetic energy of the carriers and their coupling to the radiation field. The second term is the Coulomb interaction of the many-particle system. In second-quantized notation, they are given as

$$H_1=\frac{1}{2}\sum_{\mathbf{p},j,s}\frac{[\mathbf{p}+(e_s/c)\mathbf{A}]^2}{m_s}a^{\dagger}_{\mathbf{p},j}(s)a_{\mathbf{p},j}(s) \qquad (2)$$

and

$$H_1=\frac{1}{2}\sum_{\mathbf{q},\mathbf{p},\mathbf{p}'}\sum_{j,j',s,s'}V^{s,s'}_{j,j'}(q)e_se_{s'}a^{\dagger}_{\mathbf{p}+\mathbf{q},j}(s)a^{\dagger}_{\mathbf{p}'-\mathbf{q},j'}(s')\times a_{\mathbf{p}',j'}(s')a_{\mathbf{p},j}(s)\ , \qquad (3)$$

where $\mathbf{p}$ is a two-dimensional (2D) momentum vector. $a^{\dagger}_{\mathbf{p},l}$ and $a_{\mathbf{p},l}$ represent, respectively, the electron creation and destruction operators with momentum $\mathbf{p}$ on the lth layer. We set $\hbar$ and the speed of light c equal to unity for notational convenience. The summation over s means that s can be either an electron or a hole. $V^{s,s'}_{j,j'}(q)$ is the Fourier transform of the Coulomb interaction

$$V^{s,s'}_{j,j'}(q)=\frac{2\pi}{q}e^{-q|j-j'|a} \quad (\text{for } s=e,\ s'=e \text{ or } s=h,\ s'=h) \qquad (4)$$

or

$$V^{s,s'}_{j,j'}(q)=\frac{2\pi}{q}e^{-q|(j-j')a-b|} \quad (\text{for } s=e, s'=h)\ . \qquad (4a)$$

In calculating the scattering cross section, we consider only the coupling of the incoming and outgoing radiation via the $\mathbf{A}^2$ term in the Hamiltonian. This is a good approximation when $E_F < \omega_{\rm in} < mc^2$, where E_F is the Fermi energy. This essentially nonrelativistic approximation leads to the following expression for the cross section for a photon of wave number $k_{\rm in}$, frequency $\omega_{\rm in}$, and polarization ϵ to be scattered into a state with wave number $k_{\rm out}$, frequency $\omega_{\rm out}$, and polarization ϵ (Appendix A):

$$\frac{d\sigma}{d\omega d\Omega}=e^4\,|\,\epsilon\cdot\epsilon'\,|^2\frac{\omega_{\rm out}}{\omega_{\rm in}}\int_{-\infty}^{\infty}\frac{dt}{2\pi}e^{i\omega t}\sum_{j,j',s,s'}\frac{e^{ik_z(j-j')a}}{m_s m_{s'}}\langle n_{j,s}(\mathbf{q}t)n_{j',s'}(-\mathbf{q})\rangle \ . \tag{5}$$

In Eq. (5), $n_{j,s}(\mathbf{q})$ is the Fourier transform of the density operator to the lth cell for the s species, $n_{j,s}(t)=e^{iHt}n_{j,s}(0)e^{-iHt}$, and the brackets $\langle\ \rangle$ represent the usual thermodynamic ensemble average. The factor $e^{ik_z(j-j')a}$ is a coherence term which will generate perpendicular momentum conservation. The scattering cross section is given in our Eq. (5) in terms of the time-dependent density fluctuations of the carriers in the different planes. In equilibrium Eq. (5) can be rewritten, using the fluctuation-dissipation theorem, as

$$\frac{d\sigma}{d\omega d\Omega}=e^4\,|\,\epsilon\cdot\epsilon'\,|^2\left\{\frac{\omega_{\rm out}}{\omega_{\rm in}}\right\}\frac{\rho(\omega)+1}{\pi}{\rm Im}\int_0^{\infty}dt\,e^{i\omega t}\sum_{j,j'}e^{ik_z(j-j')a}\sum_{s,s'}\frac{\Pi_{s,s'}(j,j')}{m_s m_{s'}} \ , \tag{6}$$

where $\rho(\omega)=[\exp(\beta\omega)-1]^{-1}$ and β is the inverse temperature in energy units. Here, Π is given as

$$\Pi_{s,s'}(j,j')=\theta(t)\langle[n_{j,s}(\mathbf{q},t),n_{j',s'}(-\mathbf{q},0)]\rangle \ , \tag{7}$$

where $\theta(t)$ is the standard step function and it obeys the following integral equation:

$$\Pi_{s,s'}(j,j')=\Pi_s^0\delta_{s,s'}\delta_{j,j'}+\Pi_s^0\sum_{s'',j''}e_s e_{s''}V_{j,j''}^{s,s''}\Pi_{s'',s'}(j'',j') \ , \tag{8}$$

where Π^0 is the value of Π in the absence of the Coulomb interaction:

$$\Pi_s^0(q,\omega)=\int\frac{d\mathbf{p}}{4\pi^2}\frac{f_{\mathbf{p}+\mathbf{q}}(s)-f_{\mathbf{p}}(s)}{E_{\mathbf{p}+\mathbf{q}}(s)-E_{\mathbf{p}}(s)-\omega-i\alpha}\quad(\alpha\to 0+) \ . \tag{8a}$$

Here, $f_{\mathbf{p}}(s)$ is the Fermi distribution function for the s component which is independent of the layer index

$$f_{\mathbf{p}}(s)=\frac{1}{e^{\beta[E_{\mathbf{p}}(s)-\mu_s]}+1} \ ,$$

where μ is the chemical potential and $E_{\mathbf{p}}(s)=\mathbf{p}^2/2m_s$. The dependence on q and ω is suppressed in Eq. (8). To solve this equation, we make the following Fourier transformation:

$$\Pi_{s,s'}(j,j')=\frac{1}{N}\sum_{q_z}e^{-iq_z(j-j')a}\Pi_{s,s'}(q_z) \ , \tag{9}$$

where q_z can assume the values $2\pi n/Na$. Here, N is the number of planes and $n=0,1,\ldots,N-1$. (At the end, we take the limit $N\to\infty$.) After some algebra, our result can be written as (Appendix B)

$$\frac{d\sigma}{d\omega d\Omega}=\frac{e^2}{m_e^2}\,|\,\epsilon\cdot\epsilon'\,|^2\left\{\frac{\omega_{\rm out}}{\omega_{\rm in}}\right\}\frac{q[\rho(\omega)+1]A}{\pi}{\rm Im}F(q,k_z,\omega), \tag{10}$$

where A is the area of the plane and

$$F(q,k_z,\omega)=[\,Q(1-VSB)+\alpha^2B(1-VSQ)-\alpha QBV(S'+S'^*)]\frac{V(q)}{D(q,k_z,\omega)} \ . \tag{11}$$

Here, $\alpha=m_e/m_h$, $V=2\pi e^2/q$, and Q,B refer, respectively, to the 2D density fluctuation of electrons and holes, i.e., $Q=\Pi_e^0$ and $B=\Pi_h^0$. Also, S and S' are the form factors which can be expressed, respectively, as

$$S=\sum_j e^{-q|j|a}e^{-ik_zja}\frac{\sinh(qa)}{\cosh(qa)-\cos(k_za)} \tag{12}$$

and

$$S'=\sum_j e^{-q|ja-b|}e^{-ik_zja}=\frac{\sinh[q(a-b)]+e^{-ik_za}\sinh(qb)}{\cosh(qa)-\cos(k_za)} \ . \tag{13}$$

In Eq. (11) we defined a "dielectric function"

$$D(q,k_z,\omega)=1-V(q)S[Q(q,\omega)+B(q,\omega)]+V(q)^2Q(q,\omega)B(q,\omega)(S^2-|S'|^2) \ . \tag{14}$$

From Eqs. (10) or (11), we see that the cross section is given in terms of the susceptibilities of the electrons and the holes (i.e., Q and B) of the 2D electron gas. However, the cross section is not proportional to the imaginary part of the inverse dielectric function of the LEG formed by the electron-hole system. The light scattering cross section depends on the current matrix element, while the dielectric response depends on the matrix element of the density operator. [See the mass dependence in our Eqs. (5) and (6).] We point out that the light scattering intensity will peak at the zeros of $D(q,k_z,\omega)$, which defines the resonance frequencies of the density response. However, the scattered light intensity related to these frequencies cannot be obtained from the residue off the density response but rather by using our Eqs. (10) and (11). (Nevertheless, one can calculate the cross section macroscopically using a susceptibility defined by

$n_{\text{induced}}/\phi_{\text{external}}$, provided the coupling of the external field to the charge density of the s species is made proportional to m_s^{-1} times the usual Coulomb interaction.[8])

The dispersion relation of the plasmon in LEG as well as the collective excitations observed in light scattering are given by the zeros of $D(q,k_z,\omega)$, i.e., when $D(q,k_z,\omega)=0$. This was investigated by Tselis and Quinn.[9] Sarma and Madhukar[10] had investigated the longitudinal collective spectrum of a spatially separated, two-dimensional plasma. We have generalized their results for a layered electron-hole structure.

A. High-frequency regime ($\omega \gg qv_{F1}, qv_{F2}$)

Using the high-frequency expression for Q and B, we have

$$D(q,\omega,k_z)=1-\frac{S}{\omega^2}(\omega_{pe}^2+\omega_{ph}^2)+\frac{\omega_{pe}^2\omega_{ph}^2}{\omega^4}\left[S^2-|S'|^2-\frac{3Sqm_em_h}{4e^2n_en_h}\left[\frac{n_e^2}{m_e^3}+\frac{n_h^2}{m_h^3}\right]\right], \tag{15}$$

where $\omega_{ps}^2=2\pi n_s e^2 q/m_s$ is the 2D plasma frequency for the sth component. The dispersion relation from Eq. (15) is

$$\omega_\pm^2=\frac{S(\omega_{pe}^2+\omega_{ph}^2)}{2}\pm\frac{1}{2}\left\{(\omega_{pe}^2+\omega_{ph}^2)^2S^2-4\omega_{pe}^2\omega_{ph}^2\left[S^2-|S'|^2-\frac{3Sqm_em_h}{4e^2n_en_h}\left[\frac{n_e^2}{m_e^3}+\frac{n_h^2}{m_h^3}\right]\right]\right\}^{1/2}. \tag{16}$$

Since we are only interested in the long-wavelength behavior of the collective modes, we shall consider the small-q limit, i.e., $q/k_F \ll 1$. However, we still are left with two regions: (i) the intermediate-coupling limit, i.e., $qa,qb \ll 1$, and (ii) the weak-coupling limit, i.e., $qa,qb \gg 1$.

(i) $qa,qb \ll 1$ but $k_z \neq 0$. In this situation

$$S=\frac{qa}{1-\cos(k_z a)} \tag{17}$$

and

$$|S'|^2=q^2\frac{(a-b)^2+b^2+2b(a-b)\cos(k_z a)}{[1-\cos(k_z a)]^2}. \tag{18}$$

Using Eqs. (17) and (18) in Eq. (16) we obtain following two collective modes:

$$\omega_\pm=C_\pm q. \tag{19}$$

The coefficient $C_\pm$ is given by

$$C_\pm=\left[\frac{\pi e^2 a}{1-\cos(k_z a)}\left[\frac{n_e}{m_e}+\frac{n_h}{m_h}\right](1\pm\sqrt{g})\right]^{1/2}, \tag{20}$$

with

$$g=1-\frac{m_em_hn_en_h[1-\cos(k_z a)]}{a(m_en_h+m_hn_e)^2}\times\left[2b\left[1-\frac{b}{a}\right]-\frac{3m_em_h}{4e^2n_en_h}\left[\frac{n_e^2}{m_e^3}+\frac{n_h^2}{m_h^3}\right]\right].$$

Both modes are acousticlike, and for the ω_- mode, b cannot be zero since C_- becomes purely imaginary. The ω_- mode can exist as an undamped, stable mode in the long wavelengths only in a system where the two components of the electron gas are spatially separated.[10] The separation b and the period a must satisfy the condition by the requirement ($C_- > v_{F1}=(2\pi n_e)^{1/2}/m_e$), which is given by

$$2b\left[1-\frac{b}{a}\right]>\left[\frac{4}{m_e e^2}\left[1+\frac{m_hn_e}{m_en_h}-\frac{n_em_h}{ae^2m_e^2n_h}[1-\cos(k_z a)]\right]+\frac{3m_em_h}{4e^2n_en_h}\left[\frac{n_e^2}{m_e^3}+\frac{n_h^2}{m_h^3}\right]\right]. \tag{21}$$

For a fixed a, one can find the minimum separation b_c. The ω_+ mode always exists without any restriction.

The case with $k_z=0$ is different from the above result. In such a case the term with the product QB vanishes and

$$\omega^2=\frac{4\pi e^2}{a}\left[\frac{n_e}{m_e}+\frac{n_h}{m_h}\right]. \tag{22}$$

This is just like a three-dimensional plasmon with effective plasma frequency $\Omega_{ps}=4\pi n_{Bs}e^2/m_s$; the same form as that of bulk plasmons where $n_{Bs}=n_s/a$.

(ii) $qa,qb \gg 1$. In this limit, $\omega_\pm$ are simply the respective two-dimensional plasma frequencies of the two components.

B. Low-frequency regime ($qv_{F1} > \omega > qv_{F2}$)

For a system with a small mass ratio ($\alpha \ll 1$), one may expect that there is a low-lying mode of the heavy holes screened by the electron. Using the appropriate limiting forms for the two polarizability functions [see Eqs. (7) and (8) in Ref. 10] in this regime, one can easily find that there can be just one solution of Eq. (14) satisfying $qv_{F1} > \omega > qv_{F2}$. This solution is necessarily complex (indicating the mode to be a damped one), since the polarization function of the light species has an imaginary part in this regime due to single-particle excitations. For the collective mode to be physically meaningful, damping has to be small and we shall assume this in our analysis. We write the solution of Eq. (14) in this low-frequency regime in the form

$$\omega=\omega_A - i\delta_A \,, \tag{23}$$

where

$$\omega_A=\omega_{ph}\left[\frac{S+(S^2-|S'|^2)2r_s(k_{Fe}/q)}{1+2Sr_s(k_{Fe}/q)}\right]^{1/2} \tag{24}$$

and

$$\delta_A=\frac{m_e r_s \omega_A^2}{q^2}\left[\frac{(S\omega_A^2/\omega_{ph}^2)-S^2+|S'|^2}{S+2r_s(S^2-|S'|^2)(k_{Fe}/q)}\right], \tag{25}$$

where k_{Fe} is the Fermi wave vector for the electron and $r_s=m_e e^2/k_{Fe}$ is the electronic plasma parameter. For $qa \ll 1$, ω_A and δ_A are proportional to q ($k_z \neq 0$) and their ratio is

$$\frac{\delta_A}{\omega_A}=\frac{\sqrt{\alpha}\, r_s^{3/2}(k_{Fe}a)^2}{[k_{Fe}a+4r_s k_{Fe}^2 b(a-b)]^{1/2}[1-\cos(k_z a)+2r_s k_{Fe} a]^{3/2}} \,. \tag{26}$$

Here, for $\alpha<1$ and $r_s<1$, δ_A/ω_A can be easily made smaller than unity. The proportionality coefficient C_A for ω_A must satisfy the condition $v_{Fh}<C_A<v_{Fe}$. This gives the condition for a and b,

$$1<\left[r_s k_{Fe}\frac{m_h}{m_e}\frac{a+4b(a-b)r_s k_{Fe}}{1-\cos(k_z a)+2r_s a k_{Fe}}\right]<\frac{m_h^2}{m_e^2}\frac{n_e}{n_h} \,. \tag{27}$$

For $qa,qb \gg 1$, ω_A is just ω_{ph}, which is proportional to $\sqrt{q}$, and δ_A is approximately zero. As pointed out in Ref. 9, this ion acoustic mode can even exist when the mass ratio is 1 because of the separation between two components. However, we have shown in this work that the spectral weight of the mode is rather small when the mass ratio is close to 1.

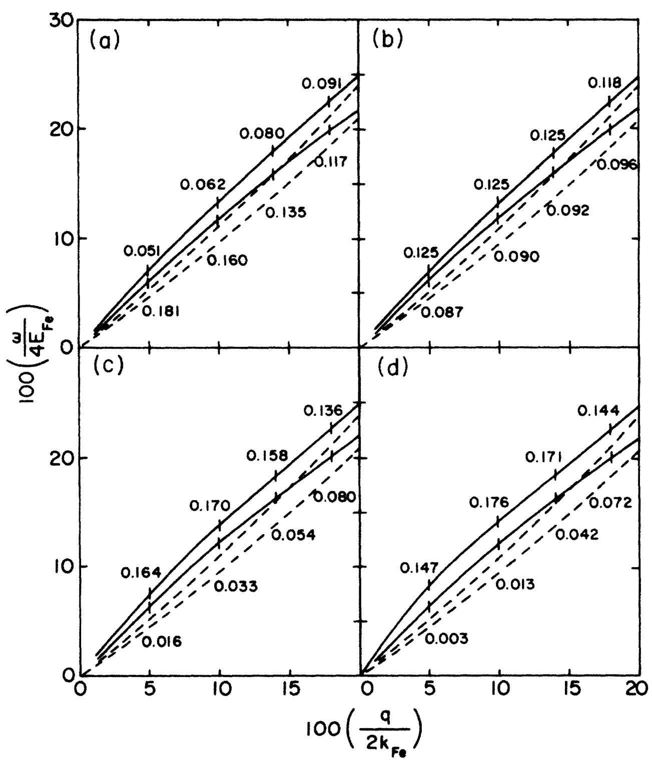

FIG. 1. Dispersion curves for two high-frequency modes ω_+ and ω_-. Here, $k_z=0.45\times10^6$ cm^{-1}, $n=3.77\times10^{11}$ cm^{-2}, $\alpha=0.87$, and $a=2b$. The region below the upper dashed line is the region for single-particle excitation of light-mass particles, and the region below the lower dashed line is the region for single-particle excitation of heavy-mass particles. The number above the vertical bar is the value of $I(q,k_z,\omega_r)$. (a) $b=300$ Å, (b) $b=350$ Å, (c) $b=400$ Å, and (d) $b=450$ Å.

All the above analytical results for the resonance frequencies are limited for a small range of parameters. We have calculated and plotted the dispersion curves for arbitrary values of qa,qb, and $k_z a$. For $a=2b$ and $\alpha=0.87$ for the InAs-GaSb system (Fig. 1), we obtain two acousticlike high-frequency modes. When $q>0.3k_{Fe}$, one of the mode drops into the single-particle excitation regime. We may note that we have the ω_- mode at separation b smaller than the critical separation defined in Eq. (21). This is because Eq. (21) is valid only under the condition $qa,qb \ll 1$, which is not accurate here. In Fig. 2, we plot the dispersion curves for the case of $\alpha=0.2$. The lower solid curve represents the ion acoustic wave.

The intensity of the Raman-scattered light as a function of its energy loss for a fixed value of in-plane momentum exchange **q** is given by the imaginary part of $F(q,k_z,\omega)$ given in Eq. (11). k_z is the z component of the wave vector of a photon inside the LEG, a negligible quantity. In our figures we have plotted the dispersion curves and the Raman intensity for several different values of **q** and mass ratio. For a small value of **q** (i.e., $q<q_s$, where q_s is the screening wave vector of the system), the scattered light only exhibits the collective spectrum. If **q** is not small, we have scattering due to collective excitation as well as single-particle excitation. If we decrease the value of the hole mass, the plasmon frequency will shift upward as expected.

We have calculated the cross section from Eq. (10). An exact expression for the 2D polarization function has been given by Stern[11] for infinite electron relaxation time τ. We use this result in our calculation. When the Landau damping is small, we introduce a phenomenological collision time τ to account for the collisional damping from background impurities and phonon scattering. Using a realistic τ,[12] we compute the dimensionless quantity $\text{Im}F(q,k_z,\omega)$. In Fig. 3, we plot this quantity as a function of frequency, the two resonant peaks are, respectively, at ω_+ and ω_-. To relate our results to experiment, we also carried out the integration over each of the resonances and define

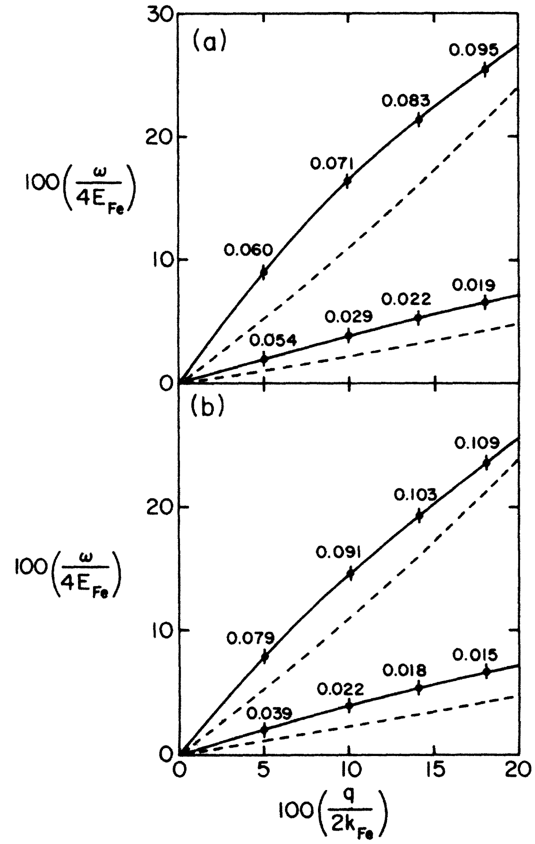

FIG. 2. Dispersion curves for two high- and low-frequency modes ω_+ and ω_A. Here, $k_z=0.45\times10^6$ cm^{-1}, $n=3.77\times10^{11}$ cm^{-2}, $\alpha=0.2$, and $a=2b$. The region below the upper dashed line is the region for single-particle excitation of light-mass particles, and the region below the lower dashed line is the region for single-particle excitation of heavy-mass particles. The number above the vertical bar is the value of $I(q,k_z,\omega_r)$. (a) $b=80$ Å, (b) $b=120$ Å.

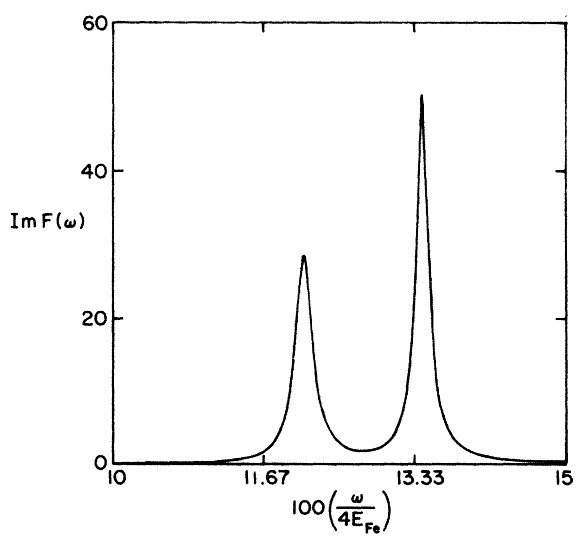

FIG. 3. Plot of function Im$F(q,k_z,\omega)$. Here, $q=0.2k_{Fe}$, $k_z=0.45\times10^6$ cm^{-1}, $n=3.77\times10^{11}$ cm^{-2}, $\alpha=0.87$, $v=0.025E_{Fe}$, and $a=2b$, $b=350$ Å.

$$I(q,k_z,\omega_r)=\int_{\omega_r-\Delta}^{\omega_r+\Delta}\text{Im}F(q,k_z,\omega)\frac{d\omega}{4E_{Fe}}\ , \tag{28}$$

which is the area under each resonant peak (ω_r is the resonant frequency ω_+ or ω_-). The integrated cross section can be written as

$$\left[\frac{d\sigma}{d\Omega}\right]_{\omega_r}=\frac{4E_{Fe}e^2}{m_e^2}\,|\epsilon\cdot\epsilon'|^2\left[\frac{\omega_{\text{out}}}{\omega_{\text{in}}}\right]\times\frac{q[\rho(\omega)+1]A}{\pi}I(q,k_z,\omega_r)\ . \tag{29}$$

We have denoted these values of $I(q,k_z,\omega_r)$ on our dispersion curves in Fig. 1 to see their dependence on q and b. We have found that I_+,I_- and the ratio $\beta=I_+/I_-$ strongly depend on the in-plane momentum transfer $q=|\mathbf{q}|$ and separation b. We choose carrier surface density $n=3.77\times10^{11}$ cm^{-2}, $k_z=0.45\times10^6$ cm^{-1}, and $\alpha=0.87$ for the InAs-GaSb system and use $a=2b$. For b smaller than 200 Å, the ω_- mode is inside the regime of single-particle excitation. For $b=300$ Å, the most spectral weight is carried by the ω_- mode and β is an increasing function of q. For $b>400$ Å, the most spectral weight is carried by the ω_+ mode and in this case β is a decreasing function of q. The interesting case is at $b=350$ Å; in this case, the spectral weights of the ω_+ and ω_- modes are about the same and β is almost independent of q. We think this is the suitable case for observing the ω_- mode in experiment. We conclude that the ω_- mode only appears when $b>b_c$, but the most spectral weight will be carried out by the ω_- mode for the separation slightly greater than b_c. For b much larger than b_c, ω_c is the plasma frequency of the holes. In Figs. 2 and 4 we present results of similar investigations for the ion acoustic mode. Here the ion acoustic mode described by the resonance at ω_A will appear for small separation b, together with the plasma mode ω_+. We found that the ω_+ mode usually carries more spectral weight than the ω_A mode for the typical mass ratio of semiconductors. We found that for $b=80$ Å and $q=0.1k_{Fe}$, I_A is very close to I_+. In Fig. 4 we see that the peak of Im$F(q,k_z,\omega)$ at ω_A is even higher than that at ω_+, but the width of the

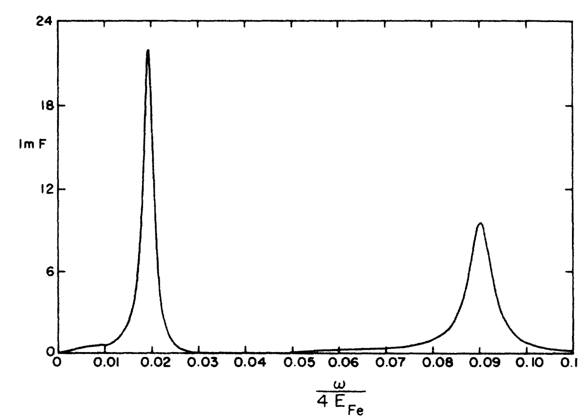

FIG. 4. Plot of function Im$F(q,k_z,\omega)$. Here, $q=0.1k_{Fe}$, $k_z=0.45\times10^6$ cm^{-1}, $n=3.77\times10^{11}$ cm^{-2}, $\alpha=0.2$, $v=0.025E_{Fe}$, and $a=2b$, $b=80$ Å.

ω_A mode is narrow compared to the width of the ω_+ mode. For the case where b is increased to 120 Å the spectral weight of the ω_+ mode increases and the spectral weight of the ω_A mode decreases since the ω_A mode is important only at small separations.

From our Figs. 1—4 we find that light scattering can provide us with experimental verification of the modes of electron-hole superlattice systems. As for the case of the two-dimensional electron-hole systems, depending on the electron-hole separation, we obtain either the high-frequency acoustic mode ω_- or the usual ion acoustic mode ω_A. The plasma mode ω_+ always exists regardless of the electron-hole separation. The advantage of scattering experiments using superlattices is that the relative intensity of the plasmon versus the acoustic mode depends strongly on b, the separation parameter. Thus it would be possible to verify the dispersion relation of ω_- or ω_A. In contrast, for the 2D electron-hole systems, most of the intensity seems to reside in the plasma (ω_+) mode.

III. CONCLUSION

In conclusion, we have calculated the Raman intensity for the electron-hole layered structure. The regimes necessary for the different modes are obtained and our calculations and plots for the spectral intensity indicate that the ω_- and the ω_A modes can be observed experimentally.

APPENDIX A

It has been shown that for the multicomponent system, the ratio of the $\mathbf{P}\cdot\mathbf{A}$ and A^2 contributions to the light scattering matrix element is[13]

$$\frac{M_{\mathbf{P}\cdot\mathbf{A}}}{M_{A^2}} \approx \frac{\omega_{\text{in}}}{mc^2} \ll 1 \; . \tag{A1}$$

Therefore, in our calculation we only consider the A^2 contribution. The matrix element of light scattering is

$$M = \langle I \,|\, H_\gamma \,|\, F\rangle \; , \tag{A2}$$

where I,F refer to initial and final states and

$$H_\gamma = \frac{e^2}{2} \sum_{s,j} \frac{A^2}{m_s} a^\dagger_{\mathbf{p},j}(s) a_{\mathbf{p},j}(s) \; .$$

Using the property of δ-function-like carrier density in a plane and by standard calculation, we obtain

$$M = \frac{2\pi e^2}{(\omega_{\text{in}}\omega_{\text{out}})^{1/2}} \sum_{j,s} \frac{e^{ik_z ja}}{m_s} \langle I \,|\, n_{j,s}(\mathbf{q}) \,|\, F\rangle \; , \tag{A3}$$

where $n_{j,s}(\mathbf{q}) = \sum_i e^{i\mathbf{q}\cdot\mathbf{r}_i(j,s)}$ and $\mathbf{r}_i(j,s)$ is the coordinate of the ith electron of the sth component on the jth cell. Now the cross section can be written as

$$\frac{d\sigma}{d\omega d\Omega} = e^4 \left[\frac{\omega_{\text{out}}}{\omega_{\text{in}}}\right] (\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}')^2 \sum_F \left| \sum_{j,s} \frac{e^{ik_z ja}}{m_s} \langle I \,|\, n_{j,s}(\mathbf{q}) \,|\, F\rangle \right|^2 \delta(E_F - E_I - \omega) \; , \tag{A4}$$

where E_F, E_I are energies are initial and final states. By using the identity

$$\delta(E_F - E_I - \omega) = \int_{-\infty}^{\infty} dt\, e^{i(E_F - E_I - \omega)t} \; . \tag{A5}$$

We finally obtain

$$\frac{d\sigma}{d\omega d\Omega} = e^4 \left[\frac{\omega_{\text{out}}}{\omega_{\text{in}}}\right] (\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}')^2 \sum_{j,j'} \sum_{s,s'} \frac{e^{ik_z(j-j')a}}{m_s m_{s'}} S^{s,s'}_{j,j'}(\mathbf{q},\omega) \; , \tag{A6}$$

$$S^{s,s'}_{j,j'}(q,\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} \int d\mathbf{r}\, d\mathbf{r}\, e^{-i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')} \langle n_{j,s}(\mathbf{r},t) n_{j',s}(\mathbf{r}) \rangle \; . \tag{A7}$$

In Eq. (A7), $\mathbf{q},\mathbf{r}$ are two-dimensional vectors in a plane.

APPENDIX B

We substitute Eq. (9) in Eq. (6) and obtain

$$\frac{d\sigma}{d\omega d\Omega} = 2\pi A e^4 |\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}'|^2 \left[\frac{\omega_{\text{out}}}{\omega_{\text{in}}}\right] \frac{\rho(\omega)+1}{\pi} \times \text{Im} \left[\frac{\Pi_{ee}(k_z)}{m_e^2} + \frac{\Pi_{hh}(k_z)}{m_h^2} + \frac{\Pi_{eh}(k_z)}{m_e m_h} + \frac{\Pi_{he}(k_z)}{m_e m_h} \right] . \tag{B1}$$

To obtain the quantities $\Pi_{ee}(k_z)$, $\Pi_{hh}(k_z)$, $\Pi_{eh}(k_z)$, and $\Pi_{he}(k_z)$, we take the Fourier transformation of Eq. (8) and have the following coupled equations:

$$\Pi_{ee}(k_z) = Q + Q[SV\Pi_{ee}(k_z) - S'V\Pi_{he}(k_z)] \; , \tag{B2}$$

$$\Pi_{he}(k_z) = B[-S'^*V\Pi_{ee}(k_z) + SV\Pi_{he}(k_z)] \; , \tag{B3}$$

$$\Pi_{hh}(k_z) = Q + Q[SV\Pi_{hh}(k_z) - S'^*V\Pi_{eh}(k_z)] \; , \tag{B4}$$

$$\Pi_{eh}(k_z) = B[-S'V\Pi_{hh}(k_z) + SV\Pi_{eh}(k_z)] \; . \tag{B5}$$

In Eqs. (B2)—(B5), the quantities V,S,S',Q,B,D are all defined in the text. The solutions for Eqs. (B2)—(B5) are

$$\Pi_{ee}(k_z) = Q(1 - SVB)D^{-1} \; , \tag{B6}$$

$$\Pi_{he} = -QBS'^*D^{-1} \; , \tag{B7}$$

$$\Pi_{hh}(k_z) = B(1 - SVQ)D^{-1} \; , \tag{B8}$$

$$\Pi_{eh} = -QBS'D^{-1} \; . \tag{B9}$$

Substituting Eqs. (B6)—(B9) into Eq. (B1), we obtain Eq. (10).

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