Magnetophonon oscillations in quasi-two-dimensional quantum wells

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The transverse dc electrical conductivity of a quasi-two-dimensional quantum well, in the presence of a magnetic field normal to the barriers of the well, is evaluated for electron-phonon interaction. For optical and polar optical phonons the conductivity oscillates as a function of the magnetic field with resonances occurring when $P\omega_0=\omega_L$, where ω_0,ω_L are the cyclotron and phonon frequencies, respectively, and where P is an integer. For elastic scattering with acoustical and piezoelectrical phonons, at low temperatures, resonances are expected when $P\hbar\omega_0=\varepsilon_F-\varepsilon_0$, where ε_F is the Fermi level and ε_0 the lowest subband energy in the direction of the magnetic field. The dependence of the evaluated conductivities, inverse scattering rates, and Landau-level widths on the magnetic field, the thickness of the well, and the temperature is shown explicitly. The results obtained here are in accordance with those available in the literature.

I. INTRODUCTION

In the past years the quasi-two-dimensional systems have been the subject of numerous experimental and theoretical investigations. Novel fabrication techniques, such as molecular-beam epitaxy or metal-organic chemical vapor deposition, have raised considerable interest in the transport properties of electrons confined between potential barriers a few tens of angstroms apart. Unusual effects associated with electronic motion, in layered structures (layer thickness less than hundred angstroms), in the direction normal to the layers[1,2] or parallel to the layer interface[3] are well known.

The influence of electron-phonon or electron-impurity interaction on electrical transport parallel to the barriers of a quantum well, in the absence of a magnetic field, has already been studied in a number of papers.[4–7] In contrast, treatments of the same subject, in the presence of a magnetic field, are limited.[8,9] Recently, magnetophonon resonances have been observed in thin (2.5–9-μm) n^+-n-n^+ GaAs structures.[10] The conductivity component σ_{xx} oscillates as a function of the magnetic field with maxima given by the relation $P\omega_0=\omega_L$, where ω_0 and ω_L are the cyclotron and phonon frequencies, respectively, and where P is an integer. These magnetophonon oscillations are the same as in conventional (three-dimensional) samples.[11] For submicron samples the results of Eaves *et al.*[10] indicate that σ_{xx} depends on the layer thickness.

Concerning magnetophonon resonances in quasi-two-dimensional quantum well structures we are not aware of theoretical work other than that of Ref. 8, in which only inverse scattering rates were calculated. The purpose of this paper is to evaluate the conductivity σ_{xx} in a quasi-two-dimensional quantum well in the presence of a magnetic field (in the z direction) perpendicular to the barriers of the well. We consider only the scattering of electrons by longitudinal phonons (optical, polar optical, acoustical, piezoelectrical) in the deformation potential model. The results show an explicit oscillatory behavior of the conductivity as a function of the magnetic field. For optical phonons the resonance condition is $\omega_L=P\omega_0$; for acoustical and piezoelectrical phonons at low temperatures the expected resonance condition is $P\hbar\omega_0=\varepsilon_F-\varepsilon_0$, where ε_F is the Fermi level and ε_0 the lowest subband energy (z direction). Since most of the results involve the Landau-level widths, we evaluate the relevant inverse scattering rates as well. These rates then are used to extract the level widths. The dependence of the results on the magnetic field, the temperature, and the thickness of the well is shown explicitly; those for the scattering rates are in agreement with the results of previous investigations.[8]

In Sec. II we present briefly the formalism. In Sec. III we present the results in detail. Remarks and conclusions are given in Sec. IV. Appendix A contains certain integrals necessary for the calculations. In Appendix B an application of the Poisson's summation formula, involving broadened δ functions, is presented. Finally, the inverse scattering rates for all types of phonons are evaluated in Appendix C.

II. PRELIMINARIES

A. Basic formulas

We consider a many-body system with a Hamiltonian given by

$$H=H^0+\lambda V-\mathscr{A}F(t)\;; \tag{2.1}$$

H^0 is the largest part of H which can be diagonalized, λV is a binary type interaction, assumed nondiagonal, and $-\mathscr{A}F(t)$ is the external field Hamiltonian with $\mathscr{A}$ being an operator and $F(t)$ a generalized force.

In Ref. 12 the Hamiltonian given by Eq. (2.1) was inserted into von Neumann's equation for the density operator ρ which was split into a diagonal (ρ_d) and a nondiagonal ($\rho_{\rm nd}$) part. Then by means of projection operators and for linear responses, two inhomogeneous master equations (diagonal and nondiagonal)[12] as well as a diagonal[12] and a nondiagonal *quantum* Boltzmann equation were obtained.[13] They are valid in the Van Hove limit, $\lambda\to 0$,

$t/\tau_{tr}\to\infty$, $\lambda^2 t=$finite, where τ_{tr} is the time for a transition between two eigenstates of H^0 to take place, $\tau_{tr}\approx\hbar/\delta\varepsilon$ (this is equivalent to the first Born approximation). In this limit the average current density J, when an electric field $\mathbf{E}(t)$ is applied, is given by

$$\langle (J_\mu)_d\rangle_t=\frac{q}{\Omega}\sum_\zeta(-\mathscr{B}_\zeta\langle n_\zeta\rangle_t\alpha_{\mu\zeta}+\langle n_\zeta\rangle_t\dot{\alpha}_{\mu\zeta}),\quad \mu=x,y,z\ , \tag{2.2}$$

where Ω is the volume, q the charge of the carriers, $\mathbf{F}(t)=q\mathbf{E}(t)$ and $\sum_i(\mathbf{r}_i-\langle \mathbf{r}_i\rangle_{eq})=\sum_i\boldsymbol{\alpha}_i$, where $\mathbf{r}_i$ are the positions of the carriers (fermions) and $\langle \mathbf{n}_i\rangle_{eq}$ their positions before the application of the electric field. Furthermore $\alpha_{\mu\zeta}=(\zeta\,|\,\alpha_\mu\,|\,\zeta)$, where $|\,\zeta)$ is the one-particle eigenstate of h^0 ($H^0=\sum h^0$), $\langle n_\zeta\rangle_t$ is the average occupancy of the state $|\,\zeta)$, and $\mathscr{B}_\zeta\langle n_\zeta\rangle_t$ is the collision integral of the Boltzmann equation. The second term of (2.2) is the usual ponderomotive current; the first term represents the many-body contribution of collisions to the current and has been termed "collisional" current.

The second term of Eq. (2.2) vanishes for Landau states;[13] it does so for the states given by Eq. (2.8), see below. In this case we have only "collisional" current, and the diagonal dc conductivity tensor is given by [cf. Ref. 13, Eq. (2.83)]

$$\sigma^d_{\mu\nu}(0)=\frac{\beta q^2}{\Omega}\sum_{\substack{\zeta,\zeta'\\ \text{spin}}}\langle n_\zeta\rangle_{eq}(1-\langle n_{\zeta'}\rangle_{eq})w_{\zeta\zeta'}(R_{\nu\zeta}-R_{\nu\zeta'})R_{\mu\zeta'}\ , \tag{2.3}$$

where $R_{\mu\zeta}=\alpha_{\mu\zeta}, \beta=1/k_BT$ with k_B being Boltzmann's constant, and T the temperature; $w_{\zeta\zeta'}$ is the binary transition rate given by the "golden rule". Formula (2.3), first derived by Argyres and Roth,[14] for $\mu=\nu=x$, is valid for both elastic ($\langle n_\zeta\rangle_{eq}=\langle n_{\zeta'}\rangle_{eq}$) and inelastic ($\langle n_\zeta\rangle_{eq}\neq\langle n_{\zeta'}\rangle_{eq}$) scattering.

As for the nondiagonal contributions (independent of the scattering due to the Van Hove limit) the relevant formula for the dc conductivity reads[13] as

$$\sigma^{nd}_{\mu\nu}(0)=\Omega\hbar i\sum_{\zeta',\zeta''}{}'\langle n_{\zeta'}\rangle_{eq}(1-\langle n_{\zeta''}\rangle_{eq})(\zeta'\,|\,j_\nu\,|\,\zeta'')(\zeta''\,|\,j_\mu\,|\,\zeta')(1-e^{-\beta(\varepsilon_{\zeta''}-\varepsilon_{\zeta'})})/(\varepsilon_{\zeta''}-\varepsilon_{\zeta'})^2\ , \tag{2.4}$$

where $j=qv/\Omega$. The prime on $\sum$ means $\zeta'\neq\zeta''$. The total conductivity is obtained by $\sigma_{\mu\nu}=\sigma^d_{\mu\nu}+\sigma^{nd}_{\mu\nu}$.

The above formulas are fairly general and not tied to a k space description. Therefore, they can be applied to situations where the semiclassical Boltzmann equation fails, such as transverse magnetoresistance (Landau levels) or conduction through localized states in amorphous materials;[15] the relevant current comes from the first term of (2.2), which is absent in a semiclassical treatment. Another example is the integer quantum Hall effect.[16]

In the case of electron-phonon interaction (we assume that the phonons remain at equilibrium), H^0, in Eq. (2.1), contains an electron part and a phonon part; λV is the usual electron-phonon interaction. In this case the transition probabilities $w_{\zeta\zeta'}$ are given by[13]

$$w_{\zeta\zeta'}=\sum_{\mathbf{q}}[Q(\zeta,\mathbf{q}\to\zeta')\langle N_{\mathbf{q}}\rangle_{eq}+Q(\zeta\to\zeta',\mathbf{q})(1+\langle N_{\mathbf{q}}\rangle_{eq})]\ , \tag{2.5}$$

where

$$\begin{aligned}Q(\zeta,\mathbf{q}\to\zeta')&=\frac{2\pi}{\hbar}\,|F(\mathbf{q})|^2\,|(\zeta'\,|\,e^{i\mathbf{q}\cdot\mathbf{r}}\,|\,\zeta)|^2\delta(\varepsilon_\zeta-\varepsilon_{\zeta'}+E_{\mathbf{q}})\ ,\\ Q(\zeta\to\zeta',\mathbf{q})&=\frac{2\pi}{\hbar}\,|F(\mathbf{q})|^2\,|(\zeta'\,|\,e^{-i\mathbf{q}\cdot\mathbf{r}}\,|\,\zeta)|^2\delta(\varepsilon_\zeta-\varepsilon_{\zeta'}-E_{\mathbf{q}})\ .\end{aligned} \tag{2.6}$$

The first and second term of Eq. (2.5) stand for the absorption and emission of a phonon of wave vector $\mathbf{q}$, respectively. $\langle N_q\rangle_{eq}=N_0$ is the average number of phonons. We consider only longitudinal phonons as treated by the deformation potential model.

B. Quantum-well characteristics

We consider the one-electron Hamiltonian ($H^0=\sum h^0$)

$$h^0=(\mathbf{p}+q\mathbf{A})^2/2m^*,\quad \mathbf{A}=(0,Bx,0) \tag{2.7}$$

where we employed the Landau gauge for the vector potential $\mathbf{A}$. The magnetic field B (in the z direction) is perpendicular to the barriers of the well. For simplicity we have assumed a spherical effective mass m^* but the results of this paper hold for $m^*_\perp\neq m^*_z$ as well. The distance between the barriers, assumed infinitely high, is L_z. Assuming that the wave function vanishes at $z=0$ and at $z=L_z$, the one-particle eigenstates $|\,\zeta)$ and eigenvalues ε_ζ are given by

$$\begin{aligned}&|\,\zeta)=(2/L_yL_z)^{1/2}\phi_N(x-x_0)e^{ik_yy}\sin(k_zz)\ ,\\ &k_z=n\pi/L_z,\quad n=1,2,3,\ldots\end{aligned} \tag{2.8}$$

$$\varepsilon_\zeta\equiv\varepsilon_{N,n}=(N+\tfrac{1}{2})\hbar\omega_0+n^2\varepsilon_0,\quad N=0,1,2,\ldots \tag{2.9}$$

where $\omega_0=|\,e\,|B/m^*$ is the cyclotron frequency, $\varepsilon_0=(\hbar^2/2m^*)(\pi/L_z)^2$, and ϕ_N represents harmonic oscillator wave functions, centered at $x_0=-\hbar k_y/m^*\omega_0$. N is the Landau level index and n denotes level quantization in the z direction. The radius of the orbit in the (x,y) plane is $l=(\hbar/m^*\omega_0)^{1/2}$. For the calculations of this paper we need the matrix elements

$$(\zeta \mid x \mid \zeta') = x_0 \delta_{NN'} \delta_{kk'} + (l'/\omega_0)[(N+1)^{1/2}\delta_{N',N+1} - N^{1/2}\delta_{N',N-1}]\delta_{kk'} \;, \tag{2.10}$$

$$(\zeta \mid \dot{\alpha}_x \zeta') = il'[-(N+1)^{1/2}\delta_{N',N+1} + (N)^{1/2}\delta_{N',N-1}]\delta_{kk'} \;, \tag{2.11}$$

$$|(\zeta \mid e^{\pm i\mathbf{q}\cdot\mathbf{r}} \mid \zeta')|^2 = |F_{nn'}(\pm q_z)|^2 |J_{NN'}(u)|^2 \delta_{k_y,k_y'\pm q_y} \;, \tag{2.12}$$

$$F_{nn'}(\pm q_z) = (2/L_z)\int e^{\pm i q_z z}\sin(n\pi z/L_z)\sin(n'\pi z/L_z)dz \;, \tag{2.13}$$

$$|J_{NN'}(u)|^2 = (N'!/N!)e^{-u}u^{N'-N}[L_N^{N'-N}(u)]^2 \;, \tag{2.14}$$

where $l' = (l/\sqrt{2})\omega_0$, $\delta_{kk'} = \delta_{k_y k_y'}\delta_{nn'}$, $u = l^2(q_x^2+q_y^2)/2 = l^2 q_\perp^2/2$, and where $L_N^M(u)$ is an associated Laguerre polynomial. The derivation of the above expressions proceeds as in the case of the usual Landau wave functions, when $\sin k_z z$ is replaced by $e^{ik_z z}$ (see Ref. 13, Sec. 4.2 and references cited therein). From Eq. (2.13) and Parseval's theorem we find

$$\int_{-\infty}^{\infty} |F_{nn'}(\pm q_z)|^2 dq_z = \frac{\pi}{L_z}(2+\delta_{nn'}) \;, \tag{2.15}$$

where we have used the fact that n is a positive integer (see also Ref. 7). Furthermore, with Eqs. (2.9) and (3.2) (see Sec. III) we find that the density of states $N(\varepsilon)$ has the form (spin included)

$$N(\varepsilon) = 2\sum_{N,n,k_y}\delta(\varepsilon - \varepsilon_{Nnk_y}) = \frac{A_0}{\pi l^2}\sum_{N,n}\delta(\varepsilon - (N+\tfrac{1}{2})\hbar\omega_0 - n^2\varepsilon_0) \;. \tag{2.16}$$

We can now proceed to the evaluation of the current in the direction of the applied electric field (x axis). Equations (2.10) and (2.11) show that the diagonal ponderomotive current $\langle (J_x)_d \rangle_t$ [second term of Eq. (2.2)] vanishes, but the "collisional" current does not, for $\mu=\nu=x$, since $\alpha_{x\zeta} = (\zeta \mid x \mid \zeta) = x_0$. Moreover, for $\mu=\nu=x$ it can be shown, by a procedure identical with that of Ref. 16, Sec. II, that the nondiagonal contribution for the current, as expressed by Eq. (2.4), vanishes identically for the states (2.8). We are thus left with formula (2.3) which, for $\mu=\nu=x$, takes the form (spin included)

$$\sigma_{xx}^d \equiv \sigma_{xx}^d(0) = \frac{\beta q^2}{\Omega}\sum_{\zeta,\zeta'}\langle n_\zeta\rangle_{\rm eq}(1-\langle n_{\zeta'}\rangle_{\rm eq}) \times w_{\zeta\zeta'}(X_\zeta - X_{\zeta'})^2 \;, \tag{2.17}$$

where $X_\zeta = (\zeta \mid x \mid \zeta)$ [cf. Ref. 13, Eq. (2.84)]. In Sec. III we evaluate this expression for various kinds of phonons.

III. MAGNETOPHONON RESONANCES

From Eq. (2.10) we find

$$X_\zeta = -l^2 k_y \;, \tag{3.1}$$

which entails that the factor $(X_\zeta - X_{\zeta'})^2$ in (2.17) varies as $(k_y - k_y')^2 = q_y^2$ due to the Kronecker delta of (2.12). Substituting (2.5), (2.6), and (2.12)–(2.14) in (2.17), we see that σ_{xx}^d depends on k_y and k_y' only through $(k_y - k_y')^2 = q_y^2$. By symmetry, σ_{yy}^d will vary as q_x^2 so that we can take $\sigma_{xx}^d = (\sigma_{xx}^d + \sigma_{yy}^d)/2$. For the summation over k_y we assume periodic boundary conditions

$$\sum_{k_y} \to \frac{L_y}{2\pi}\int_{-L_x/2l^2}^{L_x/2l^2} dk_y = \frac{A_0}{2\pi l^2} \;, \tag{3.2}$$

where A_0 is the surface area. The limits $\pm L_x/2l^2$ come from the fact that the $\phi_N(x+l^2k_y)$ are centered at $x_0 = -l^2k_y$ $(-L_x/2 \le x \le L_x/2)$. Furthermore, we set $\langle n_\zeta\rangle_{\rm eq} = f_\zeta$, $u = l^2 q_\perp^2/2$, and

$$\sum_{\mathbf{q}} \to \frac{A_0 L_z}{8\pi^3}\int d^3q = \frac{A_0 L_z}{4\pi^2 l^2}\int dq_z \int du \;. \tag{3.3}$$

We also set

$$N' - N = M, \quad M = 0,1,2,\ldots \tag{3.4}$$

in the absorption term of (2.5), and

$$N' - N = -M, \quad M = 0,1,2,\ldots \tag{3.5}$$

in the emission term. Then (2.17) takes the form

$$\sigma_{xx}^d = \frac{\beta e^2 A_0}{4\pi^2 l^2 \hbar}\sum_{N,N',n,n'} f_{Nn}(1-f_{Nn'})\int |F(\mathbf{q})|^2 |F_{nn'}(\pm q_z)|^2 dq_z \times \int u\,|J_{NN'}(u)|^2 du\,(N_0\delta(-M\hbar\omega_0 + [n^2-(n')^2]\varepsilon_0 + E_{\mathbf{q}}) + (1+N_0)\delta(M\hbar\omega_0 + [n^2-(n')^2]\varepsilon_0 - E_{\mathbf{q}}))\;. \tag{3.6}$$

We now consider various kinds of phonons.

A. Optical phonons

As usual, we take $E_q = E = \hbar\omega_L \approx$ const and

$$|F(\mathbf{q})|^2 = \frac{\hbar^2 D^2}{2\Omega\rho E} = D'/\Omega \;, \tag{3.7}$$

where D' is a constant, ω_L is the phonon frequency, and ρ is the density. The integral over q_z is given by (2.15) and that over u by (A1) of Appendix A. Then (3.6) becomes

$$\sigma_{xx}^d = CD' \sum_{N,M,n} f_{Nn}[(1-f_{N+M,n})N_0(2N+M+1)+(1-f_{N-M,n})(1+N_0)(2N-M+1)]\delta(M\hbar\omega_0-\hbar\omega_L)$$
$$+2CD' \sum_{N,M,n,n'} [f_{Nn}(1-f_{N+M,n'})N_0(2N+M+1)\delta(-M\hbar\omega_0+[n^2-(n')^2]\varepsilon_0+\hbar\omega_L)$$
$$+f_{Nn}(1-f_{N-M,n'})(1+N_0)(2N-M+1)\delta(M\hbar\omega_0+[n^2-(n')^2]\varepsilon_0-\hbar\omega_L)] , \quad (3.8)$$

where $C=(\beta e^2/\hbar l^2)(1/2L_z^2)$.

The second term of (3.8) is difficult to evaluate analytically for $n \neq n'$. An estimate of this term can be obtained by transforming the sums over n and n' into integrals and proceeding as, e.g., in Refs. 15 and 17. In what follows we will consider that L_z is so small that no transitions between the levels n can take place due to thermal excitations, or phonons. For GaAs ($m^* \approx 0.07 m_e$) ε_0 is about 50 meV, for $L_z \approx 100$ Å, $E \approx \hbar\omega_0 \approx 1.7B$ meV, with B measured in Teslas and $k_B T \approx 26$ meV at room temperature. That is, we consider that all the carriers are in the lowest subband $n=n'=1$. In this case (3.8) is simplified considerably:

$$\sigma_{xx}^d = 3CD' \sum_{N,M} f_{N1}[(1-f_{N+M,1})N_0(2N+M+1) + (1-f_{N-M1})(1+N_0)(2N-M+1)] \times \delta(M\hbar\omega_0-\hbar\omega_L) . \quad (3.9)$$

Now optical phonons are important at relatively high temperatures. In this case we approximate the factor $(1-f_{N\pm M,1})$ by 1 and f_{N1} by $e^{-\beta(\varepsilon_{N1}-\varepsilon_F)}$. Then (3.9) reduces to

$$\sigma_{xx}^d = 3CD' \sum_{N,M} e^{-\beta(\varepsilon_{N1}-\varepsilon_F)}[(2N+1)(1+2N_0)-M] \times \delta(M\hbar\omega_0-\hbar\omega_L) . \quad (3.10)$$

Equations (3.9) and (3.10) show δ-function singularities of the conductivity at resonance, $M\omega_0=\omega_L$. The sum over M can be performed with the help of Poisson's summation formula as shown in Appendix B. If N is large we can perform the sum over N as well by writing $\sum_N Ne^{-\alpha N}=(-1)(\partial/\partial\alpha)\sum_N e^{-\alpha N}$ and summing the geometric series. Using (B3) we find

$$\sigma_{xx}^d = 3CD' e^{\beta(\varepsilon_F-\varepsilon_0)} \frac{\coth(\beta\hbar\omega_0/2)}{\sinh(\beta\hbar\omega_0/2)} \left[1+2N_0-\frac{\omega_L}{2\omega_0}\right] \times \left\{1+2\sum_{s=1}^{\infty} e^{-2\pi s(\Gamma_N/\hbar\omega_0)} \cos[2\pi s(\omega_L/\omega_0)]\right\} , \quad (3.11)$$

where the damping factor Γ_N is given by (C6). At resonance $\cos[2\pi s(\omega_L/\omega_0)]=1$, the sum over s is easily evaluated, and the quantity in the curly brackets becomes equal to $\coth(\pi\Gamma_N/\hbar\omega_0)$.

B. Polar optical phonons

The only difference from Sec. III A is that

$$|F(\mathbf{q})|^2 = \frac{A}{\Omega q^2} = \frac{A}{\Omega(q_\perp^2+q_z^2)} \approx \frac{A}{\Omega q_\perp^2} , \quad (3.12)$$

where we assumed $q_\perp \gg q_z$ for transport in the (x,y) plane. This approximation allows us to do the integrals over q_z and u, in Eq. (3.6), exactly.[18] In Eq. (3.12) A is the constant of the polar interaction and $l^2 q_\perp^2/2$ will cancel the factor u in Eq. (3.6) (after the integration sign). The integral over u is standard and is listed in Appendix A, cf. (A1) and (A2). Corresponding to (3.10), that is, for high temperatures and $n=n'=1$, we now find

$$\sigma_{xx}^d = 3CA' \sum_{N,M} e^{-\beta(\varepsilon_{N1}-\varepsilon_F)} \left[N_0+\frac{(N-M)!}{N!}(1+N_0)\right] \times \delta(M\hbar\omega_0-\hbar\omega_L) , \quad (3.13)$$

where $A'=Al^2/2$. The sum over M is performed as previously. If $N \gg M$ we can perform the sum over N as well as since $(N-M)!/N! \approx 1$. In this case (3.13) becomes

$$\sigma_{xx}^d = 3CA' e^{\beta(\varepsilon_F-\varepsilon_0)} \frac{\coth(\beta\hbar\omega_0/2)}{\sinh(\beta\hbar\omega_0/2)}(1+2N_0) \times \left[1+2\sum_{s=1}^{\infty} e^{-2\pi s(\Gamma_N/\hbar\omega_0)} \cos[2\pi s(\omega_L/\omega_0)]\right] , \quad (3.14)$$

where Γ_N is given by (C10). At resonance, the peak value of the conductivity is given by (3.14) with the term in large parenthesis replaced by $\coth(\pi\Gamma_N/\hbar\omega_0)$. The period of the oscillations in Eq. (3.14), given by $\omega_L=P\omega_0$, P integer, is the same as in the experiments by Eaves *et al.*[10]

C. Acoustical phonons

In the Debye model $E_q \approx \hbar u_0 q$, where u_0 is the sound velocity. For $F(q)$, as usual, we take

$$|F(\mathbf{q})|^2 = \left[\frac{c^2}{2\Omega\rho u_0}\right] q = \frac{c'q}{\Omega'} , \quad (3.15)$$

where c' is a constant. We will also make the additional approximation $N_0 \approx (1+N_0) \approx 1/\beta\hbar u_0 q$, in order to obtain tractable integrals. All this is substituted in (3.6) with $\hbar u_0 q \approx \hbar u_0 q_\perp$; we obtain

$$\sigma_{xx}^{d}=2Cc''\sum_{N,N',n} f_{Nn}(1-f_{N'n})\int u\;|J_{NN'}(u)|^2 du\,\delta(M\hbar\omega_0-\hbar u_0\sqrt{2u}\,/l)$$
$$+2Cc''\sum_{\substack{N,N',\\ n,n'}} f_{Nn}(1-f_{N'n'})\int u\;|J_{NN'}(u)|^2 du[\delta(-M\hbar\omega_0+[n^2-(n')^2]\varepsilon_0+\hbar u_0\sqrt{2u}\,/l)$$
$$+\delta(M\hbar\omega_0+[n^2-(n')^2]\varepsilon_0-\hbar u_0\sqrt{2u}\,/l)]\;, \tag{3.16}$$

where $c''=c'/\beta\hbar u_0$.

For $n\neq n'$ the second term of (3.16) can be evaluated as in Refs. 15 and 17, after the sums over n and n' are transformed into integrals. In what follows, however, we will will limit ourselves, again, to the case $n=n'=1$ for $L_z\leq 100$ Å. In this case the second term of (3.16) is twice the first term.

1. *Inelastic scattering*

Using the property of the δ function

$$\delta(g(x))=\sum_j \frac{\delta(x-x_j)}{|g'(x_j)|}\;, \tag{3.17}$$

where $g'(x)$ is the derivative of $g(x)$ and x_j are found by solving $g(x_j)=0$, we easily find

$$\sigma_{xx}^{d}=3Cc''\frac{\omega_0}{u_0^4}\left[\frac{\hbar}{m^*}\right]^2 \times\sum_{NN'} f_{N1}(1-f_{N'1})(N-N')^3 \times\left|J_{NN'}\left[\left[\frac{(N-N')}{\sqrt{2}}\frac{l\omega_0}{u_0}\right]^2\right]\right|^2,\quad N\neq N'\;. \tag{3.18}$$

This result, which is exact for two-dimensional phonons, would be slightly more complicated if the approximation $N_0\approx 1/\beta\hbar u_0 q$ were not made. Without it, $2c''\to c'$ in (3.18) and a factor $q_\perp[1+2N(q_\perp)]$ evaluated at $q_\perp=(N-N')(\omega_0/u_0)\sqrt{2}/l$ would multiply the summand.

2. *Elastic scattering*

A much simpler result than (3.18) is obtained if $\hbar u_0\sqrt{2u}\,/l$ is neglected in (3.16). We find

$$\sigma_{xx}^{d}\approx 6Cc''\sum_N (2N+1)f_{N1}(1-f_{N1})\;. \tag{3.19}$$

Acoustical phonons are important at low temperatures, in which case the factor $f_{N1}(1-f_{N1})$ behaves like a δ function. On the other hand, the approximation $N_0\approx 1/\beta\hbar u_0 q$ requires not very low temperatures. To obtain insight, however, we make the approximation $\beta f_{N1}(1-f_{N1})\approx\delta(\varepsilon_{N1}-\varepsilon_F)$ and we use (B3). We then obtain

$$\sigma_{xx}^{d}\approx 12Cc''(\bar{\varepsilon}_F/\beta)\times\left[1+2\sum_{s=1}^{\infty}(-1)^s e^{-2\pi s(\Gamma_N/\hbar\omega_0)}\cos(2\pi s\bar{\varepsilon}_F)\right], \tag{3.20}$$

where $\bar{\varepsilon}_F=(\varepsilon_F-\varepsilon_0)/\hbar\omega_0$ and Γ_N is given by (C11). If $\Gamma_N/\hbar\omega_0\ll 1$, oscillations of the conductivity as a function of the magnetic field are expected (in pure samples) whenever $\varepsilon_F-\varepsilon_0=P\hbar\omega_0$. In this case (3.20) becomes

$$\sigma_{xx}^{d}\approx 12Cc''(P/\beta)(1-2e^{-2\pi\Gamma_N/\hbar\omega_0})\;. \tag{3.21}$$

We further notice that the approximation $\Gamma_N\approx\hbar/\tau$ [c.f. (C11)] gives a resonance value of the conductivity proportional to the magnetic field.

D. Piezoelectrical phonons

For the electron-piezoelectric phonon interaction we have $|F(\mathbf{q})|^2=\bar{P}/\Omega q$, where $\bar{P}$ is the piezoelectric constant.

1. *Inelastic scattering*

The result, corresponding to (3.18), is

$$\sigma_{xx}^{d}=(6CP'/m^*u_0^2)\times\sum_{N,N'} f_{N1}(1-f_{N'1})(N-N')\times\left|J_{NN'}\left[\left[\frac{(N-N')}{\sqrt{2}}\frac{l\omega_0}{u_0}\right]^2\right]\right|^2,\quad N\neq N' \tag{3.22}$$

where $P'=\bar{P}/\beta\hbar u_0$.

2. *Elastic scattering*

If $E_q\approx 0$ we find

$$\sigma_{xx}^{d}\approx 12CP'\sum_N f_{N1}(1-f_{N1})\;. \tag{3.23}$$

For low temperatures $\beta f_{N1}(1-f_{N1})\approx\delta(\varepsilon_{N1}-\varepsilon_F)$ and the conductivity as given by (3.23), is proportional to the density of states at the Fermi level. Corresponding to (3.20) we obtain

$$\sigma_{xx}^{d}\approx(12CP'/\beta)\left[1+2\sum_{s=1}^{\infty}(-1)^s e^{-2\pi s(\Gamma_N/\hbar\omega_0)}\times\cos(2\pi s\bar{\varepsilon}_F)\right], \tag{3.24}$$

where Γ_N is given by (C13). At resonance Eq. (3.24) becomes

$$\sigma_{xx}^{d}\approx(12CP'/\beta)(1-2e^{-2\pi\Gamma_N/\hbar\omega_0})\;. \tag{3.25}$$

The quantities CP'/β, in (3.25), and Cc''/β, in (3.21), are proportional to the temperature and to the magnetic field.

IV. REMARKS AND CONCLUSIONS

The results of this paper are valid for electron-phonon interactions treated in the first Born approximation. Moreover, the assumption has been made that only the lowest subband ($n = n' = 1$) is occupied, which limits their applicability to very thin wells ($L_z \leq 200$ Å). However, this is only a quantitative limitation. The first term of Eqs. (3.8), (3.16), and (C2) shows that the oscillations reported here remain unaffected when the sum over n is performed in the parts of the final results coming from those terms; one has to replace ε_0 by $n^2\varepsilon_0$ and sum one-third of the results over n. We do not expect the second term ($n \neq n'$) to qualitatively alter the results of the first term for large thicknesses since the sample becomes three-dimensional with a similar qualitative behavior.[11] We further note that the assumption of a spherical effective mass, which is not that realistic,[19] is not necessary for the first term, i.e., $m_\perp^*$ can be different than m_z^*.

At this point we are not aware of experimental work other than Refs. 10 and 22, in which magnetophonon oscillations are reported in superlattice structures ($L_z \leq 1\mu$). Therefore, we cannot compare our theory with the experiment more than qualitatively. Magnetophonon oscillations of the conductivity (for polar optical phonons) with the resonance condition reported here have been observed in Ref. 10; the results indicate that the conductivity depends on the thickness L_z. Our results show that the conductivity varies in all cases as $1/L_z^2$, whereas the inverse scattering rates as $1/L_z$; the latter result is in agreement with Ref. 8, the former is probably new. This dependence of the scattering rates on the thickness of the well has been obtained in the absence of the magnetic field as well.[4–7]

The temperature and magnetic field dependence of the inverse scattering rates are in agreement with the theoretical results of Ref. 8 as noted also in Appendix C. The dependence of the level widths Γ_N on magnetic field and thickness, however, is tied to the approximation $\Gamma_N \sim \hbar/\tau$ and this is done in order to have tractable algebraic equations when more than one term is kept in the expansion of $\coth(\pi\Gamma_N/\hbar\omega_0)$. For acoustical and piezoelectrical phonon scattering at low temperatures we expect oscillations, in very pure systems, when the magnetic field is varied with resonance condition $P\hbar\omega_0 = \varepsilon_F - \varepsilon_0$. To our knowledge this result is new and experimental work is needed to test its validity.

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APPENDIX A

The first three integrals below are listed in tables in Ref. 20 and the fourth one is explicitly evaluated in Ref. 15:

$$I_1 = \int_0^\infty e^{-x} x^M [L_N^M(x)]^2 dx = \frac{(N+M)!}{N!}, \quad M > 0 \tag{A1}$$

$$I_2 = \int_0^\infty e^{-x} x^M [L_{N-M}^M(x)]^2 dx = 1, \quad M > 0 \tag{A2}$$

$$I_3 = \int_0^\infty e^{-x} [L_N(x)]^2 dx = 1 , \tag{A3}$$

$$I_4 = \int_0^\infty e^{-x} x^{M+1} [L_N^M(x)]^2 dx = \frac{(N+M)!}{N!}(2N+M+1) . \tag{A4}$$

Another useful integral is

$$I_5 = \int_0^\infty e^{-x} x^{M-1} [L_N^M(x)]^2 dx, \quad M \neq 0 .$$

We set $M' = M-1$, we use the property

$$L_N^{M'+1}(x) = \sum_{N'=0}^{N} L_{N'}^{M'}(x) , \tag{A5}$$

and the orthogonality property of the Laguerre polynomials. Together with (A1) this gives

$$I_5 = \sum_{N'=0}^{N} \frac{(N'+M')!}{N'!} = \frac{1}{M}\frac{(N+M)!}{N!}, \quad M \neq 0 . \tag{A6}$$

The last step is proven by induction.

Finally, by contour integration one can prove that

$$I_6 = \int_{-\infty}^{\infty} \frac{\cos(2\pi s x)}{(x+a)(x^2+b^2)} dx = \mathrm{Re} \int_{-\infty}^{\infty} \frac{e^{2i\pi s x} dx}{(x+a)(x^2+b^2)} = \frac{a}{a^2+b^2} e^{-2\pi s b}, \quad s \geq 0 . \tag{A7}$$

Equation (A7) will be used in the evaluation of the scattering rate for polar optical phonons, cf. (C7) and (C8).

APPENDIX B

Below we evaluate the quantity $\sum_M \delta(g(M))$, where $g(M)$ is a periodic function of M. In a similar manner one can evaluate the sum $\sum_M \phi(M)\delta(g(M))$.

We use the Poisson's summation formula in the form[21]

$$\sum_{M=0}^{\infty} f(M+\tfrac{1}{2}) = \int_0^\infty f(x)dx + 2\sum_{s=1}^{\infty}(-1)^s \int_0^\infty f(x)\cos(2\pi s x)dx . \tag{B1}$$

Applying (B1) for $\delta(M - \omega_L/-\omega_0)$ we find

$$\sum_{M=0}^{\infty} \delta(M - \omega_L/\omega_0) = 1 + 2\sum_{s=1}^{\infty} \cos[2\pi s(\omega_L/\omega_0)] . \tag{B2}$$

Due to the δ function, the lower limit in (B1) can be replaced by $-\infty$. If then the δ function is approximated by a Lorentzian of width Γ_N and shift zero, the integration involved can be done analytically[20] and the result is

$$\sum_{M=0}^{\infty} \delta(M - \omega_L/\omega_0) \approx 1 + 2\sum_{s=1}^{\infty} e^{-2\pi s(\Gamma_N/\hbar\omega_0)} \cos[2\pi s(\omega_L/\omega_0)] . \tag{B3}$$

APPENDIX C

Below we evaluate the inverse scattering rates from which the damping factors Γ_N, appearing in the text, can be estimated according to $\Gamma_N \approx \hbar/\tau$. We have

$$\frac{1}{\tau}=\sum_{\zeta'} w_{\zeta\zeta'}=\sum_{N',k_y',n'} w_{Nk_ynN'k_y'n'} \; . \tag{C1}$$

When Eqs. (2.5) and (2.6) are substituted in (C1) as well as Eqs. (2.12)—(2.14), we find

$$\frac{1}{\tau}=\frac{A_0}{2\hbar l^2}\sum_{N',n'}(2+\delta_{nn'})\int |F(\mathbf{q}_\perp)|^2 J_{NN'}(u)|^2 du[N_0\delta(-M\hbar\omega_0+[n^2-(n')^2]\varepsilon_0+E\mathbf{q}_\perp)$$
$$+(1+N_0)\delta(M\hbar\omega_0+[n^2-(n')^2]\varepsilon_0-E\mathbf{q}_\perp)] \; , \tag{C2}$$

where we have neglected the component q_z in the quantities $F(\mathbf{q})$ and $E_\mathbf{q}$. In both terms of $(2+\delta_{nn'})$ we will again take $n=n'=1$.

1. Optical phonons

Using (3.7), (3.4), (3.5), and (A1) we find

$$\frac{1}{\tau}=3KD'\sum_M\left[N_0+\frac{(N-M)!}{N!}(1+N_0)\right]\delta(M\hbar\omega_0-\hbar\omega_L) \; , \tag{C3}$$

where $K=1/2\hbar l^2 L_z$. If we approximate $(N-M)!/N!$ by 1, then (B3) gives us

$$\frac{1}{\tau}=3KD'(1+2N_0)\left[1+2\sum_{s=1}^{\infty}e^{-2\pi s(\Gamma_N/\hbar\omega_0)}\times\cos[2\pi s(\omega_L/\omega_0)]\right] . \tag{C4}$$

At resonance $\omega_L=P\omega_0$, $\cos 2\pi s(\omega_L/\omega_0)=1$, and (C4) takes the form

$$\Gamma_N\approx 3\hbar KD'(1+2N_0)\coth(\pi\Gamma_N/\hbar\omega_0) \; . \tag{C5}$$

The solution (graphical) of (C5) determines Γ_N. If broadening is not included the factor, $e^{-2\pi s(\Gamma_N/\hbar\omega_0)}$ does not appear in (C4) and $1/\tau$ diverges, as found numerically in Ref. 8. For the oscillations to be clearly observed $(\pi\Gamma_N/\hbar\omega_0)<1$, cf. (C4). With $\coth x\approx 1/x+x/3-x^3/45$, we obtain from (C5) the approximate result

$$\frac{\pi\Gamma_N}{\hbar\omega_0}\approx(15\{1-3\Delta+[(1-3\Delta)^2+36]^{1/2}\}/2)^{1/2} \; ,$$
$$\Delta=\frac{\hbar^2L_z^2}{3\pi m^* D'(1+2N_0)} \; . \tag{C6}$$

Equation (C6) shows that Γ_N is independent of N and proportional to the magnetic field. Its temperature dependence is contained in N_0.

2. Polar optical phonons

We proceed as in Sec. 1 of Appendix C. With (3.12) and (A6) we find

$$\frac{1}{\tau}=\Lambda\sum_M\delta(M\hbar\omega_0-\hbar\omega_L)/M \; , \tag{C7}$$

where $\Lambda=3KA'(1+2N_0)l^2/2$. When the δ function is replaced by a Lorentzian, (B1) and (A7) give

$$\frac{1}{\tau}=\frac{\Lambda}{\hbar\omega_0}\frac{(\omega_L/\omega_0)}{(\omega_L/\omega_0)^2+(\Gamma_N/\hbar\omega_0)^2}\times\left[1+2\sum_{s=1}^{\infty}e^{-2\pi s(\Gamma_N/\hbar\omega_0)}\times\cos[2\pi s(\omega_L/\omega_0)]\right] . \tag{C8}$$

At resonance (C8) gives the equation for Γ_N:

$$x\approx\hbar\Lambda\left[\frac{\pi}{\hbar\omega_0}\right]^2\frac{\delta}{\delta^2+x^2}\coth x \; , \tag{C9}$$

where $x=\pi\Gamma_N/\hbar\omega_0$ and $\delta=\pi\omega_L/\omega_0$. If broadening is not included $1/\tau$ is proportional to the quantity in large parenthesis in (C8) with e^{-x} replaced by 1 and it diverges at the resonance (see also, Ref. 8). If $\coth x$ is approximated by $1/x$, Eq. (C9) gives us

$$\frac{\pi\Gamma_N}{\hbar\omega_0}=\left[\frac{\delta(\delta^2+4\Lambda')^{1/2}-\delta}{2}\right]^{1/2} , \quad \Lambda'=\pi\Lambda/\omega_L\omega_0 \; . \tag{C10}$$

Again, Γ_N is independent of N but its dependence on the magnetic field is more complex than that of the preceding Γ_N, [Eq. (C6)].

3. Acoustical phonons

a. Elastic scattering

With (3.4), (3.5), (3.15), (A1), and $N_0\approx 1+N_0\approx 1/\beta\hbar u_0 q$ we find

$$1/\tau\approx 6Kc'' \; . \tag{C11}$$

Equation (C11) shows that $1/\tau$ varies linearly with temperature and magnetic field in close agreement with Ref. 8.

b. Inelastic scattering

Repeating the steps in Sec. 3 a, we find

$$\frac{1}{\tau}=(6Kc''/m^*u_0^2)\times\sum_{N'}(N-N')\left|J_{NN'}\left[\left(\frac{(N-N')l}{\sqrt{2}}\frac{\omega_0}{u_0}\right)^2\right]\right|^2 . \quad \text{(C12)}$$

4. Piezoelectrical phonons

a. Elastic scattering

If we replace q_z in $F(\mathbf{q})$ and $N_0 \approx (1+N_0) \approx 1/\beta\hbar u_0 q$ by some average value $\bar{q}_z$ we obtain

$$\frac{1}{\tau}\approx 6KP'l^2\int_0^\infty \frac{|J_{NN}(u)|^2du}{2u+q_0},\quad q_0=l^2\bar{q}_z^2 . \quad \text{(C13)}$$

This gives $1/\tau$ in terms of exponential integrals [if q_z is neglected in $F(\mathbf{q})$ the integral over u diverges]. Particular cases $(N=0,1,2,\ldots)$ can be worked out. For $N=0$, (C13) takes the form

$$\frac{1}{\tau}\approx 6KP'l^2 e^{q_0/2}E_i(q_0/2),\quad E_i(\alpha)=\int_\alpha^\infty \frac{e^{-u}}{u}du . \quad \text{(C14)}$$

b. Inelastic scattering

In this case $M=N-N'\neq 0$ and the result is

$$\frac{1}{\tau}=6KP'/m^*\omega_0^2\times\sum_{N'}(N-N')^{-1}\left|J_{NN'}\left[\left(\frac{(N-N')l}{\sqrt{2}}\frac{\omega_0}{u_0}\right)^2\right]\right|^2 . \quad \text{(C15)}$$

Notice that all scattering rates are inversely proportional to the thickness of the well $(K=1/2\hbar l^2 L_z)$.

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