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Effect of magnetic-field-induced frustration on the superconducting transition of proximity-coupled arrays

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The magnetic field dependence of the superconducting transition has been studied in triangular arrays of Pb-Sn proximity-coupled junctions. The transition temperature T_c and the array resistance $R(T)$ are found to vary periodically in a perpendicular magnetic field with a period corresponding to one flux quantum per unit cell of the array. In addition, both quantities show substructure at rational fractions of a flux quantum $f=\Phi/\Phi_0=\frac{1}{4},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{3}{4}$. The current-voltage characteristics of the arrays are nonlinear at temperatures near the transition temperature T_c, with voltage increasing as a power of the current $V\sim I^{a(T,f)}$. As a function of reduced temperature $t=[T_c(f)-T]/T_c(f)$, the power-law exponent $a(t,f)$ is insensitive to f.

Weakly coupled Josephson arrays form a new class of superconductors whose critical properties differ markedly from both bulk and thin film superconductors. In zero external field, Josephson arrays having $LI_c \ll \Phi_0$ (L is the inductance and I_c is the critical current of a primitive cell) fall in the same universality class as the XY model, with a resistive transition modeled as a Kosterlitz-Thouless vortex unbinding transition.[1]

In a weak magnetic field, however, Josephson arrays exhibit properties unlike other superconducting systems. In particular, the low-field magnetoresistance of square arrays has been shown to be periodic in an external magnetic field, with pronounced minima occurring when the external flux threading a primitive cell of the array equals an integer or half-integer multiple of the flux quantum, $\Phi_0=hc/2e$.[2] Furthermore, there is growing speculation that the universality class of Josephson arrays in a magnetic field may not be that of the XY model, with the detailed critical behavior a function of magnetic field. This paper addresses these issues by reporting on the magnetic field dependence of the electrical resistance, current-voltage characteristics, and superconducting transition temperature of triangular arrays of proximity-coupled junctions.

Recent theoretical treatments of Josephson arrays have predicted a periodic variation of array properties with substructure at a large number of rational f, where $f=\Phi/\Phi_0$.[3-5] Although these calculations have focused on the ground-state energy, $E_0(f)$, the critical temperature, $T_c(f)$, and the zero-temperature critical current, $I_c(T=0,f)$, simple physical arguments suggest that the resistance of arrays at $T>T_c$ should exhibit analogous periodic behavior.

Teitel and Jayaprakash[4] have calculated $E_0(f)$ exactly for a square lattice of Josephson junctions at several rational f, and Shih and Stroud[5] have calculated $E_0(f)$ for triangular and honeycomb lattices. In all cases $|E_0(f)|$ is found to have a periodicity of 1 and to be symmetric at about $f=\frac{1}{2}$. As f increases from 0 to $\frac{1}{2}$, $|E_0(f)|$ is generally decreasing but has sharp upward features at several f.

Similar periodic behavior is expected for $T_c(f)$. Although explicit calculations of $T_c(f)$ have proved difficult, Teitel and Jayaprakash[4] have shown that T_c is related to $E_0(f)$ through the inequality $T_c(f)<(\pi/k_b)|E_0(f)|(1/q)$ for values of f satisfying the relationship $f=p/q$, with p and q being integers having no common factor. Monte Carlo simulations[4,5] have been used to estimate T_c at a few values of f for square, triangular, and honeycomb lattices. In addition, Shih and Stroud[5] have developed a mean-field theory that allows the calculation of T_c at arbitrary rational f. Although not strictly applicable to two-dimensional systems, mean-field theory allows one to examine the qualitative structure of $T_c(f)$ over a broad range of f. The mean-field results indicate that T_c is generally decreasing on $f\in[0,\frac{1}{2}]$, with sharp upward features at various rational f, and that the relative magnitudes of these features are lattice dependent. For both a square lattice and a triangular lattice the largest feature occurs at $f=\frac{1}{2}$. However, the second largest feature is at $f=\frac{1}{3}$ in a square lattice but at $f=\frac{1}{4}$ in a triangular lattice, where it is of nearly the same magnitude as the $f=\frac{1}{2}$ feature. In contrast, the largest feature in a honeycomb lattice is predicted to occur at $f=\frac{1}{3}$ rather than $f=\frac{1}{2}$.

For Josephson junction arrays, only features at $f=\frac{1}{2}$ have been resolved[2,3,6,7] to date. However, in honeycomb lattices of superconducting wires, Pannetier, Chaussy, Rammal, and Villegier[8,9] have observed local minima in the magnetoresistance at $f=\frac{1}{3},\frac{2}{3}$ but not at $f=\frac{1}{2}$. They have also measured the transition temperature as a function of magnetic field in square arrays of wires and see T_c maxima at $f=\frac{1}{4},\frac{1}{3},\frac{2}{5},\frac{1}{2},\frac{3}{5},\frac{2}{3}$, and $\frac{3}{4}$. The largest of these maxima is at $f=\frac{1}{2}$ and the second largest is at $f=\frac{1}{3}$, in accord with theory.

The details of the phase transition in Josephson arrays at nonzero f are generally not well understood. For $f=\frac{1}{2}$, Monte Carlo simulations[4,5] of both square and triangular lattices have shown a size-dependent peak in the specific heat, indicative of a second-order phase transition rather than a Kosterlitz-Thouless (KT) transition. Furthermore, mean-field theory—with some corroboration from Monte Carlo simulations—suggests that a triangular lattice at $f=\frac{1}{4}$ and $\frac{3}{8}$ may exhibit an unusual double transition.[10] Because the specific-heat structure arising from the Josephson coupling is immeasurably small, the only obvious manifestation of the phase transition would be the temperature dependence of the electrical resistance and the current-voltage characteristics. Unfortunately, no theoretical prediction for these properties has been made except for $f=0$.

EXPERIMENTAL DETAILS

The samples used in this study consist of triangular arrays of Pb islands approximately 1500 Å thick with an overlay of Sn approximately 1000 Å thick. The six-sided asterisk shape of the Pb islands, fabricated using electron beam lithography techniques, is shown in the inset to Fig. 1. A typical array has an area of 1 cm^2 and contains 10^6 Pb islands, with spacing between the edges of adjacent islands of 0.8 μm. The Sn overlay is deposited in an rf sputtering system after briefly etching the surface of the Pb at low rf power to remove oxides formed during previous processing steps.

Measurements of the resistivity and I-V characteristics were obtained between 4.0–8.0 K using a bridge circuit with a SQUID galvanometer as a null detector. The resolution of the bridge was about 10^{-12} V and typical sample currents were 1–10 μA. A magnetic field up to 10 G could be applied perpendicular to the plane of the samples using a small superconducting solenoid. The cryostat was vibration isolated and enclosed in a double-walled μ-metal shield in order to reduce ambient magnetic fields to less than 1 mG.

EXPERIMENTAL RESULTS

As shown in Fig. 1, the resistance of the triangular arrays falls to zero in a broad transition, characterized by both a resistive tail in which the resistance becomes exponentially small and a crossover from linear (Ohmic) to nonlinear power-law I-V characteristics as the exponential region is approached. In zero magnetic field, the resistive tail and the non-Ohmic behavior have been previously explained in terms of a KT vortex unbinding transition.[1]

Figure 1 also illustrates the influence of an external magnetic field on the resistive transition for several values of f. All data were taken by cooling the sample from the normal state to the desired temperature in a constant external magnetic field. (This "field cooling" step appeared to be necessary to inhibit the formation of metastable vortex states, corresponding to local minima in the free energy.) It is clear from the figure that the transition temperature T_c is an oscillatory function of magnetic field. As f is increased from 0, T_c decreases by about 0.25 K at $f=\frac{1}{2}$ and then returns nearly to its original value at $f=1$. The departure from strict periodicity in f may result from the depression of the Josephson coupling energy with external magnetic field. Granato and Kosterlitz[11] have also suggested that weak disorder arising from nonuniform areas of the superconducting islands can also suppress the perfect periodicity in f.

Figure 2(a) shows that the I-V power-law exponent $a(T)$ tracks the displacement of T_c and is otherwise not strongly affected by the magnetic field. To illustrate this point, Fig. 2(b) shows the exponent as a function of reduced temperature $t=(T_c-T)/T_c$. [Because the broad nature of the transition makes it difficult to define T_c with precision, we have arbitrarily defined $T_c(f)$ to be the lowest temperature at which linear I-V characteristics could be observed.] Although a "universal jump" in $a(T)$ from 1 to 3 at T_c is expected for a KT transition, the array system appears to display only a poorly defined jump, with an increase from $a(T)=1$ to $a(T)\approx 2$ as T_c is approached from above. Note that the insensitivity of the I-V characteristics to magnetic field does not support the idea that the $f=\frac{1}{2}$ and

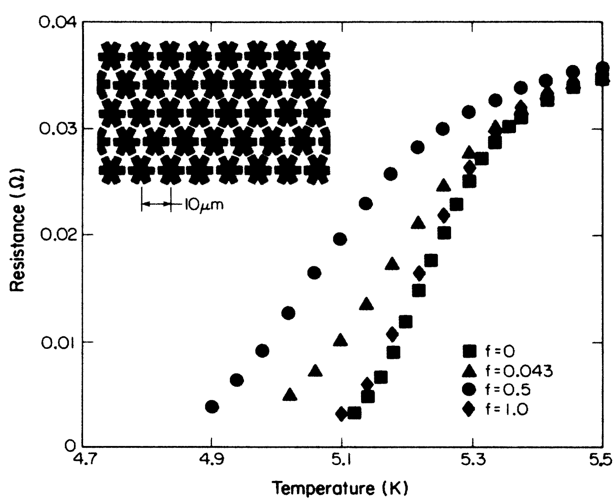

FIG. 1. The resistive transition $R(T)$ of a triangular array of Pb-Sn-Pb proximity-coupled junctions at different values of the normalized magnetic field $f=\Phi/\Phi_0$. The inset shows the six-sided asterisk shapes of the triangular array elements.

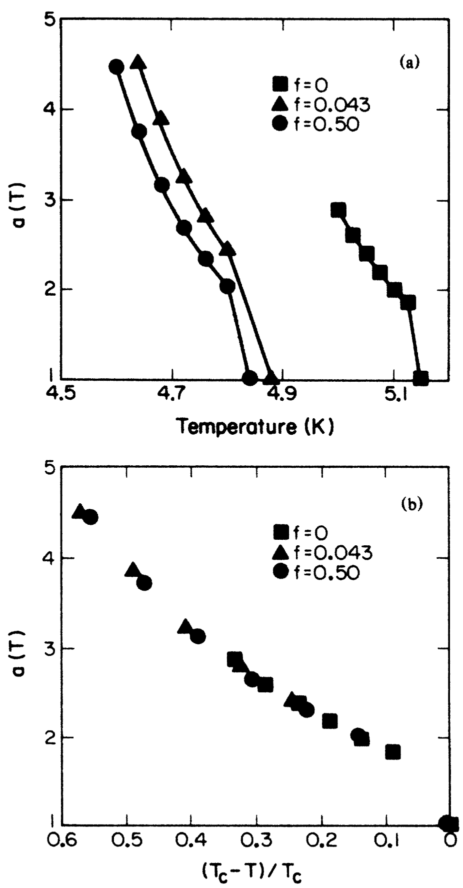

FIG. 2. (a) The current-voltage exponent $a(T)$ for different values of f; (b) the exponent $a(T)$ as a function of reduced temperature $t=(T-T_c)/T_c$.

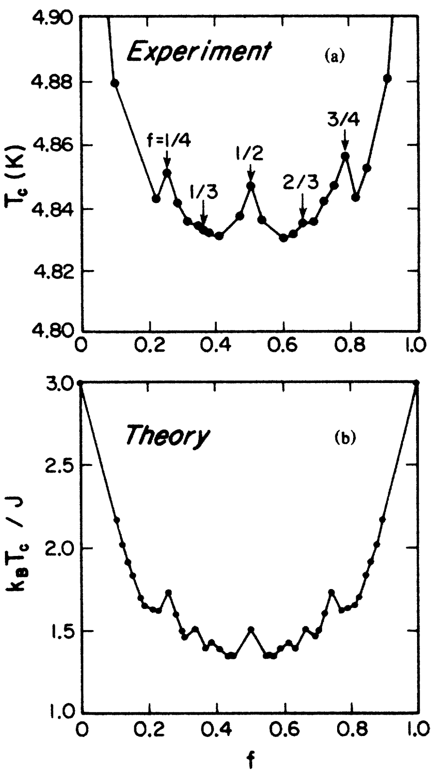

FIG. 3. (a) The transition temperature T_c of a triangular array of proximity-coupled junctions as a function of f; (b) theoretical prediction of T_c as a function of f for a triangular Josephson junction array, from Ref. 5.

$f=\frac{1}{4}$ transitions in triangular lattices fall in a different universality class than the Kosterlitz-Thouless–like $f=0$ transition.

Figure 3(a) shows the variation of T_c with external magnetic field, while Fig. 3(b) shows the mean-field results of Shih and Stroud,[5] plotted in the reduced units $k_b T_c/J$, where $J=(\hbar/2e)I_c$ and I_c is the single junction critical current. The data are in qualitative agreement with theory, with T_c generally decreasing as f increases from 0 to $\frac{1}{2}$, and with sharp upward features at several rational f. The largest of these features is at $f=\frac{1}{2}$ and the second largest, which is nearly of equal magnitude, occurs at $f=\frac{1}{4}$. A possible feature at $f=\frac{1}{3}$ is also noted in the figure. The data are nearly symmetric at about $f=\frac{1}{2}$, with features at $f=\frac{2}{3}$ and $\frac{3}{4}$.

Similar features were observed in measurements of the resistivity, although the substructure tended to wash out at temperatures more than 100 mK above T_c; the absence of substructure at higher temperatures indicates that the periodicity in $R(f)$ cannot be viewed as a rigid translation of the R vs T curve along the temperature axis. This conclusion is also evident from inspection of Fig. 1.

In conclusion, we have studied the resistive transition of triangular arrays of proximity-coupled junctions in a weak magnetic field. We find that both the resistance $R(T)$ and the transition temperature T_c vary periodically in a magnetic field, with a major period corresponding to integer multiples of Φ_0 per unit cell. There is additional substructure in both $R(T)$ and T_c at $f=\frac{1}{4}$, $\frac{1}{3}, \frac{1}{2}, \frac{2}{3}$, and $\frac{3}{4}$. The relative magnitudes of these features are in qualitative agreement with theory. Measurements of the I-V characteristics at various f do not show features attributable to different classes of phase transitions.

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