# Anderson localization and delocalization in an electric field

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The possibility of Anderson localization of electrons in a disordered solid in $d$ dimensions in the presence of a finite, uniform electric field is discussed. The self-consistent diagrammatic theory of localization developed for zero fields is generalized to treat the case of a finite electric field. In one-dimensional systems this theory is shown to reproduce the exact results of Prigodin except for some minor differences. For weak fields, or strong disorder, there is power-law localization and for stronger fields there is a mobility edge past which the states are extended. In higher dimensions the self-consistent theory leads to the conclusion that Anderson localization is not possible in finite electric fields. Simple arguments indicate that this conclusion is independent of the self-consistent theory.

## I. INTRODUCTION

In this paper, I examine the possibility of Anderson localization of electrons in a disordered solid in $d$ dimensions in the presence of a finite, uniform electric field. In zero electric fields it is believed that for $d \leq 2$ all quantum-mechanical states are localized, while for $d > 2$ the states are extended for weak disorder, but it is also known that for stronger disorder there is a mobility edge past which the states are localized.[1,2]

For the case of a one-dimensional disordered solid in a finite electric field there are rigorous results available both for smooth[3] and for discontinuous[4] electron-impurity scattering potentials. The rigorous results for these two types of scattering centers are quite different for reasons that will be discussed below. Here, I consider only disordered solids where the electron-impurity scattering is "hard" or discontinuous. For this case Delyon *et al.*[4] showed, rigorously, that for a small but finite field the states are still localized but that the localization is power-law localization rather than exponential localization typical for electron systems in zero field. For large electric fields, Delyon *et al.* showed that the states are extended. This implies that there is a mobility edge at some critical value of the electric field and impurity density. These results supported earlier[5] computer simulations, which indicated that in a finite electric field the states were power-law localized.

In addition to these rigorous results in $d = 1$, there are exact results for the one-dimensional case. Prigodin[6] generalized the Berezinskii diagrammatic technique[7] for zero field to finite fields. He explicitly calculated the density-density correlation function (DDCF) and showed that for small fields there was power-law localization; he calculated the exponents of the power-law decay. He also obtained the value of the critical field (or impurity density), above which the states (or DDCF) are extended. His results are consistent with the rigorous results of Delyon *et al.*[4]

In this paper I use an approximate theory to discuss Anderson localization in finite electric fields in $d$ dimensions. The basic idea is to generalize the self-consistent diagrammatic theory of Vollhardt and Wölfle[1,2,8] to treat the case of finite electric fields. I show that this approximate theory leads to qualitatively correct results in $d = 1$ by comparing the results with those of Prigodin.[6] It should be remarked that this is interesting by itself since the calculations presented here are quite simple while the Berezinskii technique[7] used by Prigodin is quite complicated. Furthermore, the one-dimensional calculation given here suggests that the self-consistent diagrammatic approach can be reliably used in finite electric fields in higher dimensions. The Berezinskii technique cannot be generalized to higher dimensions.

Two-dimensional electron systems are then considered. By examining the structure of the theory and by explicit calculation I show that within the approximate theory used here that the states are always extended in $d = 2$ for finite electric fields. I then interpret this result physically, and argue that it is independent of the approximate theory used here. The general conclusion is that Anderson localized states are not possible in $d = 2$ in a finite electric field. The same arguments lead to the conclusion that localized states are also not possible in $d = 3$.

It should be remarked, that it is not clear *a priori* whether or not Anderson localization is possible in $d$ dimensions in a finite electric field. It is clear that an electric field will cutoff the singularity in the maximally crossed diagrams that cause electron localization in $d \leq 2$ in zero field for any impurity density. However, this cutoff is proportional to the electric field times the diffusion coefficient. Therefore, it is possible that the self-consistent theory of localization could lead to an Anderson transition at finite fields and impurity densities where the diffusion coefficient, and hence the cutoff mentioned above, vanishes. I show here how the possibility of an Anderson transition in $d = 2$ (and $d = 3$) is removed by a quite different mechanism. This mechanism is not operative in $d = 1$ for reasons discussed below.

It is also interesting to compare the effects of an elec-

 

tric field to those of a magnetic field[1] on Anderson localization. In $d=2$ a magnetic field removes the singularity associated with the maximally crossed diagrams but does not effect possible singularities arising from the ordinary ladder diagrams. This distinction is due to the breaking of the time-reversal symmetry by the magnetic field. The net result appears to be that for small magnetic fields the states are still localized. An electric field does not break time-reversal symmetry and removes the possible singularities from both the maximally crossed diagrams and ladder diagrams in the same way. As mentioned above, this leaves open the possibility of an Anderson transition at finite-impurity density.

The paper is organized as follows. In Sec. II the general formalism of the self-consistent diagrammatic theory of localization for finite electric fields is developed. The basic idea is to obtain a closed equation for the effective diffusion coefficient, or generalized relaxation time, in terms of the DDCF. In Sec. III the resulting nonlinear equations are solved for $d=1$ and in Sec. IV they are solved for $d=2$. In Sec. V this paper is concluded with a discussion.

## II. GENERALIZED KINETIC EQUATION FOR AN ELECTRON IN A DISORDERED SOLID IN AN ELECTRIC FIELD

In this section we first define the basic quantities of interest in the theory of Anderson localization used here. A Bethe-Salpeter equation is then given for the DDCF. For the inhomogeneous system considered here it is more natural to work in configuration space than in momentum space as is the case for homogeneous disordered solids. The Bethe-Salpeter equation is then transformed to a generalized kinetic equation in real space by introducing Wigner variables. An approximate formal solution to this equation is then obtained in the approximation where only the lowest order in the impurity density diagrams and the maximally crossed diagrams are retained. The idea here is that we take into account the leading diagrams as well as the most important hydrodynamic pole diagrams, i.e., the maximally crossed diagrams.

### A. Basic definitions and the general kinetic equation

We consider an electron with energy $E$ in a $d$-dimensional disordered solid in a uniform electric field directed along the negative $x$ axis so that the force, $\mathbf{F}$, on the electron is directed along the positive $x$ axis. The basic quantity of interest in electron localization theory is the DDCF:

$$\psi_E(\mathbf{x}_1,\mathbf{x}_2,t)=\langle\langle E\,|\,\delta[\mathbf{x}_1-\hat{\mathbf{q}}(t)]\delta(\mathbf{x}_2-\hat{\mathbf{q}})\,|\,E\rangle\rangle\ , \tag{2.1a}$$

or its one-sided Fourier transform

$$\psi_E(\mathbf{x}_1,\mathbf{x}_2,\omega)=\int_0^\infty dt\, e^{i(\omega+i0)t}\psi_E(\mathbf{x}_1,\mathbf{x}_2,t)\ . \tag{2.1b}$$

Here $|E\rangle$ denotes an energy eigenstate, $\hat{\mathbf{q}}(t)$ is the quantum-mechanical position operator at time $t$, $\hat{\mathbf{q}}(0)=\hat{\mathbf{q}}$, circumflexes denote quantum-mechanical operators, and the bold angular brackets in Eq. (2.1a) denote an average over the randomness. By standard manipulations[1-2,6-8] $\Psi_E(\mathbf{x}_1,\mathbf{x}_2,\omega)$ can be written as

$$\Psi_E(\mathbf{x}_1,\mathbf{x}_2,\omega)=\frac{\hbar}{2\pi}\left\langle\left\langle\mathbf{x}_1\left|\frac{1}{E-\hbar\omega-\hat{H}-i0}\right|\mathbf{x}_2\right\rangle\right. \times\left\langle\mathbf{x}_2\left|\frac{1}{E-\hat{H}+i0}-\frac{1}{E-\hat{H}-i0}\right|\mathbf{x}_1\right\rangle\right\rangle. \tag{2.1c}$$

It is known that only the average of the product of an advanced and retarded Green's function exhibits localized behavior in the long-time or small-frequency limit. We define

$$\Phi_2(1\,|\,4)\equiv\left\langle\left\langle\mathbf{x}_1\left|\frac{1}{E-\hbar\omega-\hat{H}-i0}\right|\mathbf{x}_2\right\rangle\right. \times\left\langle\mathbf{x}_2\left|\frac{1}{E-\hat{H}+i0}\right|\mathbf{x}_4\right\rangle\right\rangle \tag{2.1d}$$

and for $\omega\to 0$, neglecting the second term in Eq. (2.1c),

$$\Psi_E(\mathbf{x}_1,\mathbf{x}_2,\omega)=\frac{\hbar}{2\pi}\Phi_2(1\,|\,1)\ . \tag{2.1e}$$

In Eqs. (2.1), $\hat{H}$ is the Hamiltonian operator,

$$\hat{H}=\frac{\hat{p}^2}{2m}-F\hat{q}_x+\hat{V}(\hat{\mathbf{q}})\ , \tag{2.2a}$$

where $\hat{\mathbf{p}}$ is the momentum operator and $\hat{V}(\hat{\mathbf{q}})$ is the random potential operator. For convenience we consider here the model of a disordered solid where in the coordinate representation $V(\mathbf{x})$ is a Gaussian random potential that is $\delta$ function correlated in space with a strength $U_0$,

$$\langle V(\mathbf{x})V(\mathbf{x}')\rangle=U_0\delta(\mathbf{x}-\mathbf{x}')\ . \tag{2.2b}$$

It must be stressed, however, that we expect our results to be model dependent at least as far as details are concerned. Physically, this follows since for large values of $x$ the momentum or kinetic energy of the particles can be arbitrarily large for fixed $E$ and $\mathbf{F}\neq 0$. As a consequence of this, the detailed nature of the potential is important. This point and the sensitivity of our results to the particular electron-impurity potential considered will be discussed further below.

In the absence of impurities the free-particle advanced (and retarded) Green's function for a particle with energy $E$ is given by[9]

$$G_0^+(1,2)=\int_0^\infty dt\, e^{i(E+i0)t/\hbar}G_0(\mathbf{x}_1,\mathbf{x}_2,t)\ , \tag{2.3a}$$

where

$$G_0(x_1,x_2,t)=\frac{-i}{\hbar}\left[\frac{m}{2\pi i\hbar t}\right]^{d/2} \times\exp\left[\frac{i}{\hbar}\left[\frac{mr_{12}^2}{2t}+\frac{Ft}{2}(x_1+x_2) -\frac{F^2t^3}{24m}\right]\right], \tag{2.3b}$$

with

$$r_{12}^2=(x_1-x_2)^2+(y_1-y_2)^2+\cdots$$

in $d$ dimensions. More explicit representations of $G_0^{\pm}(1,2)$ will be given later. Here we note that the generalized coordinate

$$\xi_i=E+Fx_i \tag{2.3c}$$

will play an important role in our discussion.[6] For $\xi_i<0$, $G_0^{\pm}$ is strongly exponentially damped since this corresponds to a classically inaccessible region. Consequently, our discussion of delocalization caused by the finite electron field will be restricted to $\xi_i>0$ and large since if the electron escapes at long times it is to this region.

A Bethe-Salpeter equation can be constructed for $\Phi_2(1\,|\,4)$ in the usual way[1,2,8]

$$\Phi_2(1\,|\,4)=G^-(1,2)G^+(2,4)+\int d5\,d6\,d7\,d8\,G^-(1,5)G^+(8,4)\times U(5,6\,|\,7,8)\Phi_2(6\,|\,7)\,. \tag{2.4a}$$

Here, $U(5,6\,|\,7,8)$ is a four-point irreducible vertex function in configuration space. $G^{\pm}$ are retarded and advanced averaged Green's functions that satisfy Dyson equations. If $\Sigma^{\pm}(1,3)$ are the irreducible two-point vertex functions or self-energies then the Dyson equation for $G^+(1,2)$ can be written as

$$\left[E+i0-H_0(1)-\int d3\Sigma^+(1,3)P_{13}\right]G^+(1,2)=\delta(1-2)\,, \tag{2.4b}$$

with $\hat{H}_0(1)$ the free-particle Hamiltonian,

$$H_0(1)=\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x_{1\alpha}\partial x_{1\alpha}}-Fx_1 \tag{2.4c}$$

and $P_{13}$ a permutation operator that interchanges the labels one and three. $G^-(1,2)$ satisfies an identical equation with $\Sigma^+$ replaced by $\Sigma^-$, $E\to E-\hbar\omega$, and $+i0$ replaced by $-i0$.

Next a generalized Boltzmann equation is obtained for $\Phi_2$. We first multiply Eq. (2.4a) by $\int d4[G^+(3,4)]^{-1}$, where $[G^+]^{-1}$ is the formal inverse of $G^+$ defined by

$$\int d1[G^{\pm}(3,1)]^{-1}G^{\pm}(1,2)=\delta(3-2)\,, \tag{2.5a}$$

We then replace 1 by 4 and interchange 3 and 4. From this equation we subtract Eq. (2.4a) multiplied by $\int d1[G^-(3,1)]^{-1}$ to obtain

$$\left[\hbar\omega+H_0(3)-H_0(4)-\int d5\Sigma^+(4,5)P_{45}+\int d5\Sigma^-(3,5)P_{35}\right]\Phi_2(3\,|\,4)=\delta(2-4)G^-(3,4)-\delta(2-3)G^+(3,4)+\int d6\,d7\,d8[G^-(3,8)U(8,6\,|\,7,4)-G^+(8,4)U(3,6\,|\,7,8)]\Phi_2(6\,|\,7)\,. \tag{2.5b}$$

To obtain a generalized Boltzmann equation we next transform to Wigner variables

$$\mathbf{x}_3=\mathbf{X}+\frac{\mathbf{x}}{2}\,,\quad \mathbf{x}_4=\mathbf{X}-\frac{\mathbf{x}}{2}\,,$$
$$\Phi_2(3\,|\,4)\to\Phi_2(\mathbf{X},\mathbf{x})\,, \tag{2.5c}$$

and derive an equation for

$$\Phi_2(\mathbf{X},\mathbf{p})\equiv\int d\mathbf{x}\,e^{i\mathbf{x}\cdot\mathbf{p}/\hbar}\Phi_2(\mathbf{X},\mathbf{x})\,. \tag{2.5d}$$

We note that the quality of interest in this representation is [cf. Eq. (2.1e) for $\mathbf{x}_1=\mathbf{X}$] the zeroth-momentum moment of Eq. (2.5d),

$$\Phi_2(\mathbf{X})=\Phi_2(\mathbf{X}\,|\,\mathbf{X})=\int\frac{d\mathbf{p}}{(2\pi\hbar)^d}\Phi_2(\mathbf{X},\mathbf{p})\,. \tag{2.5e}$$

The kinetic equation for $\Phi_2(\mathbf{X},\mathbf{p})$ is

$$\left[-i\omega+\frac{\mathbf{p}}{m}\cdot\frac{\partial}{\partial\mathbf{X}}+\mathbf{F}\cdot\frac{\partial}{\partial\mathbf{p}}+\Delta\Sigma(\mathbf{p},\mathbf{X})^*-U(\mathbf{p},\mathbf{X})^*\right]\Phi_2(\mathbf{X},\mathbf{p})=\frac{1}{i\hbar}\int d\mathbf{x}\,e^{i\mathbf{p}\cdot\mathbf{x}/\hbar}\left[\delta\left(\mathbf{x}_2-\mathbf{X}+\frac{\mathbf{x}}{2}\right)G^-\left(\mathbf{X}+\frac{\mathbf{x}}{2},\mathbf{X}-\frac{\mathbf{x}}{2}\right)-\delta\left(\mathbf{x}_2-\mathbf{X}-\frac{\mathbf{x}}{2}\right)G^+\left(\mathbf{X}+\frac{\mathbf{x}}{2},\mathbf{X}-\frac{\mathbf{x}}{2}\right)\right]\,, \tag{2.6a}$$

where

$$\Delta\Sigma(\mathbf{p},\mathbf{X})^*\Phi_2(\mathbf{X},\mathbf{p})=\frac{i}{\hbar}\int\frac{d\mathbf{p}'}{(2\pi\hbar)^d}\int d\mathbf{x}\,d\mathbf{x}'\,d\mathbf{X}'\,e^{i\mathbf{x}\cdot(\mathbf{p}/\hbar)-i\mathbf{x}'\cdot(\mathbf{p}'/\hbar)}\times\Phi_2(\mathbf{X}',\mathbf{p}')\left[\delta\left(\mathbf{X}+\frac{\mathbf{x}}{2}-\mathbf{X}'-\frac{\mathbf{x}'}{2}\right)\Sigma^+\left(\mathbf{X}-\frac{\mathbf{x}}{2},\mathbf{X}'-\frac{\mathbf{x}'}{2}\right)-\delta\left(\mathbf{X}-\frac{\mathbf{x}}{2}-\mathbf{X}'+\frac{\mathbf{x}'}{2}\right)\Sigma^-\left(\mathbf{X}+\frac{\mathbf{x}}{2},\mathbf{X}'+\frac{\mathbf{x}'}{2}\right)\right] \tag{2.6b}$$

and

$$U(\mathbf{p},\mathbf{X})^*\Phi_2(\mathbf{X},\mathbf{p})=\frac{i}{\hbar}\int\frac{d\mathbf{p}'}{(2\pi\hbar)^d}\int d\mathbf{x}\,d\mathbf{x}'d\mathbf{X}'d\mathbf{x}_8 e^{i\mathbf{x}\cdot\mathbf{p}/\hbar-i\mathbf{x}'\cdot\mathbf{p}'/\hbar}\times\Phi_2(\mathbf{X}',\mathbf{p}')\left[G^+\left[\mathbf{x}_8,\mathbf{X}-\frac{\mathbf{x}}{2}\right]U\left[\mathbf{X}+\frac{\mathbf{x}}{2},\mathbf{X}'+\frac{\mathbf{x}'}{2}\middle|\mathbf{X}'-\frac{\mathbf{x}}{2},\mathbf{x}_8\right]-G^-\left[\mathbf{X}+\frac{\mathbf{x}}{2},\mathbf{x}_8\right]U\left[\mathbf{x}_8,\mathbf{X}'+\frac{\mathbf{x}'}{2}\middle|\mathbf{X}'-\frac{\mathbf{x}'}{2},\mathbf{X}-\frac{\mathbf{x}}{2}\right]\right]. \tag{2.6c}$$

Equation (2.6a) has the form of a kinetic equation for a particle in an external field, $\mathbf{F}$, with $\Delta\Sigma$ and $U$ determining the collision operator. Finally, we note that if we integrate Eq. (2.6a) over momentum and use a Ward identity[8] expressing particle conservation then we obtain, with Eq. (2.5d),

$$-i\omega\Phi_2(\mathbf{X})+\frac{1}{m}\frac{\partial}{\partial X_\alpha}\Phi_{2\alpha}(\mathbf{X})=\frac{\delta(\mathbf{x}_2-\mathbf{X})\Delta G(\mathbf{X})}{i\hbar}\,. \tag{2.7a}$$

where

$$\Delta G(\mathbf{X})=G^-(\mathbf{X},\mathbf{X})-G^+(\mathbf{X},\mathbf{X}) \tag{2.7b}$$

and

$$\Phi_{2\alpha}(\mathbf{X})=\int\frac{d\mathbf{p}}{(2\pi\hbar)^d}p_\alpha\Phi_2(\mathbf{X},\mathbf{p})\,. \tag{2.7c}$$

### B. Approximate solution of Eq. (2.6a)

Our goal here is to obtain an approximate expression for $\Phi_{2\alpha}(\mathbf{X})$ in terms of $\Phi_2(\mathbf{X})$ so that with Eq. (2.7a) a closed equation is obtained for the quantity of interest, $\Phi_2(\mathbf{X})$. First multiply Eq. (2.6a) by $p_\alpha$ and integrate over momentum

$$-i\omega\Phi_{2\alpha}(\mathbf{X})+\int\frac{d\mathbf{p}}{(2\pi\hbar)^d}p_\alpha\times[\Delta\Sigma(\mathbf{p},\mathbf{X})-U(\mathbf{p},\mathbf{X})]^*\Phi_2(\mathbf{X},\mathbf{p})=F_\alpha\Phi_2(\mathbf{X})-\frac{1}{m}\frac{\partial}{\partial X_\beta}\int\frac{d\mathbf{p}}{(2\pi\hbar)^d}p_\alpha p_\beta\Phi_2(\mathbf{X},\mathbf{p})\,, \tag{2.8}$$

where we have neglected the right-hand side of Eq. (2.6a) in the equation for $\Phi_{2\alpha}(\mathbf{X})$. This can be justified by noting that if this term is retained in the equation for $\Phi_{2\alpha}(\mathbf{X})$ then it leads to higher order in the gradient corrections to the right-hand side of Eq. (2.7a). As for the case where $\mathbf{F}=0$,[8] Eq. (2.6a) can be approximately solved by expanding $\Phi_2(\mathbf{X},\mathbf{p})$ in spherical tensors of

$$\Phi_2(\mathbf{X},\mathbf{p})=\Delta G(\mathbf{p},\mathbf{X})\left[\frac{\Phi_2(\mathbf{X})}{\int\frac{d\mathbf{p}}{(2\pi\hbar)^d}\Delta G(\mathbf{p},\mathbf{X})}+\frac{p_\alpha\Phi_{2\alpha}(\mathbf{X})}{\int\frac{d\mathbf{p}}{(2\pi\hbar)^d}p_\alpha^2\Delta G(\mathbf{p},\mathbf{X})}+\cdots\right], \tag{2.9a}$$

where

$$\Delta G(\mathbf{p},\mathbf{X})=\int d\mathbf{x}\,e^{i\mathbf{p}\cdot\mathbf{x}/\hbar}\left[G^-\left[\mathbf{X}+\frac{\mathbf{x}}{2},\mathbf{X}-\frac{\mathbf{x}}{2}\right]-G^+\left[\mathbf{X}+\frac{\mathbf{x}}{2},\mathbf{X}-\frac{\mathbf{x}}{2}\right]\right]. \tag{2.9b}$$

The ellipsis in Eq. (2.9a) denotes terms of higher order in the gradients than $\Phi_2$ and $\Phi_{2\alpha}$. As for the $\mathbf{F}=0$ case we assume here that such an expansion is justified in the description of the large distance (or long-wavelength) behavior of the DDCF. To justify Eq. (2.9a) one needs the reciprocal relation $G^\pm(\mathbf{x}_1,\mathbf{x}_2)=G^\pm(\mathbf{x}_2,\mathbf{x}_1)$ to be valid even for finite $F$.

To proceed we need to approximate the vertex functions in Eqs. (2.6) and (2.8). Here, the same approximations used in the self-consistent diagrammatic theory of localization when $\mathbf{F}=0$ are used for finite fields. The approximations are given diagrammatically in Fig. 1; analytically one has[10]

$$\Sigma^\pm(1,2)=U_0\delta(1-2)G_0^\pm(1,2) \tag{2.10a}$$

and

$$U(1,2\,|\,3,4)=U_0\delta(1-2)\delta(3-4)\delta(1-3)+\delta(1-3)\delta(2-4)C(1,2)\,, \tag{2.10b}$$

where $C(1,2)$ is the sum of the maximally crossed diagrams important in the theory of localization for $\mathbf{F}=0$. Using time-reversal symmetry, $C(1,2)$ can be related to

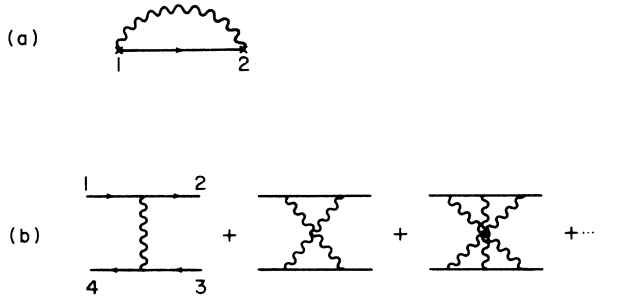


FIG. 1. (a) Lowest-order contribution to $\Sigma(1,2)$. (b) Lowest-order and maximally crossed-diagram contribution to $U(1,2\,|\,3,4)$.

the DDCF in the low-density approximation.[2,8] The theory of localization used here replaces this DDCF by one self-consistently given by Eqs. (2.7), (2.8), (2.10a), and (2.10b):

$$C(1,2)=U_0^2\Phi_2(\mathbf{x}_1)\ . \tag{2.10c}$$

Inserting Eqs. (2.10) and (2.9) in Eq. (2.8), and using approximations identical to those used for $\mathbf{F}=0$ yields a nonlinear equation for $\Phi_{2\alpha}(\mathbf{X})$ in terms of $\Phi_2(\mathbf{X})$. These approximations are consistent with either the expansion given by Eq. (2.9a) or with the idea of taking only the most important part of the maximally crossed diagrams into account. The resulting equation and Eq. (2.7a) can be written, for $\omega\to 0$, as

$$\left[-i\omega+\frac{1}{m}\frac{\partial}{\partial X_\alpha}\frac{1}{\gamma_\alpha(X,\omega)}\left[F_\alpha-\frac{\partial}{\partial X_\alpha}\Delta(X)\right]\right]\Phi_{X'}(\mathbf{X})=\frac{\Delta G_0(\mathbf{X})}{i\hbar}\delta(\mathbf{X}'-\mathbf{X}) \tag{2.11a}$$

with

$$\gamma_\alpha(X,\omega)\simeq\gamma_0(X)-2i\hbar U_0^2\Phi_X(X)\times\left[\int\frac{d\mathbf{p}}{(2\pi\hbar)^d}p_\alpha^2\Delta G_0(\mathbf{p},X)\right]^{-1}\times\mathrm{Re}\int d\mathbf{x}\,G^-\left[\mathbf{X}+\frac{\mathbf{x}}{2},\mathbf{X}-\frac{\mathbf{x}}{2}\right]\frac{\partial^2}{\partial x_\alpha\partial x_\alpha}\times G^+\left[\mathbf{X}+\frac{\mathbf{x}}{2},\mathbf{X}-\frac{\mathbf{x}}{2}\right], \tag{2.11b}$$

where

$$\gamma_0(\mathbf{X})=\frac{iU_0}{\hbar}[G_0^+(\mathbf{X},\mathbf{X})-G_0^-(\mathbf{X},\mathbf{X})] \tag{2.11c}$$

and

$$\Delta(\mathbf{X})=\frac{1}{m}\left[\int\frac{d\mathbf{p}}{(2\pi\hbar)^d}\Delta G_0(\mathbf{p},X)\right]^{-1}\times\int\frac{d\mathbf{p}}{(2\pi\hbar)^d}p_\alpha^2\Delta G_0(\mathbf{p},X)\ . \tag{2.11d}$$

Here, $\gamma_\alpha(X,\omega)$ is a frequency-dependent relaxation time that is determined self-consistently from Eq. (2.11) and $\gamma_0$ is the bare or Boltzmann relaxation time.

## III. ONE-DIMENSIONAL CASE

In this section the self-consistent theory of localization in a finite electric field is applied to the one-dimensional case. We first give the needed properties of the free and averaged Green's function. The Boltzmann or lowest-order result for the DDCF is then given. Following this, the self-consistent equations of localization are solved and discussed.

### A. Green's-function properties and the Boltzmann approximation

From Eqs. (2.3), (2.7b), and (2.9b) we obtain, for $E+FX>0$, and large,

$$\Delta G_0(X)=\frac{i}{\hbar}\left[\frac{2m}{E+FX}\right]^{1/2} \tag{3.1a}$$

and

$$\int\frac{dp}{2\pi\hbar}p^2\Delta G_0(p,X)=\frac{1}{\hbar}(2m)^{3/2}(E+FX)^{1/2}\ . \tag{3.1b}$$

These results and Eqs. (2.11c) and (2.11d) yield

$$\gamma_0(X)=\frac{U_0}{\hbar^2}\left[\frac{2m}{E+FX}\right]^{1/2} \tag{3.1c}$$

and

$$\Delta(X)=2(E+FX)\ . \tag{3.1d}$$

Equation (3.1c) is the Boltzmann approximation for the relaxation time $\gamma_\alpha(X,\omega)$ and with Eq. (3.1d) in Eq. (2.11a) yields the DDCF in the Boltzmann approximation. We note that these results effectively define a diffusion coefficient that diverges as $E+FX\to\infty$.

In order to discuss the localization caused by the second term in Eq. (2.11b) we need the average Green's functions to evaluate the integral in Eq. (2.11b). From Eqs. (2.3) and saddle-point techniques one obtains, for $\xi_i\to\infty$,

$$G_0^\pm(x_1,x_2)\simeq\mp\frac{i}{\hbar}(m/2)^{1/2}\frac{1}{(\xi_1\xi_2)^{1/4}}\left\{\exp\left[\pm\frac{2i}{3}\left[\frac{2m}{\hbar^2F^2}\right]^{1/2}|\xi_1^{3/2}-\xi_2^{3/2}|\right]\pm i\exp\left[\pm\frac{2i}{3}\left[\frac{2m}{\hbar^2F^2}\right]^{1/2}|\xi_1^{3/2}+\xi_2^{3/2}|\right]\right\}. \tag{3.2a}$$

The second term in Eq. (3.2a) is a contribution due to reflections from the boundary of the classically accessible region. In Eq. (2.11b) we are interested in the large-$X$ region and it is easily shown that in this region the second term can be neglected. The lowest-order average Green's function is obtained from the self-energy given in Eqs. (2.10a) and (2.4b). For $x_1$ and $x_2\to\infty$, we obtain

$$G^\pm(\mathbf{x}_1,\mathbf{x}_2)\simeq G_0^\pm(\mathbf{x}_1,\mathbf{x}_2)\left[\left[\frac{\xi_2}{\xi_1}\right]^{1/4\alpha}\Theta(x_1-x_2)+\left[\frac{\xi_1}{\xi_2}\right]^{1/4\alpha}\Theta(x_2-x_1)\right], \tag{3.2b}$$

with $G_0^\pm(\mathbf{x}_1,\mathbf{x}_2)$ given by the first term in Eq. (3.2a) and

$$\alpha=\frac{|F|\hbar^2}{2mU_0}\ , \tag{3.2c}$$

i.e., the average Green's functions decay as a power law rather than exponentially as when $F=0$. This is the first indication that for $F\neq 0$, localization, if it exists, is a power law for $F\neq 0$. Note that as $F\to 0$, Eqs. (3.2) reduce to the usual exponentially damped Green's functions.

With Eq. (3.2b) the integral in Eq. (2.11b) can be evaluated for $\omega\to 0$ and $X\to\infty$ as

$$\mathrm{Re}\int dx\, G^-\left[X+\frac{x}{2},X-\frac{x}{2}\right]\frac{\partial^2}{\partial x^2}G^+\left[X+\frac{x}{2},X-\frac{x}{2}\right]$$
$$=-\frac{2m^2}{\hbar^4 F}(E+FX)\left[\left[\frac{1}{2}+\frac{1}{2\alpha}\right]^{-1}{}_2F_1\left[1,\frac{1}{2}+\frac{1}{2\alpha},\frac{3}{2}+\frac{1}{2\alpha},-1\right]+\left[1+\frac{1}{2\alpha}\right]^{-1}{}_2F_1\left[1,\frac{1}{2\alpha},2+\frac{1}{2\alpha},-1\right]\right], \tag{3.3}$$

where ${}_2F_1$ is the hypergeometric function[11] of the indicated arguments.

### B. Self-consistent theory of localization in $d=1$

From Eqs. (3.3), (3.1), and (2.11) the self-consistent equations for the relaxation time $\gamma(X,\omega)$ are

$$\gamma(X,\omega)=\gamma_0(X)+\frac{\sqrt{2m(E+FX)}}{F\hbar^2}U_0^2\Phi_X(X)$$
$$\times\left[\left[\frac{1}{2}+\frac{1}{2\alpha}\right]^{-1}{}_2F_1\left[1,\frac{1}{2}+\frac{1}{2\alpha},\frac{3}{2}+\frac{1}{2\alpha},-1\right]+\left[1+\frac{1}{2\alpha}\right]^{-1}{}_2F_1\left[1,\frac{1}{2\alpha},2+\frac{1}{2\alpha},-1\right]\right] \tag{3.4a}$$

and

$$\left[-i\omega-\frac{\partial}{\partial X}\frac{F}{m\gamma(X,\omega)}-\frac{\partial}{\partial X}\frac{2(E+FX)}{m\gamma(X,\omega)}\frac{\partial}{\partial X}\right]\Phi_{X'}(X)=\frac{1}{\hbar^2}\left[\frac{2m}{E+FX}\right]^{1/2}\delta(X-X'). \tag{3.4b}$$

The plan here is to first show that, for sufficiently small fields or large impurity densities, these equations have a localized solution of the form

$$\Phi_{X'}(X)=\frac{H(X,X')}{-i\omega}. \tag{3.5}$$

That is, a solution where the DDCF does not decay on time. We then show that as $F$ increases, or $U_0$ decreases, this solution ceases to exist. Next the quantities that characterize this mobility edge are calculated.

Motivated by the fact the $G^{\pm}$ decays only as a power law we expect $\Phi_{X'}(X)$ to also decay as a power law in the localized region. This implies a relaxation time of the form

$$\gamma(X,\omega)=\frac{F^2}{-i\omega(E+FX)m\beta}, \tag{3.6a}$$

where $\beta$ is the unknown and the constants in Eq. (3.6a) have been chosen for convenience. Using this and Eq. (3.5) in Eq. (3.4b) and changing variables to $\xi=E+FX$ and $\xi'=E+FX'$ yields

$$\left[1-\frac{\partial}{\partial\xi}\beta\xi-\frac{\partial}{\partial\xi}2\beta\xi^2\frac{\partial}{\partial\xi}\right]H(\xi,\xi')=\frac{\sqrt{2m}}{\hbar^2}\frac{F\delta(\xi-\xi')}{\xi^{1/2}}. \tag{3.6b}$$

For $\omega\to 0$, Eq. (3.4a) yields

$$\frac{F^2}{\xi m\beta}=\frac{\sqrt{2m\xi}}{F\hbar^2}U_0^2H(\xi,\xi)$$
$$\times\left[\left[\frac{1}{2}+\frac{1}{2\alpha}\right]^{-1}{}_2F_1\left[1,\frac{1}{2}+\frac{1}{2\alpha},\frac{3}{2}+\frac{1}{2\alpha},+1\right]+\left[1+\frac{1}{2\alpha}\right]^{-1}{}_2F_1\left[1,\frac{1}{2\alpha},2+\frac{1}{2\alpha},-1\right]\right]. \tag{3.6c}$$

These equations yield a localized solution if $\beta>0$ and $H(\xi,\xi)\sim 1/\xi^{3/2}$.

Equation (3.6b) can be easily solved, and one obtains

$$H(\xi>\xi')=\frac{\sqrt{2m}F}{\hbar^2(\xi')^{3/2}}\frac{1}{2\beta(\mu_+-\mu_-)}\left[\frac{\xi'}{\xi}\right]^{\mu_+} \tag{3.7a}$$

and

$$H(\xi<\xi')=\frac{\sqrt{2m}F}{\hbar^2(\xi')^{3/2}}\frac{1}{2\beta(\mu_+-\mu_-)}\left[\frac{\xi'}{\xi}\right]^{\mu_-} \tag{3.7b}$$

with

$$\mu_{\pm}=\frac{3}{4}\pm\frac{1}{2}\left[\frac{1}{4}+\frac{2}{\beta}\right]^{1/2}. \tag{3.7c}$$

Equations (3.7) and (3.6c) yield an equation for the unknown quantity $\beta$,

$$\left[\frac{1}{4}+\frac{2}{\beta}\right]^{1/2} = \frac{1}{4\alpha^2}\left[\left[\frac{1}{2}+\frac{1}{2\alpha}\right]^{-1} {}_2F_1\left[1,\frac{1}{2}+\frac{1}{2\alpha},\frac{3}{2}+\frac{1}{2\alpha},-1\right] + \left[1+\frac{1}{2\alpha}\right]^{-1} {}_2F_1\left[1,\frac{1}{2\alpha},2+\frac{1}{2\alpha},-1\right]\right]. \tag{3.8}$$

This equation has a physically allowable $\beta$ ($\beta>0$) as long as the right-hand side is greater than $\frac{1}{2}$. For small $\alpha$ one obtains

$$\frac{1}{(2\beta)^{1/2}} = \frac{1}{4\alpha}\frac{mU_0}{2F\hbar^2} \tag{3.9a}$$

and

$$\mu_{\pm} = \pm\frac{mU_0}{2F\hbar^2}+\tfrac{3}{4}+O\left[\frac{F\hbar^2}{mU_0}\right]. \tag{3.9b}$$

Except for logarithmic correction terms, Eq. (3.9) agrees with Prigodin's results except $mU_0/2F\hbar^2$ is replaced by $mU_0/4F\hbar^2$. The precise reason for this discrepancy is not clear but is should be noted that with Eqs. (3.9) and (3.7a) one correctly obtains Berezinskii's result[7] in the limit $F\to 0$. Prigodin's result does not reduce to Berezinskii's in this limit.

At the mobility edge $\beta\to\infty$ and

$$\mu_+^*=1\ ,\quad \mu_-^*=\tfrac{1}{2} \tag{3.10a}$$

in precise agreement with the results of Prigodin. The equation for the critical value of $\alpha$ ($\equiv\alpha^*$) is, from Eq. (3.8),

$$2(\alpha^*)^2 = \left[\frac{1}{2}+\frac{1}{2\alpha}\right]^{-1} {}_2F_1\left[1,\frac{1}{2}+\frac{1}{2\alpha},\frac{3}{2}+\frac{1}{2\alpha},-1\right] + \left[1+\frac{1}{2\alpha}\right]^{-1} {}_2F_1\left[1,\frac{1}{2\alpha},2+\frac{1}{2\alpha},-1\right]. \tag{3.10b}$$

We note that the condition $\beta\to\infty$ at the mobility is similar to the condition that the localization length$\to\infty$ at the mobility edge when $F=0$ (for $d>2$).[1,2,8] Furthermore, we note that all moments of $H(\xi,\xi')$ diverge as the mobility edge is approached. I have solved Eq. (3.10b) approximately and obtained $\alpha^*\simeq 0.73$. This should be compared to the value $\alpha^*=1$ by Prigodin.[6] These results will be discussed in more detail in Sec. V.

## IV. TWO-DIMENSIONAL CASE

In this section, I first give some relevant properties of the free-particle Green's functions in $d=2$ and obtain the Boltzmann approximation for the DDCF. Following this I show that the self-consistent equations for $\gamma_\alpha(X,\omega)$, Eq. (2.11), do not possess a localized solution for finite $F$ in $d=2$.

### A. Green's-function properties and the Boltzmann approximation

From Eqs. (2.3), (2.7b), and (2.9b) we obtain, for $E+FX>0$, and large,

$$\Delta G_0(X)=\frac{im}{\hbar^2} \tag{4.1a}$$

and

$$\int\frac{d\mathbf{p}}{(2\pi\hbar)^2}p_\alpha p_\beta\Delta G_0(p,X)=\frac{im^2}{\hbar^2}(E+FX)\delta_{\alpha\beta}\,. \tag{4.1b}$$

These results and Eqs. (2.11c) and (2.11d) yield

$$\gamma_0(X)=\frac{U_0 m}{\hbar^3} \tag{4.1c}$$

and

$$\Delta(X)=E+FX\,. \tag{4.1d}$$

Equation (4.1c) is the Boltzmann approximation for the relaxation time $\gamma_\alpha(X,\omega)$, and with Eq. (4.1d) in Eq. (2.11a) yields the DDCF in the Boltzmann approximation. As in $d=1$, these results effectively define a diffusion coefficient that diverges as $E+FX\to\infty$.

In order to discuss whether or not the second term in Eq. (2.11b) can cause localization for $F\neq 0$ (it does for $F=0$), we need the average Green's function to evaluate the integral in Eq. (2.11b). I have not succeeded in obtaining the average Green's function in $d=2$ for all $x$ and $y$, but I have been able to evaluate the integral in Eq. (2.11b) for large $X$ and for $F$ not too large. The basic idea is to first Fourier transform the Dyson equation for $G^{\pm}(\mathbf{x}_1,\mathbf{x}_2)$ in the $y$ direction with momentum variable $p_y$. The classically accessible region is then $E+Fx_i-p_y^2/2m>0$ which gives a maximum value for $p_y$. The $X$ dependence of the average Green's function, with the self-energy given by Eq. (2.10a), can then be worked out as in $d=1$. For large $X$ the integral in Eq. (2.11b) can be evaluated for small $F$ with the result

$$\mathrm{Re}\int d\mathbf{x}\,G^-\left[\mathbf{X}+\frac{\mathbf{x}}{2},\mathbf{X}-\frac{\mathbf{x}}{2}\right]\frac{\partial^2}{\partial x_\alpha\partial x_\alpha} \times G^+\left[\mathbf{X}+\frac{\mathbf{x}}{2},\mathbf{X}-\frac{\mathbf{x}}{2}\right]\simeq-\frac{m}{U_0\hbar^2}(E+FX)\,. \tag{4.2}$$

The result for larger $F$ will not be needed since I will show that even for small, but finite $F$, the DDCF does not exhibit localized behavior.

### B. Self-consistent theory for $d=2$

From Eqs. (4.1), (4.2), and (2.11) the self-consistent equations for the relaxation time $\gamma_\alpha(X,\omega)$ for small $F$ and $E+FX\to\infty$ are

$$\gamma_\alpha(X,\omega)=\gamma(X,\omega)=\gamma_0(X)+\frac{2\hbar U_0}{m}\Phi_X(X) \tag{4.3a}$$

and

$$\left[-i\omega-\frac{\partial}{\partial X_\alpha}\frac{(E+FX)}{m\gamma(X,\omega)}\frac{\partial}{\partial X_\alpha}\right]\Phi_{\mathbf{X}'}(\mathbf{X})=\frac{m}{\hbar^3}\delta(\mathbf{X}-\mathbf{X}')\ . \tag{4.3b}$$

The plan here is to first show that these equations consistently have a solution of the form for $\omega\to 0$;

$$\gamma(X,\omega)=\gamma_0(X)\left[1+O\left[\frac{\ln X}{X}\right]\right] \tag{4.4}$$

for $X\to\infty$. That is, I consistently show that the second term in Eq. (4.3a) vanishes for large $X$. Furthermore, it can be shown that this is the only self-consistent solution for large $X$. The conclusion is that, for large $X$, one is always driven to the Boltzmann result for $\gamma(X,\omega)$ and that localization is not possible in $d=2$ for $F\neq 0$. This implies that the electron escapes to $X\to\infty$. The simple physics of delocalization at finite $F$ will be discussed below.

To proceed, we look for a solution to Eqs. (4.3), where $\gamma(X,\omega)\to\gamma(\omega)$. Transforming to the variables $\xi=E+FX$ and $\xi'=E+FX'$ and Fourier transforming in the $y$ direction, with wave number $k$, Eq. (4.3b) yields

$$\left[\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi}\frac{k^2\xi}{F^2}+\frac{i\omega m}{F^2}\gamma(\omega)\right]\Phi_k(\xi,\xi')=-\frac{m^2\gamma(\omega)}{F\hbar^3}\delta(\xi-\xi')\ , \tag{4.5a}$$

with

$$\Phi_{\mathbf{X}'}(\mathbf{X})=\int'\frac{dk}{2\pi}e^{ik(y-y')}\Phi_k\Phi_k(\xi,\xi')\ . \tag{4.5b}$$

Equations (4.3a) and (4.5b) yield

$$\gamma(X,\omega)=\gamma_0(X)+2\frac{\hbar U_0}{m}\int'\frac{dk}{2\pi}\Phi_k(\xi,\xi)\ . \tag{4.5c}$$

The primes on the wave number integrals denote $|k|<k_0$, where $k_0$ is a hydrodynamic cutoff wave number.

Equation (4.5a) can be solved by the substitution $\Phi_k(\xi,\xi')=H(\xi,\xi')/\xi^{1/2}$. The resulting equation for $H(\xi,\xi')$ is Whittaker's differential equation which has solutions in terms of confluent hypergeometric functions.[11] The final solution to Eq. (4.5a) can be written as

$$\Phi_k(\xi>\xi')=\frac{m^2\gamma(\omega)}{F\hbar^3}e^{i|k|(\xi+\xi')/F}\Gamma\left[\frac{1}{2}-\frac{i\omega m\gamma(\omega)}{2F|k|}\right]\times M\left[\frac{1}{2}-\frac{i\omega km\gamma(\omega)}{2F|k|},1,2\frac{|k|\xi'}{F}\right]U\left[\frac{1}{2}-\frac{i\omega m\gamma(\omega)}{2F|k|},1,2\frac{|k|\xi}{F}\right], \tag{4.6}$$

where $\Gamma$ is the $\gamma$ function and $M$ and $U$ are the usual confluent hypergeometric functions of the indicated arguments. $\Phi_k(\xi<\xi')$ is obtained from Eq. (4.6) by interchanging $\xi$ and $\xi'$. From the asymptotic properties of $M$ and $U$ and assuming $\omega\gamma(\omega)\to 0$ as $\omega\to 0$ and using Eq. (4.5c), one then obtains Eq. (4.4). This implies that the self-consistent solution to Eqs. (2.11) for $d=2$ and $X\to\infty$ is just the Boltzmann approximation and that Anderson localization is not possible in $d=2$ for finite $F$.

## V. DISCUSSION

First, the main results of this paper will be reviewed. In Sec. III I demonstrated that the maximally crossed diagrams important in the theory of electron localization in zero field lead to power-law localization of the DDCF in one dimension in a finite, but small, electric field. For larger fields, or lower impurity density, we obtained an Anderson transition to extended state behavior and a mobility edge. The location of the mobility edge and the properties of the DDCF as one approached it were calculated in detail. The results were in reasonable agreement with the exact calculations of Prigodin. In Sec. IV I demonstrated that the solution of the self-consistent equations for $d=2$ was that as $X\to\infty$, the effective relaxation time is given by its Boltzmann approximation and that there was no localization. Physically this can be interpreted as the electron escaping to infinity in $d=2$.

These results can be interpreted in the following way. First note that, in general, for all $d$, the contribution of the maximally crossed diagrams to the diffusion coefficient is finite for finite electric field and for finite diffusion coefficient. They are, however, singular for zero diffusion coefficient. This implies the possibility of an Anderson transition, where the diffusion coefficient is zero in any dimension. Secondly, I note that in the absence of a field the contribution of the maximally crossed diagrams to the diffusion coefficient is of order $(\lambda/l_d)^{d-1}$, where $\lambda$ is the de Broglie wavelength, $\lambda=\hbar/\sqrt{2mE}$, and $l_d$ is the mean free path in $d$ dimensions. The mean free path is given in terms of the Boltzmann diffusion coefficient $D_B$, by $D_B\sim l_d(E/m)^{1/2}$. In finite electric fields the results of Secs. III and IV imply $l_d\sim(E+FX)^{3-d/2}$ (for $d\leq 3$) and $E\to E+FX$. From this I conclude that in $d=1$ the maximally crossed diagrams are of $O(1)$, i.e., essentially all diagrams are of the same order in $d=1$. In $d=2$ these diagrams are of $O(1/X)$ for large $X$. Equation (4.4) then follows if we note that the maximally crossed diagrams are cutoff logarithmically by the electric field. The general conclusion is that the basic wave interference effects that cause Anderson localization, and which are of $O(\lambda/l)$, vanish in a finite electric field for large $X$ and that the electron can escape to $X\to\infty$ and that there is no localization. The power-law localization in $d=1$ is an exception to this general argument due to the fact that all diagrams are at the same order in $\lambda/l$ in $d=1$.

Clearly this argument is independent of the approximations used in the theory so that, in general, one does not expect localized behavior in $d>1$. In $d=3$ the maximally crossed diagrams are of $O(1/X)$ and hence vanish as $X\to\infty$.

The power-law localization in $d=1$ can be heuristically related to the energy dependence of the zero-field localization length. In zero field the long-time limit of the DDCF is $\sim\exp[-x/\xi_0(E)]$ for $x\to\infty$ with $\xi_0(E)$ $(\sim E)$ the zero-field localization length. For finite fields $\xi_0(x)$ is effectively replaced by the spatial average of $\xi_0(E+Fx)$. The DDCF is then given by

$$\exp\left[-\int_0^x \frac{dx}{\xi_0(E+Fx)}\right],$$

which leads to power-law localization.

This paper is concluded with a few remarks.

(1) The presence of an electric field introduces a new length $l_F=E/F$ into the discussion of Anderson localization. Physically $l_F$ is the length scale, where the kinetic energy of the electron due to the electric field is comparable to the total energy of the electron.

If $L$ $(\sim\xi_0=$zero-field localization length) is a typical length then for $L<l_F$ one expects the electric field to be of only minor importance. This is the weak-field region. For $l_F<L$ the electric field is effectively very strong and it determines the transport properties of the system. The theory given here can be used to describe the crossover from weak electric field to strong electric field as the length scale increases.

(2) In real experimental situations the problem of joule heating[12] will invalidate the model used here unless the number of charge carries is very small and the electric field is not too large. If these conditions are satisfied, then the consideration of this paper should be valid if $l_F$ is less than the length associated with the inelastic scattering mechanisms. The two-dimensional case is of particular interest since $\xi_0$ can be made arbitrary large so that $\xi_0>l_F$ is possible. In one-dimensional systems $\xi_0\sim l$, which is usually quite small.

(3) In Sec. I it was mentioned that the rigorous results for $d=1$ are very different for smooth versus discontinuous or hard electron-impurity interaction potentials. Furthermore, it was implied that the Gaussian white-noise model used here was in the class of hard or discontinuous potentials. The crucial physical point is whether or not the electron-impurity scattering potential remains nonzero for large momentum since arbitrarily large momenta can occur for $X\to\infty$ in a finite electric field. Both the usual $\delta$-function Kronig-Penny model and the Gaussian model used here are hard potentials since the scattering due to these potentials is independent of momentum. For smooth potentials, the scattering due to impurities vanishes for large momentum and even power-law localization is not possible in $d=1$ for finite fields.[3]

In general, this implies a sensitivity of our results to details of the electron-impurity interaction potential. However, since $\lambda\sim 1/E^{1/2}\to 1/X^{1/2}$ for large $X$, we expect localization is not possible in $d>1$ according to the arguments given at the beginning of Sec. V.

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