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Paramagnon versus lowest-order Hartree-Fock contributions in disordered nearly magnetic two- and three-dimensional itinerant fermion systems and the renormalization of the Stoner enhancement by disorder

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We reexamine weakly disordered itinerant fermion systems, interacting via a Hubbard-type dimensionless contact repulsion $\bar{I}$ close to a magnetic instability, in the diffusive regime. The relative weights of the paramagnon and the first-order $\bar{I}$ Hartree-Fock contributions to the spin susceptibility χ are compared using perturbation theory. It is pointed out that, in two dimensions, the paramagnon contribution to the $\ln T$ singularity of χ vanishes identically and the only contribution is due to the first-order $\bar{I}$ Hartree-Fock term. In contrast, in three dimensions, both contributions enter equally in the $T^{1/2}$ dependence of χ. It is also emphasized that the most singular dependences in T are the least singular in the Stoner factor $(1-\bar{I})^{-1}$ and vice versa. This is important for calculating the main corrections to the interaction induced by the disorder. For that purpose, important $T=0$ contributions are involved, coming from diagrams usually neglected in the literature (their T dependence being not singular). These are responsible for pushing the system closer to, or farther away from, the magnetic instability.

I. INTRODUCTION

In a recent paper,[1] weakly disordered itinerant fermions were reexamined in three dimensions (3D) in the diffusive regime. The particular case of a strong contact repulsion among opposite spins was considered, $\bar{I} \lesssim 1$, for which the system is close to a magnetic instability and exhibits strong spin fluctuations, often called "paramagnons."[2] Most researchers have studied the so-called weakly localized (or rather, the weakly disordered) regime, in the case where the fermions interact through screened Coulomb interactions.[3] Reference 4 however, also briefly examined the Hubbard contact interaction in 3D and 2D, and Ref. 1 was closely connected with Ref. 4. Reference 1 studied the low-temperature dependence of the spin susceptibility $\chi(T)$ given by the diagrams containing disordered paramagnons and only particle-hole diffusion processes since these are the ones which drive the system away from the metal-insulator transition, when the disorder $(\epsilon_F\tau)^{-1}$ increases[5] (ϵ_F is the Fermi energy and τ the fermion lifetime due to impurity scattering). For sake of clarity, we recall here the definition of the particle-hole diffusion propagators, called phDP's in Ref. 1: a n-phDP diagram is a diagram which contains n vertex corrections, each of them involving a propagator formed by an infinite ladder of impurity scattering in the particle-hole channel. Two main points were made in Ref. 1, for the three-dimensional case.

(i) Diagrams containing 2-phDP and one paramagnon, which cancel if a screened Coulomb interaction[6] replaces the paramagnon, do not cancel in the Hubbard case (although they were assumed to be negligible in Ref. 4, even for a Hubbard interaction). They give a contribution to $\chi(T)$ proportional to $T^{3/2}/(1-\bar{I})^{1/2}$.

(ii) The same 2-phDP diagrams also modify $\chi(0)$ by a term proportional to $(1-\bar{I})^{-1/2}$, which pushes the system closer to the magnetic instability than in absence of disorder.

Diagrams with 3-phDP plus one paramagnon, and diagrams with 4-phDP plus two paramagnons of equal frequencies, were shown in Ref. 1 to add to $\chi(T)$ terms proportional to $T^{1/2}$ in 3D, with nondivergent coefficients when $\bar{I}\to 1$ (similar to those found in Ref. 4 with only minor differences in the numerical coefficients; this will be clarified here).

The above remarks made in Ref. 1 in 3D were based upon a detailed diagrammatic analysis taking into account spin constraints imposed by the very nature of the interaction (of contact type among opposite spins). Reference 1 restricted itself to 3D since the two-dimensional case reveals pathological singularities in the absence of disorder [Ref. 7(a)], and that, in the presence of disorder, nondiffusive regimes must be considered together with the diffusive one near the magnetic instability [Ref. 7(b)]. More precisely, in the three-dimensional case only vanishing moments are relevant both for the magnetic instability and for the weak localization; in contrast in 2D, all moments from 0 to $2k_F$ are relevant for the magnetic instability, so that the diffusive regime $k_Fq\tau < 1$, as well as the one $1 < k_Fq\tau < 4\epsilon_F\tau$, must be considered on equal footing. This yields different temperature and impurity concentration dependences in all the physical properties of the system.

With this warning in mind, we will still examine here the *partial* contributions to $\chi(T)$ due to the *diffusive regime* in weakly disordered two-dimensional fermion systems with strong contact repulsion among opposite spins. This will be done in order to make a comparison with the three-dimensional case of Ref. 1 and with previous work[4] with the same hypotheses. We will thus restrict ourselves to the regime $k_Fq\tau < 1$ or $Dq^2 < \tau^{-1}$, where D is the diffusion coefficient. [Note that the regime $k_Fq\tau > 1$ would not contain any $\ln T$: for instance when $\tau^{-1}\sim 0$, $\chi(T)$ contains only T^2 terms when $T\to 0$.] The detailed structures of the diagrams are the same in $2d$ and $3d$; they have been displayed in Ref. 1 and thus will not be duplicated here. The main differences will come out from different dimensional integrations over the momenta. A comparison between the two- and three-dimensional cases will follow.

It will be pointed out that the paramagnon contribution arising from the sum of diagrams with, on one hand, 3-phDP and one paramagnon, $\delta\chi^{(P)}_{\text{3-phDP,1par}}$ and, on the other hand, 4-phDP and two paramagnons of equal frequencies $\delta\chi^{(P)}_{\text{4-phDP,2par}}$, identically vanishes in 2D. Therefore, the unique contribution to the logarithmic singularity $\ln T$ in $\chi(T)$ in 2D is due to the first-order ($\bar{I}$) Hartree-Fock (HF) term. In contrast, in 3D both contributions happen to be equally important, the paramagnon one not being enhanced by the approach to the magnetic instability. The minor differences between Refs. 1 and 4 will thus be clarified. Indeed Ref. 1 contained only the paramagnon contribution in analogy with the pure case, where the first $\bar{I}$ HF diagrams are reducible[8] and thus can be discarded; but in the presence of disorder, these last diagrams become irreducible[8] and must be retained. Taking them into account allows one to recover the result of Ref. 4.

It will also be shown that in 2D, as well as in 3D,[1] the paramagnon contribution in diagrams with 2-phDP plus one paramagnon and in diagrams with 3-phDP plus two paramagnons of equal frequencies, are strongly enhanced by the approach to the magnetic instability and thus dominates, by far, over the first-order ($\bar{I}$) Hartree-Fock contribution. Such terms, usually neglected in the literature,[4] are important for computing the renormalized value of the interaction which will show whether the system moves closer to magnetism than in the absence of disordered (as was suggested in Refs. 1 and 9), or farther away from it. Actually, even diagrams with 1-phDP plus one paramagnon and those with 2-phDP plus two paramagnons participate in such a renormalization. It thus appears more promising and less ambiguous in the diagrammatic counting, even within perturbation theory, to recast the problem and compute χ directly from the magnetic field dependence of the free-energy closed diagrams with a redefined interaction function of magnetic field and disorder.

II. PARAMAGNONS VERSUS LOWEST $\bar{I}$ HARTREE-FOCK CONTRIBUTIONS

The demonstration concerning the absence of $\ln T$ from the paramagnon contribution will be done, in the following, in the specific case of diagrammatic combinations of interest in the present problem. But we first wish to note that such a demonstration involves more generally specific behaviors occurring in 2D for integrals over momenta of the form

$$J=2\int_0^\infty \bar{q}^{\,d-1}F(\bar{q}^{\,2})d\bar{q}\ , \tag{1}$$

where $\bar{q}$ is a dimensionless variable related to the momentum q by

$$\bar{q}^{\,2}=\frac{Dq^2}{|\omega_\nu|}=\frac{1}{d}\frac{(k_F q\tau)^2}{|\omega_\nu|\tau}=Q\ . \tag{2}$$

Here D is the diffusion constant $D=k_F^2\tau/d$ in a.u. and $|\omega_\nu|$ the corresponding Matsubara frequency whose summation will be responsible for the T dependence. Switching to $Q=\bar{q}^{\,2}$, (1) becomes

$$J=\int_0^\infty Q^{(d-2)/2}F(Q)dQ\ . \tag{3}$$

The upper limit of Q in (3) is of order $(2\pi T\tau)^{-1}$ [i.e., the maximum of $(|\omega_\nu|\tau)^{-1}$], which is approximated as usual[3,4,6] by ∞. We will come back to that point later. Under the condition that the integrand is well behaved and that $F(Q)$ may be written as

$$F(Q)=\frac{dG(Q)}{dQ} \tag{4}$$

the integration by parts of (3) gives

$$J=K-[(d-2)/2]L\ ,$$
$$K=[Q^{(d-2)/2}G(Q)]_0^\infty\ , \tag{5}$$
$$L=\int_0^\infty Q^{(d-4)/2}G(Q)dQ\ ,$$

where, again, the integrand in L must be well behaved. If that condition is achieved, and supposing that the condition

$$K=0 \tag{6}$$

is fulfilled, then J in (5) will reduce to its second term $[(d-2)/2]L$; this last term will contribute only for $d\neq 2$, whereas it will vanish at 2D and so will J. It is the same kind of peculiarity arising only when $(d-2)=0$, which, in pure nearly magnetic fermion systems[10] yielded a q-independent spin-correlation function between 0 and $2k_F$, the q dependence in d dimensions entering into an hypergeometric function whose coefficient is just $(d-2)$.

A. 3-phDP plus one paramagnon diagrams and 4-phDP plus two paramagnons (of equal frequencies) diagrams

Back to the problem of interest here, the paramagnon contribution to χ from diagrams containing each of them 3-phDP plus one paramagnon, and those containing 4-phDP plus two paramagnons, was given in formula (28) of Ref. 1 as[11]

$$\delta\chi^{(P)}_{\text{3-phDP,1par}}+\delta\chi^{(P)}_{\text{4-phDP,2par}}=2^{(2-d)}\pi^{-d/2}\Gamma^{-1}(d/2)\bar{I}^{\,2}T\sum_\nu|\omega_\nu|\left[2\int\frac{q^{d-1}dq}{(Dq^2+|\omega_\nu|)^3}\frac{Dq^2}{[(1-\bar{I})Dq^2+|\omega_\nu|]}-|\omega_\nu|\int\frac{q^{d-1}dq}{[Dq^2+|\omega_\nu|]^2[(1-\bar{I})Dq^2+|\omega_\nu|]^2}\right]\ , \tag{7}$$

where Γ is the gamma function.[12] We have in (7) generalized the result of Ref. 1 to d dimensions. Using (2) and (3), one obtains

$$\delta\chi^{(P)}_{3\text{-phDP},1\text{par}}+\delta\chi^{(P)}_{4\text{-phDP},2\text{par}}=2^{(2-d)}\pi^{-d/2}\Gamma^{-1}(d/2)D^{-d/2}\bar{I}^{2}T \times\sum_{\nu}|\omega_\nu|^{(d-4)/2}\frac{1}{2\bar{I}}\int Q^{(d-2)/2}dQ\left[\frac{1}{\bar{I}}\frac{1}{(Q+1)^2}-\frac{(1-\bar{I})^2}{\bar{I}[(1-\bar{I})Q+1]^2}-\frac{2}{(Q+1)^3}\right]. \quad (8)$$

The integrand here is well behaved for 2D and 3D. Comparing with (3) and (4), we have here the explicit for $F(Q)$, $F_{(3+4)}$ from the above 3- and 4-phDP processes:

$$F_{(3+4)}(Q)=\frac{1}{\bar{I}}\left[\frac{1}{(Q+1)^2}-\frac{(1-\bar{I})^2}{[(1-\bar{I})Q+1]^2}\right]-\frac{2}{(Q+1)^3}=\frac{dG_{(3+4)}(Q)}{dQ} \quad (9)$$

with

$$G_{(3+4)}(Q)=\frac{1}{\bar{I}}\left[-\frac{1}{Q+1}+\frac{1-\bar{I}}{(1-\bar{I})Q+1}\right]+\frac{1}{(Q+1)^2} \quad (10a)$$

$$=\frac{-\bar{I}Q}{(Q+1)^2[(1-\bar{I})Q+1]}\,. \quad (10b)$$

Now (6) will be verified in 2D, as well as in 3D. On the other hand, the second term of J in (5), L, will also be well behaved for $d=2$ and 3. Therefore, going back to (8), with (5), (6), and (10), one finds

$$\delta\chi^{(P)}_{3\text{-phDP},1\text{par}}+\delta\chi^{(P)}_{4\text{-phDP},2\text{par}}=\begin{cases}\dfrac{3\sqrt{3}}{2\pi}\bar{I}\dfrac{N_0}{(\epsilon_F\tau)^2}(1-\sqrt{2\pi T\tau})[C(\bar{I})]_{d=3},\ d=3 & (11a)\\[2ex] \dfrac{\bar{I}N_0}{\pi(\epsilon_F\tau)}\ln\left[\dfrac{1}{2\pi T\tau}\right][C(\bar{I})]_{d=2},\ d=2\,, & (11b)\end{cases}$$

$$[C(\bar{I})]_d=-\frac{d-2}{2}L(\bar{I})=\begin{cases}\dfrac{\pi}{2\bar{I}}\left[1-\dfrac{\bar{I}}{2}-\sqrt{1-\bar{I}}\right],\ d=3 & (12a)\\ 0,\ d=2\,, & (12b)\end{cases}$$

where N_0 is the density of states at ϵ_F. The results (11a) and (12a) were already found in Ref. 1. In 2D, although the function $L(\bar{I})$ can be shown to be perfectly well defined, the coefficient $(d-2)$, in front, makes the entire contribution to vanish and, thus, the logarithmic singularity to disappear from this *paramagnon* contribution. To be rigorous, one can avoid the approximation $\max Q=\infty$ and retain as the upper cutoff for Q, $(|\omega_\nu|\tau)^{-1}$ [following from (2) when Dq^2 reaches its upper limit τ^{-1}]; then the algebra follows straightforwardly from (8) and one finds again that in 2D the paramagnon contribution does not contain any $\ln T$ term when $T\tau\to 0$ (more precisely when $2\pi T\tau \ll 1-\bar{I}$). The function of $\bar{I}$ in (11a) starts with $\bar{I}^2$ when $\bar{I}\to 0$ by analogy with the pure case.[8,13] In other words, in Fig. 6 of Ref. 1, where the pure case was recalled, diagrams (a) and (b) contain at least two dotted lines, i.e., twice the interaction I (their lowest order is I^2); analogously diagram (e) contains at least three dotted lines (its lowest order in I is I^3). However, there is a difference with the pure case: only one dotted line (I) is excluded in the pure case[8,13] since such diagrams would be reducible,[8] while all those of Fig. 6 of Ref. 1 are irreducible. However in presence of disorder these extra diagrams [starting with one dotted line in diagrams (a), (b), and (e) of Fig. 16 in Ref. 1] become irreducible[8] and must be retained: they correspond to the lowest order in $\bar{I}$ Hartree-Fock contribution (first $\bar{I}$ HF). Note however that all the other diagrams of that same figure contain necessarily, both in the pure and in the disordered cases, at least two dotted lines, due to spin constraints. The above remarks amount, in order to account for first $\bar{I}$ HF, to change in Ref. 1, the last bracket of formula (22): $-I^3\tilde{\chi}_0/(1-I^2\tilde{\chi}_0)$ into $-I/(1-I^2\tilde{\chi}_0^2)$, i.e.,

$$\left[-\left[\frac{I^3\tilde{\chi}_0}{1-I^2\tilde{\chi}_0^2}+I\right]\right],$$

and in formula (24) to change $I^2\tilde{\chi}_0/(1-I\tilde{\chi}_0)$ into $[I^2\tilde{\chi}_0/(1-I\tilde{\chi}_0)]+I$; so in formula (25) of Ref. 1, $I^2\tilde{\chi}_0/(1-I\tilde{\chi}_0)$ should instead read as

$$\left[\frac{I^2\tilde{\chi}_0}{1-I\tilde{\chi}_0}+I\right]=\frac{I}{1-I\tilde{\chi}_0}\,.$$

Then straightforward algebra using all the integrals computed in Ref. (1) yields, for the combined contributions due to paramagnons and first $\bar{I}$ HF [respectively (P) and ($\bar{I}$) as the upper indices of the $\delta\chi$'s],

$$\delta\chi^{(P)+(\bar{I})}_{3\text{-phDP},1\text{par}}+\delta\chi^{(P)+(\bar{I})}_{4\text{-phDP},2\text{par}}=\begin{cases}\dfrac{3\sqrt{3}}{4}\dfrac{N_0}{(\epsilon_F\tau)^2}(1-\sqrt{2\pi T\tau})\left[\left[1-\dfrac{\bar{I}}{2}-\sqrt{1-\bar{I}}\right]_{\text{par}}+\left[\dfrac{\bar{I}}{2}\right]_{\text{HF}}\right],\ d=3 & (13a)\\[2ex] \dfrac{2}{\pi}\dfrac{N_0}{(\epsilon_F\tau)}\left[\ln\left[\dfrac{1}{2\pi T\tau}\right]\right]\{[1-(1-\bar{I})-\bar{I}]_{\text{par}}+(\bar{I})_{\text{HF}}\},\ d=2\,. & (13b)\end{cases}$$

In the large square brackets of (13a) and in the curly brackets of (13b), we have separated the paramagnon contribution from the lowest $\bar{I}$ Hartree-Fock one, to compare with Ref. 1. One thus recovers (up to a factor of 2 in 2D) the results briefly quoted in Ref. 4 for the Hubbard model (where $F/2$ must be replaced by $\bar{I}/(1-\bar{I})$ in absence of T_{matrix}).

It is interesting to note, on formulas (13), that near the magnetic instability $\bar{I}\sim 1$, the paramagnon contribution in 3D $(1-\bar{I}/2-\sqrt{1-\bar{I}})\sim\frac{1}{2}$ is equal to the first $\bar{I}$ HF, $\bar{I}/2\sim\frac{1}{2}$ (the result of Ref. 1 is thus only off by a factor of 2). In contrast, in 2D the paramagnon contribution $[\bar{1}-(1-\bar{I})-\bar{I}]$ is identical to 0, while the first $\bar{I}$ HF is proportional to $\bar{I}\sim 1$ and is the unique contribution to the logarithmic singularity ln $(1/2\pi T\tau)$ in $\chi(T)$. Renormalization-group arguments[5] indeed indicate that in 2D Fermi liquid effects do not play a role.[14] However, the very different relative weights of paramagnon versus first $\bar{I}$ HF in 3D and 2D is not clear; one ought to remember though that only the diffusive regime is considered here which may not be sufficient.[7b]

B. 2-phDP plus one paramagnon diagrams and 3-phDP plus two paramagnons (of equal frequencies) diagrams

We now return first to diagrams with 2-phDP plus one paramagnon. From formula (21) in Ref. 1, we get for the paramagnon contribution

$$\delta\chi^{(P)}_{2-\text{phDP},1\text{par}}=2^{3-d}\pi^{-d/2}\Gamma^{-1}(d/2)\bar{I}^{2}\tau T\sum_{\nu}|\omega_\nu|\int\frac{q^{d-1}dq}{(Dq^2+|\omega_n|)^2}\frac{Dq^2}{[(1-\bar{I})Dq^2+|\omega_\nu|]}\ . \tag{14}$$

Using (2) and (3) again, we obtain

$$\delta\chi^{(P)}_{2-\text{phDP},1\text{par}}=2^{2-d}\pi^{-d/2}\Gamma^{-1}(d/2)D^{-d/2}\bar{I}^{2}\tau T \times\sum_{\nu}|\omega_\nu|^{(d-2)/2}\int Q^{(d-2)/2}dQ\left[-\frac{1}{\bar{I}}\right]\left[\frac{1}{\bar{I}}\left[-\frac{1}{Q+1}+\frac{1-\bar{I}}{(1-\bar{I})Q+1}\right]+\frac{1}{(Q+1)^2}\right] \tag{15}$$

so that here the function F involved in (3) is $F_{(2)}(Q)$:

$$F_{(2)}(Q)=-\frac{1}{\bar{I}}\left[\frac{1}{\bar{I}}\left[-\frac{1}{Q+1}+\frac{1-\bar{I}}{(1-\bar{I})Q+1}\right]+\frac{1}{(Q+1)^2}\right] \tag{16a}$$

$$=\frac{Q}{(Q+1)^2[(1-\bar{I})Q+1]}\ . \tag{16b}$$

In addition, note that

$$F_{(2)}(Q)=-\bar{I}^{-1}G_{(3+4)}(Q) \tag{17}$$

or

$$F_{(3+4)}(Q)=-\bar{I}\frac{dF_{(2)}(Q)}{dQ}\ . \tag{18}$$

The integrand in (15) is well behaved in 2D and 3D. Here also, $F_{(2)}(Q)$ is indeed the derivative with respect to Q of a function $G_{(2)}(Q)$:

$$G_{(2)}(0)=-\frac{1}{\bar{I}}\left[-\frac{1}{\bar{I}}\ln\left[\frac{Q+1}{(1-\bar{I})Q+1}\right]-\frac{1}{Q+1}\right]\ . \tag{19}$$

However, the integration by part of the form (5) does not hold because the functions K and L that one would get from it, in (5), would diverge, although the overall J in (3) [with $F_{(2)}(Q)$ given by (16)] is perfectly finite in 2D as well as in 3D. For 2D, (3) yields

$$J_{(2)}(Q)=\int_0^\infty F_{(2)}(Q)dQ=[G_{(2)}(Q)]_0^\infty,\ d=2 \tag{20a}$$

$$=-\frac{1}{\bar{I}}\left[-\ln\left[\frac{1}{1-\bar{I}}\right]+1\right]\ . \tag{20b}$$

On the other hand, in 3D the calculation was done in Ref. 1 and one finally gets

$$\delta\chi^{(P)}_{2\text{-phDP},1\text{par}}=\begin{cases}\dfrac{N_0\sqrt{3}}{(\epsilon_F\tau)^2}[1-(2\pi T\tau)^{3/2}]\left[\dfrac{1}{\sqrt{1-\bar{I}}}-1-\dfrac{\bar{I}}{2}\right],\ d=3 & (21a)\\[2ex] \dfrac{2N_0}{\pi(\epsilon_F\tau)}[1-(2\pi T\tau)]\left[\ln\left[\dfrac{1}{1-\bar{I}}\right]-\bar{I}\right],\ d=2\ . & (21b)\end{cases}$$

Compare to (21a), which has been obtained in Ref. 1, (21b) is the new result that is found here for 2D. It gives a linear temperature contribution to $\chi(T)$ strongly enhanced by $\ln[(1-\bar{I})^{-1}]$, although the perturbative treatment of the present paper as well as of Ref. 1 imposes that

$$[(\epsilon_F\tau)^{-1}\ln(1-I)^{-1}]<1\ .$$

However, here too, for the same reasons that explained in Sec. II A, one has to take into account the first $\bar{I}$ HF contribution although, as we will see, it will turn out to be a minor one, both in 2D and 3D, compared to the paramagnon contribution. Therefore going back to the first line of formula (21) in Ref. 1, one should again read $I/(1-\bar{I}\tilde{\chi}_0)$ instead of $I^2\tilde{\chi}_0/(1-I\tilde{\chi}_0)$. On the other hand one should also take into account all other 2-phDP plus one paramagnon diagrams obtained in suppressing one 1-phDP in all possible allowed ways from 3-phDP plus one paramagnon diagrams studied above in Sec. II A. No new algebra is involved and one checks easily that this just amounts to change the numerical factor of 4 in the first line of formula (21) in Ref. 1, into a factor of 8. Moreover, one should also add diagrams obtained by suppressing 1-phDP in all possible allowed ways in the 4-phDP plus two paramagnons diagrams considered in Sec. II A. (An easy dimensional analysis shows that they give similar contributions; we will come back to that point later.) Collecting all terms together we obtain, for the total (paramagnon plus first $\bar{I}$ Hartree-Fock) contribution,

$$\delta\chi^{(P)+(\bar{I})}_{\text{2-phDP,1par}}+\delta\chi^{(P)+(\bar{I})}_{\text{3-phDP,2par}} = 8N_0\tau^3 T\sum_\nu |\omega_\nu| \int \frac{d^dq}{(2\pi)^d}\Lambda^2\frac{I}{1-I\tilde{\chi}_0} - 4N_0^2\tau^4 T\sum_\nu \omega_\nu^2 \int \frac{d^dq}{(2\pi)^d}\Lambda^3\frac{I^2}{(1-I\tilde{\chi}_0)^2} \tag{22}$$

with, as before,[1] $\Lambda=\tau^{-1}(Dq^2+|\omega_\nu|)$. Thus one obtains

$$\delta\chi^{(P)+(\bar{I})}_{\text{2-phDP,1par}}+\delta\chi^{(P)+(\bar{I})}_{\text{3-phDP,2par}} = \begin{cases} \dfrac{\sqrt{3}N_0}{(\epsilon_F\tau)^2}[1-(2\pi T\tau)^{3/2}]\left[\left[\dfrac{2+\bar{I}}{\sqrt{1-\bar{I}}}-2-2\bar{I}\right]_{\text{par}}+(2\bar{I})_{\text{HF}}\right], & d=3 \quad (23a)\\[2ex] \dfrac{2N_0}{\pi(\epsilon_F\tau)}[1-2\pi T\tau]\left[\left[\ln\dfrac{1}{1-\bar{I}}-\bar{I}\right]_{\text{par}}+(2\bar{I})_{\text{HF}}\right], & d=2 \quad (23b)\end{cases}$$

where in the large square brackets we have separated the paramagnon contribution and the first $\bar{I}$ Hartree-Fock one.

It is clear that for the diagrams involved here and neglected in Ref. 4, the paramagnon contributions, being respectively enhanced by the factors $\sim(1-\bar{I})^{-1/2}$ in 3D and $\ln(1-\bar{I})^{-1}$ in 2D, dominate by far the first $\bar{I}$ Hartree-Fock ones, equal at most to 2 in 3D and in 2D when $\bar{I}\to 1$. Note that here the factor $(1-\bar{I})^{-1/2}$ is multiplied by $(2+\bar{I})\sim 3$ instead of 1 as in Ref. 1, due to all the other 2-phDP plus one paramagnon and 3-phDP plus two paramagnons diagrams not present in Ref. 1 and which have been collected here. However, the main point is unchanged: the sum of all these diagrams *does not vanish* but is, on the contrary, *strongly* enhanced when $\bar{I}$ is close to 1. In particular in 3D, as already emphasized in Ref. 1, the processes involved in Sec. II A or those involved in Sec. II B will dominate the T dependence of $\chi(T)$ depending whether T is smaller or larger than $\sqrt{1-\bar{I}}$.[15] But at $T=0$ the processes involved in Sec. II B will predominate by far when $\bar{I}\sim 1$, with consequences examined in Sec. III. Actually, at $T=0$ and following the same lines, one must also consider other diagrams, for instance the contributions from diagrams containing 1-phDP plus one paramagnon and those with 2-phDP plus two paramagnons of equal frequencies; we will include them in Sec. III since they play a crucial role at $T=0$, where they contribute to the renormalization of the interaction.

III. THE INTERACTION RENORMALIZED BY DISORDER AND THE CLOSENESS TO THE MAGNETIC INSTABILITY AT $T=0$ IN 3D

In this section we confine ourself to the calculation of $\chi(T=0)$ in three dimensions. The various diagrams involved here and in Ref. 1 represent perturbative corrections $\delta\chi$, due to weak localization and disordered paramagnons, to the bare electron-hole spin-correlation function (the bare bubble χ^0); they contribute in the random phase approximation (RPA) to the total enhanced spin susceptibility through the Bethe-Salpeter equation

$$\chi(T,\tau,I)=\tilde{\chi}^0(T,\tau,I)+\tilde{\chi}^0(T,\tau,I)I\chi(T,\tau,I)$$

yielding

$$\chi=\frac{\tilde{\chi}^0}{1-I\tilde{\chi}^0} \tag{24}$$

with

$$\tilde{\chi}^0=\chi^0+\delta\chi\ . \tag{25}$$

The uniform static limit of χ^0 is N_0, the density of states at the Fermi level proportional to the Pauli susceptibility at $T=0$ K. $\delta\chi$ includes mode-mode coupling corrections, i.e., here, one and two paramagnons insertions (including the disorder) in the bare bubble.[16] Therefore in the denominator of (24), one obtains the new value for $\bar{I}$ renormalized by the disorder and by disordered paramagnon contributions:

$$\bar{I}_{\text{eff}}=I(\chi^0+\delta\chi)_{T=0}=\bar{I}\left[1+\frac{\delta\chi}{N_0}\right]_{T=0}\ . \tag{26}$$

If $\delta\chi$ is positive $\bar{I}_{\text{eff}}>\bar{I}$ and the disorder brings the system closer to the magnetic instability as was suggested in Ref. 9 and found in Ref. 1. Obviously as announced at the end of Sec. II B in the contribution to $(\delta\chi)_{T=0}$ from (13a) and (23a), the main term arises from (23a) through the factor $(2+\bar{I})/\sqrt{1-\bar{I}}\sim 3/\sqrt{1-\bar{I}}$, which was essentially the result (36) of Ref. 1. Two remarks must take place at this stage.

(i) As stated in the abstract, at low temperature when $2\pi T\tau < \sqrt{1-\bar{I}}$, the main temperature correction to $\chi(T)$ comes from the $\sqrt{T\tau}$ of (13a), while the one in (23a) is minor; in contrast when $T=0$, the main correction to $\chi(0)$ comes from the $(1-\bar{I})^{-1/2}$ of (23a), while the $\bar{I}$ dependence in (13a) is negligible in comparison (at variance with what was recently proposed[17]). In other words

$$(\delta\chi_{(23a)+(13a)})_{T=0} = \frac{\sqrt{3}N_0}{(\epsilon_F T)^2}\left[\frac{1+\bar{I}}{\sqrt{1-\bar{I}}} - 1 + \tfrac{3}{4}(1-\sqrt{1-\bar{I}}) + \cdots\right] \simeq \frac{2\sqrt{3}N_0}{(\epsilon_F T)^2}\frac{1}{\sqrt{1-\bar{I}}} + \cdots \,. \quad (27)$$

(ii) However, one can check that one will have, added to (27), other powers of $1/(1-\bar{I})^n$, $n=\frac{3}{2}$ and $\frac{5}{2}$. Actually one obtains the contributions (we will come back to the details later) listed in Table I. The contribution of the second column in Table I can be obtained from a simple dimensional analysis. If one defines in the various diagrams the quantities n_{ff} equals the number of independent fermion frequencies, and n_{phDP} equals the number of phDP then the power laws of T involved are

$$T^{(d/2+1+n_{\mathrm{ff}}-n_{\mathrm{phDP}})}$$

and the power laws of $(1-\bar{I})^{-1}$ are

$$(1-\bar{I})^{-(d/2+n_{\mathrm{ff}}-n_{\mathrm{phDP}})}$$

the factor $d/2$ ($=\frac{3}{2}$ in 3D, here), comes from the integral over the momentum q; the extra factor of 1 in the power of T involves the sum over the Matsubara frequency ω_ν; the number of paramagnons, if it is linked to the number of independent fermion frequencies, does not enter in the above dimensional analysis since

$$\tilde{\chi}_0 = N_0 Dq^2/(Dq^2 + |\omega_\nu|)$$

is dimensionless in $|\omega_\nu|$ through the change of variable (2). At the order in perturbation that we examine, the number of paramagnon frequencies is only 1 since the cases containing two paramagnons means two paramagnons with the same frequency. Such a dimensional analysis is often used.[18] It proved useful in the dimensional analysis of the singularities exhibited in the local paramagnon problem,[19] which may be the one where the present case would eventually switch for stronger disorder, as conjectured elsewhere.[20]

Going back to Table I, the first column adds extra terms due to the necessity of using in some integrals over Q, the actual upper cutoff, equal to $(|\omega_\nu|\tau)^{-1}$ for sake of

TABLE I. The various contributions to $\delta\chi_{(23a+(13a)}$.

Diagrams	Extra contribution	Main contribution	Other contributions
[1-phDP plus 2par]	$-\dfrac{1-(2\pi T\tau)^3}{(1-\bar{I})^2}$	$\dfrac{1-(2\pi T\tau)^{7/2}}{(1-\bar{I})^{5/2}}$	nondivergent terms in $(1-I)^{-1}$
[1-phDP plus 1par], [2-phDP plus 2par]	$\dfrac{1-(2\pi T\tau)^2}{1-\bar{I}}$	$-\dfrac{1-(2\pi T\tau)^{5/2}}{(1-\bar{I})^{3/2}}$	nondivergent terms in $(1-I)^{-1}$
[2-phDP plus 1par], [3-phDP plus 2par]		$\dfrac{1-(2\pi T\tau)^{3/2}}{(1-\bar{I})^{1/2}}$	nondivergent terms in $(1-I)^{-1}$
[3-phDP plus 1par], [4-phDP plus 2par]		$-\{(1-\bar{I})^{1/2}[1-2\pi T\tau)^{1/2}]\}$	$1-(2\pi T\tau)^{1/2}$

convergence. These terms however do not play any role in the renormalization of $\bar{I}$ at $T=0$ since the constant term in the second column predominates over the one of the first column.

Finally the nondivergent term in $(1-\bar{I})^{-1}$ in the third column is shown explicitly for the last line, since it becomes the dominant one in that line (the one in the second column being vanishingly small when $\bar{I}\sim 1$). In contrast in the preceding lines, the third column terms play no role since the contributions in the first two columns contain powers of $(1-\bar{I})^{-1}$.

From the table it is clear that the *last line* predominates as far as the *temperature* dependence is concerned, at least when $T\tau<(1-\bar{I})<\sqrt{1-\bar{I}}$; in contrast, as far as the $T=0$ is concerned, the *first* line predominates. In that case ($T=0$) one should obviously replace (27) by

$$\delta\chi_{T=0}\simeq\frac{N_0}{(\epsilon_F T)^2}\left[\frac{a}{(1-\bar{I})^{5/2}}-\frac{b}{(1-\bar{I})^{3/2}}+\frac{c}{(1-\bar{I})^{1/2}}-d(1-\bar{I})^{1/2}+e\cdots\right], \tag{28}$$

where a, b, c and e are positive numbers of order 1. Therefore, putting (28) back in (26) it appears that $\bar{I}_{\text{eff}}>1$ and the three-dimensional system seems closer to the magnetic instability than in the absence of disorder as was already found from previous partial studies;[1,9,17] Ref. 17, using Ref. 4, accounts only for d and e, and Ref. 1 accounted for c, d, and e. From (28) the main correction actually comes from the first term. However, since the signs of the various $(1-\bar{I})^{-n}$ contributions to $(\delta\chi)_{T=0}$ alternate it looks curcial to reexamine more carefully which one wins, since, depending on its sign, the system will be closer to or farther away from the magnetic instability when disorder is present.

Actually, in diagrams containing two paramagnons, as exhibited in Fig. 8 of Ref. 1, an important question arises: Should or should not one introduce at least 1-phDP in each of the two triangles? In other words is it or is it not allowed to keep one of the two triangles free of phDP (the fermion lines still being renormalized by the mean free path)? The answer is not trivial. Indeed if one may keep one of the two triangles free of phDP, then the first line of the preceding table exists with the important consequence that, its contribution being the most important one in (28) (the first term), the system is brought *closer* to the magnetic instability under the influence of disorder.[21] In contrast, if each one of the two triangles should contain at least 1-phDP, then the first line of the table is absent and so is the first term in (29); in that case one is left with the second term in (29) as being the main correction to $\delta\chi$, with its minus sign. The system then becomes *farther* from being magnetic under the influence of disorder. Such a tendency has been observed in Pd films,[22] where diminishing the film thickness (i.e., increasing the disorder) yields the effective Stoner factor to decrease; however these films are two-dimensional systems rather than three-dimensional and, as indicated earlier, the problem of disordered two-dimensional nearly magnetic systems is far from being understood yet.[7(b)] On the other hand, an increase of $\bar{I}$, which would bring the system closer to magnetism, would agree better with the conclusions of Ref. 5 and with the perturbative tendency for the paramagnon peak to become sharper and stronger than in absence of disorder.[9]

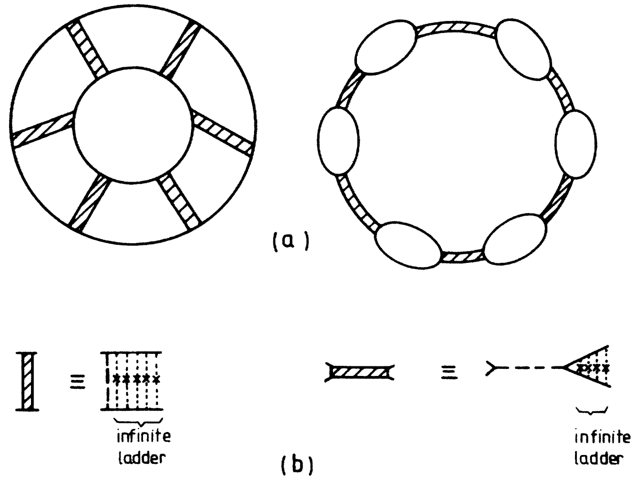

FIG. 1. (a) Free-energy closed diagrams reconsidered, with refined interactions (the hatched pieces) described diagrammatically on (b) where the dashed line is the bare interaction I, the dotted lines are scatterings on impurities (the crosses). The continuous lines in the free-energy diagrams are fermion Green's functions including the mean free path as self-energy corrections.

At that stage the only unambiguous way to compute χ, in particular $\chi(0)$ even within RPA, would be through a direct differentiation with respect to the applied magnetic field H of the free-energy closed diagrams. One should, to start with, redefine a renormalized interaction as indicated on Fig. 1. One would thus compute a new interaction $\bar{I}'$ which would be momentum and frequency dependent and function of the magnetic field H and the lifetime τ. Then one could calculate $\chi(T)$ as in the pure case of Ref. 13 with differentiating twice with respect to H to get $\delta\chi$ (i.e., by cutting two fermion lines in all possible ways). Note that if the (nontrivial) computation of $\bar{I}'$ is feasible, the remainder of the calculation would be easier since the surviving fermion bubbles will only contain the disorder in the fermion Green's function self energies. Such a calculation is beyond the scope of the present paper. However, unless one does this, it appears difficult at that stage to claim for sure whether disorder increases or decreases the tendency towards magnetism. It might also be that contrary to what was thought so far, perturbation theory is a bad tool to handle disorder in 3D as it is for 2D, although for different reasons.

IV. CONCLUSION

In the present paper we have shown that the respective roles of the paramagnons versus the lowest order (in the interaction) Hartree-Fock contributions are very different in 3D and 2D as far as the singular temperature dependences of the spin susceptibility are concerned. On the other hand, we have also shown that diagrams usually neglected in the literature [they indeed do not contribute a singular T dependence of $\chi(T)$, neither in 2D or in 3D] become important in 3D at least, as far as the interaction value renormalized by disorder effects is concerned. Such a value is indeed crucial to know whether the system becomes closer to, or farther away from, the magnetic instability in presence of disorder, compared to the pure system.

As far as this last point is concerned we have collected, in 3D, at the lowest order in disorder [in $(\epsilon_F\tau)^{-2}$ and at the lowest order in the (disordered) paramagnon insertion (i.e., insertions of one paramagnon or two paramagnons of equal frequencies which contribute equally), the correction in perturbation to the bare interaction $\bar{I}$. We pointed out that this correction contains various powers of $(1-\bar{I})^{-1}$ [times $(\epsilon_F\tau)^{-2}$] with alternate signs. Either a reinforcement of $\bar{I}$ through a term to proportional to

$$\left[+\frac{1}{(\epsilon_F\tau)^2}\frac{1}{(1-\bar{I})^{5/2}}\right],$$

or a decrease of $\bar{I}$, through a term proportional to

$$\left[-\frac{1}{(\epsilon_F\tau)^2}\frac{1}{(1-\bar{I})^{3/2}}\right],$$

is found, depending on the way the disorder is taken care of. Since this is a crucial point we emphasize that the best way to compute χ, even in RPA, would be to recast the calculation and calculate directly the free-energy closed diagrams after redefining new basic interactions renormalized by disorder.

Note added in proof. According to a recent discussion with P. Nozières, which is gratefully acknowledged here, diagrams with 1phDP plus two paramagnons (and those with 0phDP plus one paramagnon) should exist, so that the positive first term in (28) prevails to render $\bar{I}_{\rm eff}$ greater than $\bar{I}$.

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