Anisotropic superconductors containing transition-metal impurities: Local spin fluctuations

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The effects of transition-metal (TM) impurities on the transition temperature of an anisotropic superconductor when the TM impurities give rise to local spin fluctuations (LSF's) are studied. The effective electron-electron interaction between the Cooper-pair electrons is taken to be the sum of a separable anisotropic electron-phonon potential and the repulsive Coulomb potential existing between the electrons. The multiplicative renormalization technique is used to treat the Coulomb repulsion between the d electrons of opposite spins which localize about an impurity site. It is found that the LSF's influence the anisotropic superconductors in two ways. They cause a decrease in the anisotropy and an increase in the Coulomb pseudopotential. Numerical calculations of the decrease in T_c are carried out using parameters appropriate to the dilute AlMn superconductors.

I. INTRODUCTION

The enhancement of the superconducting transition temperature T_c due to an anisotropy in the effective electron-electron interaction between the electrons belonging to a Cooper pair has been studied by many authors. Markowitz and Kadanoff[1] (MK) showed that the inclusion of an anisotropy in the pairing interaction leads to an increase in T_c given by

$$\frac{T_c}{T_{c0}}=\exp[\langle a^2\rangle/N(0)V] , \quad (1)$$

where $\langle a^2\rangle$ is the mean square of the anisotropy parameter, V is the averaged pairing interaction, $N(0)$ is the density of states at the Fermi surface, and T_{c0} is the transition temperature of the corresponding isotropic superconductor. The criterion for superconductivity in the MK model is the same as that in the Bardeen-Cooper-Schrieffer (BCS) theory, i.e., the averaged pairing interaction must be attractive. Recently, Whitmore and Carbotte[2] studied the ehancement of T_c due to the anisotropy when a repulsive Coulomb interaction was included in the electron-electron interaction. They found that superconductivity could exist even if the averaged electron-electron interaction was repulsive as long as the anisotropy existed. These two studies lead to further studies on the influence of impurities on the anisotropic superconductors. Markowitz and Kadanoff[1] showed that nonmagnetic impurities could reduce the enhancement of T_c through a valence effect and a mean-free-path effect (reduction of the anisotropy by scattering by impurities). Zuckermann and Singh[3] showed that the resonant scattering by nonmagnetic transition-metal (TM) impurities gave rise to a third mechanism (cross term) by which the enhancement of T_c could be affected. Zuckermann and Singh believed that the cross term was responsible for the nonlinear behavior of the T_c versus n_i (impurity concentration) curves for the A1 $3d$ and Zn $3d$ TM superconductors (the linear behavior being a prediction of the MK model). Whitmore and Carbotte[4] have also studied the effects of nonmagnetic impurities on the properties of anisotropic superconductors with repulsive average interactions. They found that the decrease in T_c due to the impurity scattering depended on the relative strengths of the attractive electron-phonon interaction and the repulsive Coulomb interaction present in their effective electron-electron interaction.

The effects of paramagnetic impurities on the MK anisotropic superconductor have been studied by Fulde.[5] Using the Abrikosov-Gorkov (AG) treatment of the spin-flip scattering by magnetic impurities, he showed that T_c could be depressed by both non-spin-flip scattering and spin-flip scattering and that the initial decrease of T_c depended on the ratio of the relaxation times due to the two types of scattering. Warier and Nagi,[6] Okabe and Nagi,[7] and Reithofer and Schachinger[8] have also studied the effects of the paramagnetic impurities for the case where the interaction between the magnetic moments and the conduction electrons is strong. Since the second-order Born approximation used in the AG treatment is not valid for strongly interacting systems, the T-matrix approach of the Shiba-Rusinov treatment of paramagnetic impurities in superconductors was used in these three studies. Okabe and Nagi[7] found that the expression for the decrease of T_c was identical to that of Fulde[5] except for a redefinition of the ratio between the relaxation times, but that their expression for the specific-heat jump at T_c was different from that obtained by Fulde. Reithofer and Schachinger[8] found that the anisotropy increased with increasing impurity concentration and that the increase depends on the position of the impurity state within the energy gap. The enhancement of T_c due to the anisotropy and the effects of impurities on the ehancement has also been studied using the Eliashberg formalism. An excellent review of work done in this area was given recently by Allen and Mitrovic.[9]

The present paper is concerned with the effects of the transition-metal impurities on the anisotropic superconductors when the TM impurities give rise to local spin fluctuations (LSF's). In an earlier study[10] on the effects of the LSF's on anisotropic superconductors, the

Hartree-Fock approximation was used to treat the Coulomb repulsion between the d electrons of opposite spins which were localized at an impurity site. Instead of using the Hartree-Fock treatment which is known to give erroneous results in the LSF region in the normal phase, we have used, in this paper, the multiplicative renormalization method of Schlottmann[11] to treat the Coulomb repulsion. The multiplicative-renormalization-method treatment of the LSF's in the normal phase of Iche[12] (on which Schlottmann's method is based) is exact in the limit of small g ($=U/\pi\Gamma_d$, U being the strength of the Coulomb repulsion and Γ_d the half-width of the impurity state formed by the TM impurity). The effective electron-electron interaction introduced by Whitmore and Carbotte[2,4] will be used in our study.

II. RENORMALIZATION RELATIONS

Whitmore and Carbotte[2] (WC) added a Coulomb repulsion term to the separable potential proposed by Markowitz and Kadanoff[1] for anisotropic superconductors, so that the effective electron-electron interaction between electrons of opposite momentum and spin became

$$V_{kk'}=[1+a_k(\Omega)]V_{e\text{-ph}}[1+a_{k'}(\Omega')]-V_c\ , \qquad (2)$$

where $V_{e\text{-ph}}$ and V_c are the strengths of the attractive electron-phonon coupling and the Coulomb repulsion between the electrons belonging to the Cooper pair, respectively. The anisotropy of the superconductor is represented by the parameter $a_k(\Omega)$ which when averaged over the Fermi surface is zero. The double Fermi-surface average of Eq. (2) can lead to either an averaged attractive interaction or an averaged repulsive interaction depending on the relative strengths of $V_{e\text{-ph}}$ and V_c. As we pointed out, superconductivity is possible even if the averaged interaction is repulsive as will be seen in the steps leading to Eq. (38). The off-diagonal elements of the self-energy correction due to Eq. (2) can be obtained by standard methods. These corrections are

$$\Sigma_{\rm od}=[1+a_k(\Omega)]\epsilon_0+\epsilon_1\ , \qquad (3)$$

where

$$\epsilon_0=N(0)V_{e\text{-ph}}\pi k_B T\sum_{\omega_n=-\omega_D}^{\omega_D}\int\frac{d\Omega}{4\pi}[1+a_k(\Omega)]\times\frac{\tilde{\Delta}_n(\Omega)}{[\omega_n^2+\tilde{\Delta}_n^2(\Omega)]^{1/2}} \qquad (4)$$

and

$$\epsilon_1=-N(0)V_C\pi k_B T\sum_{\omega_n=-\omega_D}^{\omega_D}\int\frac{d\Omega}{4\pi}\frac{\tilde{\Delta}_n(\Omega)}{[\omega_n^2+\tilde{\Delta}_n(\Omega)]^{1/2}}\ , \qquad (5)$$

with $\omega_n=(2n+1)\pi k_B T$ and $\tilde{\Delta}_n(\Omega)$ the renormalized anisotropic energy gap.

The effect of transition-metal impurities can be incorporated by adding the Anderson-like Hamiltonian

$$\sum_{i,k,\sigma}V_{ik}C_{k\sigma}^{\dagger}d_{i\sigma}+\text{H.c.}+E_d\sum_{i,\sigma}d_{i\sigma}^{\dagger}d_{i\sigma}+U\sum_{i,\sigma}d_{i\sigma}^{\dagger}d_{i\sigma}d_{i,-\sigma}^{\dagger}d_{i,-\sigma} \qquad (6)$$

to the Hamiltonian used in the WC model of the anisotropic superconductor. In the above equation, $c_{k\sigma}^{\dagger}$ ($c_{k\sigma}$) is the creation (destruction) operator for a host-metal electron of momentum k and spin σ; $d_{i\sigma}^{\dagger}$ ($d_{i\sigma}$) is the creation (destruction) operator for a d electron of spin σ localized at an impurity site R_i; V_{ik} is the potential that mixes the conduction electrons with d electrons; E_d is the orbital energy of the d electrons; and U is the strength of the Coulomb repulsion between the d electrons of opposite spin which are localized at the same impurity site. The presence of the mixing potential leads to the formation of an impurity state which can scatter the conduction electrons. This scattering leads to a further self-energy correction given by (for the diagonal part)

$$\Sigma_d=n_i V^2 G_d(i\omega_n)\ , \qquad (7)$$

and (for the off-diagonal part)

$$\Sigma_{\rm od}=n_i V^2 F_d(i\omega_n)\ , \qquad (8)$$

where n_i is the concentration of the impurities in the host system and where $G_d(i\omega_n)$ and $F_d(i\omega_n)$ are the normal and anomalous propagators for the d electrons.

The magnetic nature of the impurity state is determined by a complicated interplay of the mixing potential V_{ik} and the Coulomb repulsion energy U. When the ratio $U/\pi\Gamma_d\to 0$, the impurity state is nonmagnetic, while for $U/\pi\Gamma_d\to\infty$, the state is magnetic. The crossover from nonmagnetic to magnetic behavior is thought to occur somewhere around $g=U/\pi\Gamma_d\sim 1$, the exact point being in doubt. The local-spin-fluctuation (LSF) region occurs just on the nonmagnetic side of the transition region. The previous study of the LSF's in anisotropic superconductors[10] was based on a Hartree-Fock treatment of the Coulomb term in Eq. (6). The Hartree-Fock treatment of the Coulomb term in the normal phase is known to give erroneous results close to the nonmagnetic-to-magnetic transition region. We have, therefore, used the multiplicative renormalization method (MRM) of Iche[12] to treat the Coulomb term. The MRM, which gives exact results for the LSF region in the normal phase, is based on the fact that there are quantities in nature which are invariant under a change in the energy scale. This invariance leads to the requirement that the renormalization parameter (constant) be a solution of a Lie differential equation. We refer the reader to Iche's paper for the details of the MRM.

To treat the LSF's in the superconducting phase, Schlottmann[11] realized that the multiplicative renormalization of the anomalous propagator for the d electrons could only be achieved by separating the perturbative expansion of the propagator into two sets and then requiring each subset to obey the multiplicative renormalization requirement. The details of these calculations can be found in Schlottmann's paper[11] or in two recent papers by the present author.[13,14] The presence of the anisotropy factor $1+a_k(\Omega)$ in Eqs. (4) and (5) requires a slight modification of Schlottmann's approach as we shall presently see. Again using standard techniques, we find that the unrenormalized energy gap for the d-electron propagators is given by

$$\phi_d(i\omega_n)=\Delta_d+N(0)V^2\int\frac{d\Omega}{4\pi}\frac{\tilde{\Delta}_n(\Omega)}{[\omega_n^2+\tilde{\Delta}_n^2(\Omega)]^{1/2}}\ , \tag{9}$$

where we have assumed that the mixing potential V_{ik} is isotropic. The Coulomb gap is defined as

$$\Delta_d=-U(2\sigma)\langle d_\sigma d_{-\sigma}\rangle=-g\pi\Gamma_d(2\sigma)\langle d_\sigma d_{-\sigma}\rangle\ , \tag{10}$$

where the correlation function $(2\sigma)\langle d_\sigma d_{-\sigma}\rangle$ is related to the anomalous d-electron propagator by the relation

$$(2\sigma)\langle d_\sigma d_{-\sigma}\rangle=\pi k_B T\sum_{\omega_n}F_d(i\omega_n)$$

$$=\pi k_B T\sum\frac{\phi_d(i\omega_n)}{z_d^2\omega_n^2+E_d^2+\phi_d^2(i\omega_n)}\ , \tag{11}$$

where $z_d=1+\Gamma_d/|\omega_n|$ for $T\sim T_c$. Combining Eqs. (9), (10), and (11), we get

$$(2\sigma)\langle d_\sigma d_{-\sigma}\rangle=\frac{1}{1+g}\pi k_B T\sum_{\omega_n=\omega_D}^{\omega_D}\frac{\Gamma_d}{(|\omega_n|+\Gamma_d)^2+E_d^2+\phi_d^2(i\omega_n)}\int\frac{d\Omega}{4\pi}\frac{\tilde{\Delta}_n(\Omega)}{[\omega_n^2+\tilde{\Delta}_n^2(\Omega)]^{1/2}}\ . \tag{12}$$

Assuming that $\Gamma_d>\omega_D$, we can neglect the frequency term appearing in the denominator $(|\omega_n|+\Gamma_d)^2+E_d^2+\phi_n^2$. For temperatures close to T_c, the energy-gap term can also be neglected. This allows us to pull out the factor $\Gamma_d/\pi(\Gamma_d^2+E_d^2)$ from the summation. Doing this, we find that the terms remaining inside the summation when multiplied by $N(0)V_C$ comprise the gap function ϵ_1 defined by Eq. (5). Putting everything together, we get

$$(2\sigma)\langle d_\sigma d_{-\sigma}\rangle=\frac{1}{1+g}\frac{\Gamma_d^2}{(\Gamma_d^2+E_d^2)}\frac{1}{N(0)V_C}\epsilon_1\ . \tag{13}$$

For the isotropic superconductor, the correlation function is proportional to the order parameter and to the recipocal of the electron-phonon coupling constant. Substituting Eq. (13) into Eq. (10) and then the resulting expression into Eq. (9), we find that the anomalous propagator which obeys the multiplicative renormalization is given by

$$F_d(i\omega_n)=\frac{U}{1+g}\frac{1}{N(0)V_C}\chi(0)\frac{\epsilon_1}{(|\omega_n|+\Gamma_d)^2+E_d^2+\phi_d^2(i\omega_n)}$$

$$+\frac{\Gamma_d}{\pi[(|\omega_n|+\Gamma_d)^2+E_d^2+\phi_d^2(i\omega_n)]}\int\frac{d\Omega}{4\pi}\frac{\tilde{\Delta}_n(\Omega)}{[\omega_n^2+\tilde{\Delta}_n^2(\Omega)]^{1/2}}\ . \tag{14}$$

Schlottmann showed that the following expression,

$$G_d(i\omega_n)=\frac{|\omega_n|+\Gamma_d}{(|\omega_n|+\Gamma_d)^2+E_d^2+\phi_d^2(i\omega_n)}\ , \tag{15}$$

is the form for the normal propagator which obeys multiplicative renormalization.

III. TRANSITION TEMPERATURE

The unrenormalized energy gap of the anisotropic superconductor is obtained by inspection of the off-diagonal parts of the self-energy corrections, Eqs. (3) and (8), i.e.,

$$\phi_n(\Omega)=[1+a(\Omega)]\epsilon_0+\left[1+n_iV^2\frac{1}{N(0)V_C}U_{\text{eff}}\chi(0)\frac{1}{z_d^2|\omega_n^2|+E_d^2+\phi_d^2(i\omega_n)}\right]\epsilon_1$$

$$+n_iV^2\frac{\Gamma_d}{\pi[z_d^2|\omega_n^2|+E_d^2+\phi_d^2(i\omega_n)]}\int\frac{d\Omega'}{4\pi}\frac{\tilde{\Delta}_n(\Omega')}{[\omega_n^2+\tilde{\Delta}_n^2(\Omega')]^{1/2}}\ , \tag{16}$$

where $U_{\text{eff}}=U/(1+g)$, $\tilde{\Delta}_n(\Omega)$ is the renormalized energy gap, and z_d is the ratio between the renormalized frequency for the d electrons and ω_n. Close to T_c, Eq. (16) yields the following equation for the renormalized energy gap:

$$\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]z_d\right]\tilde{\Delta}_n(\Omega)=[1+a(\Omega)]\epsilon_0+\left[1+n_iV^2\frac{U_{\text{eff}}\chi(0)}{N(0)V_C}\frac{1}{z_d^2|\omega_n|^2+E_d^2}\right]\epsilon_1$$

$$+n_iV^2\frac{\Gamma_d}{\pi(z_d^2|\omega_n|^2+E_d^2)}\int\frac{d\Omega'}{4\pi}\frac{\tilde{\Delta}_n(\Omega')}{[\omega_n^2+\tilde{\Delta}_n^2(\Omega')]^{1/2}}\ , \tag{17}$$

where $z_d = 1+\Gamma_d/|\omega_n|$ and $N_d(0)$ is the density of states for the d electrons. Following MK, WC, and Zuckermann and Singh, we shall assume[1-3]

$$\tilde{\Delta}_n(\Omega)=\Delta_0+a(\Omega)\Delta_1\;, \tag{18}$$

which leads to

$$\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\right]\Delta_0=\epsilon_0+\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\frac{1}{N(0)V_C}U_{\text{eff}}\chi(0)\right]\epsilon_1 \tag{19}$$

and

$$\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\right]\Delta_1=\epsilon_0\;, \tag{20}$$

where ϵ_0 and ϵ_1 are the parameters defined by Eqs. (4) and (5), respectively. Substituting Eqs. (19) and (20) into Eq. (18) and then substituting the resulting renormalized energy gap into Eqs. (4) and (5), we obtain the following set of coupled equations:

$$\left\{1-N(0)V_{e\text{-ph}}[A(T)+\langle a^2\rangle B(T)]\Big/\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\right]\right\}\epsilon_0 - \left\{N(0)V_{e\text{-ph}}\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\frac{1}{N(0)V_C}U_{\text{eff}}\chi(0)\right]A(T)\Big/\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\right]\right\}\epsilon_1=0 \tag{21}$$

and

$$\left\{N(0)V_C A(T)\Big/\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\right]\right\}\epsilon_0 + \left\{1+N(0)V_C\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\frac{U_{\text{eff}}\chi(0)}{N(0)V_C}\right]A(T)\Big/\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\right]\right\}\epsilon_1=0\;, \tag{22}$$

where

$$A(T)=\pi k_B T\sum_{\omega_n=-\omega_D}^{\omega_D}\frac{1}{|\omega_n|} \tag{23}$$

and

$$B(T)=\pi k_B T\sum_{\omega_n=-\omega_D}^{\omega_D}\frac{1}{|\omega_n|+n_i[N_d(0)/N(0)](\Gamma_d/\{1+n_i[N_d(0)/N(0)]\})}\;. \tag{24}$$

Nonzero solutions for ϵ_0 and ϵ_1 are possible only if the determinant of their coefficients is zero. Imposing this condition, we get

$$\begin{aligned}0=&1-N(0)V_{e\text{-ph}}[A(T)+\langle a^2\rangle B(T)]\Big/\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\right]\\ &+N(0)V_C A(T)\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\frac{U_{\text{eff}}\chi(0)}{N(0)V_C}\right]\Big/\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\right]\\ &-N^2(0)V_{e\text{-ph}}V_C\langle a^2\rangle A(T)B(T)\left[1+n_1\left[\frac{N_d(0)}{N(0)}\right]\frac{U_{\text{eff}}\chi(0)}{N(0)V_C}\right]\Big/\left[1+n_i\left[\frac{N_d(0)}{N(0)}\right]\right]\;.\end{aligned} \tag{25}$$

Before we consider the general case of $\langle a^2\rangle\neq0$ and $n_i\neq0$, let us first look at some limiting cases. The first case to be considered is when $n_i=0$ (no impurities) and $\langle a^2\rangle=0$ (no anisotropy). Equation (25) yields the BCS-like result

$$k_B T_{C0}=1.13\hbar\omega_D\exp\left[-\frac{1}{N(0)V_{e\text{-ph}}-\mu}\right], \tag{26}$$

where $\mu=N(0)V_C$. When impurities are added to the superconductor (but with $\langle a^2\rangle$ still zero), Eq. (25) yields

$$A(T)=\ln\left[\frac{1.13\hbar\omega_D}{k_B T_C}\right]=\frac{1+n_i[N_d(0)/N(0)]}{N(0)V_{e\text{-ph}}-\mu-n_i[N_d(0)/N(0)]U_{\text{eff}}\chi(0)}\;. \tag{27}$$

Combining Eqs. (26) and (27), we get

$$\frac{T_c}{T_{c0}}=\exp\left[-n_i[N_d(0)/N(0)]\frac{1}{N(0)g_{\text{eff}}}\frac{1+[1/N(0)g_{\text{eff}}]U_{\text{eff}}\chi(0)}{1-n_i[N_d(0)/N(0)][1/N(0)g_{\text{eff}}]U_{\text{eff}}\chi(0)}\right], \tag{28}$$

where the effective electron-phonon coupling $N(0)g_{\text{eff}}=N(0)V_{e\text{-ph}}-\mu$. Equation (28) is just the result obtained by Schlottmann.[11] We have drawn the concentration dependence of T_c as predicted by Eq. (28) for several different values of the LSF parameter $U_{\text{eff}}\chi(0)$. To obtain the curves appearing on Fig. 1 we have taken $N(0)g_{\text{eff}}=0.1702$, $N_d(0)=2.13$ states/eV atom, and $N(0)=0.286$ states/eV atom. These values are those for aluminum superconductors and for Mn impurities in the Al superconductor. Along with the curves predicted by Eq. (28), we also show the observed decrease in the dilute *Al*Mn superconductors as measured by Huber and Maple.[15]

The next case to be considered is that of the pure anisotropic superconductor. By setting $n_i=0$ in Eq. (25) we will obtain the results of Whitmore and Carbotte.[2] This is easily seen by noting that for $n_i=0$, $A(T)=B(T)$ and that Eq. (25) now has as its solution

$$A(T)=\frac{N(0)V_{e\text{-ph}}(1+\langle a^2\rangle)-\mu\pm\{[N(0)V_{e\text{-ph}}(i+\langle a^2\rangle)-\mu]^2+4N(0)V_{e\text{-ph}}\mu\langle a^2\rangle\}^{1/2}}{-2N(0)V_{e\text{-ph}}\mu\langle a^2\rangle}, \tag{29}$$

which is the same as Eq. (15) of Ref. 2. By setting $N(0)V_C$ to zero, we obtain the results as in Ref. 1.

Our discussion of the general case begins with the observation that for very low concentration of impurities, the functions $A(T)$ and $B(T)$ are related through

$$B(T)=A(T)-n_i\left[\frac{N_d(0)}{N(0)}\right]\frac{\Gamma_d}{2\pi T\{1+n_i[N_d(0)/N(0)]\}}\left[\psi'\left[\frac{1}{2}\right]-\psi'\left[\frac{1}{2}+\frac{\omega_D}{2\pi T}\right]\right], \tag{30}$$

where $\psi'(x)$ is the trigamma function. Equation (30) can be rewritten as $B(T)=A(T)-\delta\psi$, where

$$\delta\psi=n_i\left[\frac{N_d(0)}{N(0)}\right]\frac{\Gamma_d}{2\pi T\{1+n_i[N_d(0)/N(0)]\}}\left[\psi'\left[\frac{1}{2}\right]-\psi'\left[\frac{1}{2}+\frac{\omega_D}{2\pi T}\right]\right] \tag{31}$$

is a positive quantity. Substituting this into Eq. (25), we get

$$1+N(0)V_{e\text{-ph}}\langle a^2\rangle\Delta\psi+\frac{A(T)}{1+n_i[N_d(0)/N(0)]}[\mu_{\text{eff}}-N(0)V_{e\text{-ph}}(1+\langle a^2\rangle_{\text{red}})]$$
$$-\left[\frac{A(T)}{1+n_i[N_d(0)/N(0)]}\right]^2N(0)V_{e\text{-ph}}\mu_{\text{eff}}\langle a^2\rangle=0, \tag{32}$$

where

$$\mu_{\text{eff}}=\mu+n_i\left[\frac{N_d(0)}{N(0)}\right]U_{\text{eff}}\chi(0) \tag{33a}$$

and

$$\langle a^2\rangle_{\text{red}}=\langle a^2\rangle(1-\mu_{\text{eff}}\Delta\psi), \tag{33b}$$

with

$$\Delta\psi=\delta\psi/[1+n_iN_d(0)/N(0)].$$

Equations (33a) and (33b) show that the impurities influence the anisotropic superconductors in two ways, i.e., they increase the strength of the Coulomb pseudopotential and they decrease the anisotropy. Solving for $A(T)$, we get

$$\frac{A(T)}{1+n_i[N_d(0)/N(0)]}=[N(0)V_{e\text{-ph}}(1+\langle a^2\rangle_{\text{red}})]-\mu_{\text{eff}}$$
$$\pm\{[N(0)V_{e\text{-ph}}(1+\langle a^2\rangle_{\text{red}})-\mu_{\text{eff}}]^2$$
$$+4N(0)V_{e\text{-ph}}\mu_{\text{eff}}\langle a^2\rangle[1+N(0)V_{e\text{-ph}}\Delta\psi]\}^{1/2}[-2N(0)V_{e\text{-ph}}\mu_{\text{eff}}\langle a^2\rangle]^{-1}. \tag{34}$$

The above expression is very similar to Eq. (30) of Ref. 4.

Rather simple expressions for the change in the transition temperature due to the LSF's can be obtained if we assume that

$$\frac{N(0)V_{e\text{-ph}}\mu_{\text{eff}}\langle a^2\rangle[1+N(0)V_{e\text{-ph}}\langle a^2\rangle\Delta\psi]}{[N(0)V_{e\text{-ph}}(1+\langle a^2\rangle_{\text{red}})-\mu\text{eff}]^2} \ll 1 . \tag{35}$$

For

$$N(0)V_{e\text{-ph}}(1+\langle a^2\rangle_{\text{red}})>\mu_{\text{eff}} ,$$

Eq. (34) along with Eq. (35) gives

$$T_c/T_{c0}=\exp\left[-\left[\frac{n_i}{N(0)g^*_{\text{eff}}}\frac{\left[\frac{N_d(0)}{N(0)}\right]\left[1+\frac{U_{eff}\chi(0)}{N(0)g^*_{\text{eff}}}[1+N(0)V_{e\text{-ph}}\langle a^2\rangle\delta]\right]+N(0)V_{e\text{-ph}}\langle a^2\rangle\delta}{1-n_i\left[\frac{N_d(0)}{N(0)}\right]\frac{U_{\text{eff}}\chi(0)}{N(0)g^*_{\text{eff}}}[1+N(0)V_{e\text{-ph}}\langle a^2\rangle\delta]}\right]\right] , \tag{36}$$

where

$$N(0)g^*_{\text{eff}}=N(0)V_{e\text{-ph}}(1+\langle a^2\rangle)-\mu$$

and

$$\delta=\left[\frac{N_d(0)}{N(0)}\right]\frac{\Gamma_d}{2\pi T_c}\left[\psi'\left[\frac{1}{2}\right]-\psi'\left[\frac{1}{2}+\frac{\omega_D}{2\pi T_c}\right]\right] . \tag{37}$$

Equation (36) reduces to Eq. (28) when $\langle a^2\rangle$ goes to zero. Like Eq. (28), Eq. (36) also predicts that there is a finite concentration at which the LSF's can suppress the superconductivity in the host system. The predicted critical concentration for the anisotropic superconductor would be much less than that of the isotropic superconductor due to the extra $1+N(0)V_{e\text{-ph}}\langle a^2\rangle\delta$ factor appearing in the denominator of Eq. (36). For the case where

$$\mu_{\text{eff}}>N(0)V_{e\text{-ph}}(1+\langle a^2\rangle_{\text{red}}) ,$$

we get

$$\frac{T_c}{T_{c0}}=\exp\left\{-\left[n_i\left[\frac{N_d(0)}{N(0)}\right]\left[\frac{\mu-N(0)V_{e\text{-ph}}(1+\langle a^2\rangle)}{N(0)V_{e\text{-ph}}\mu\langle a^2\rangle}+\frac{U_{\text{eff}}\chi(0)[\langle a^2\rangle(1+\delta)+1]}{\mu^2}\right]\right]\right\} . \tag{38}$$

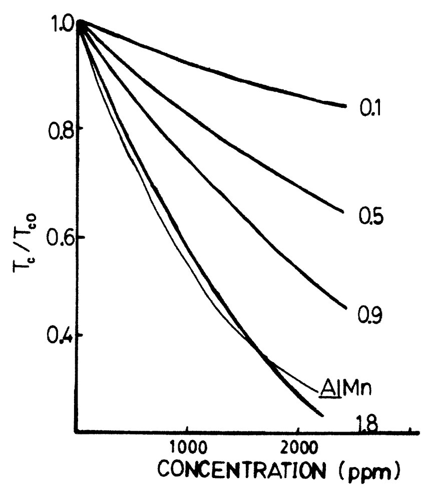

FIG. 1. Decrease of the transition temperature of an isotropic superconductor containing LSF's. The curves represent the decrease of T_c due to the presence of local spin fluctuations as predicted by Eq. (28). The curves are labeled with the numerical values of the LSF parameter $U_{\text{eff}}\chi(0)$. The numerical values of the other parameters are those for the *Al*Mn superconductors and are given in the text. We have also plotted the observed decrease in T_c of the *Al*Mn superconductors.

Equation (38) predicts an exponential drop in the transition temperature without a critical concentration. It also shows that superconductivity can exist even if the averaged electron-electron interaction is repulsive as long as the anisotropy remains.

It should be emphasized that Eqs. (36) and (38) are valid only for very low concentrations. For higher concentrations the functions $A(T)$ and $B(T)$ are not related in the simple manner indicated by Eq. (30). To find the concentration dependence of T_c for the higher concentrations, Eq. (25) would have to be solved numerically. As we will see in the next section, Eq. (36) predicts a too rapid drop in T_c at the higher concentrations.

IV. NUMERICAL CALCULATIONS AND DISCUSSION

To see the effects of the local spin fluctuations on the transition temperature of the anisotropic superconductors, we have performed some numerical calculations of the decrease in T_c as predicted by Eq. (36). As we pointed out in the preceding section, the values of the effective pairing interaction, the density of states of the host system, and the density of states of the d electrons were chosen to give the transition temperature of a pure aluminum supercon-

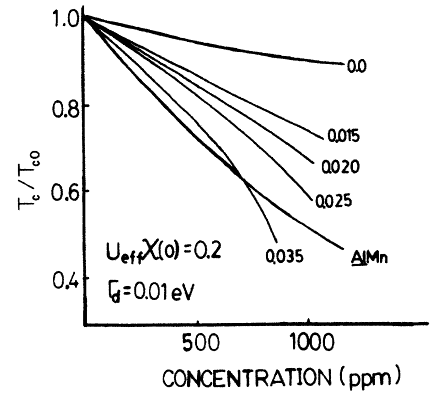

FIG. 2. Decrease of T_c of an anisotropic superconductor containing LSF's. The curves represent the decreases of T_c as predicted by Eq. (36) for several different values of the anisotropy. The curves are labeled by the values of $\langle a^2\rangle$. The T_{c0} in this figure is the transition temperature of the pure anisotropic superconductor. The values of the parameters other than those given on the curves are the same as those used in Fig. 1.

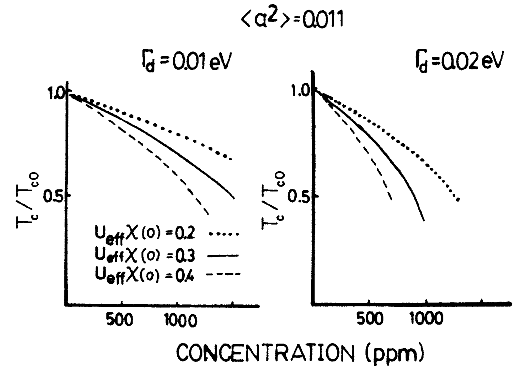

FIG. 3. Decrease of T_c of an anisotropic superconductor containing LSF's. The curves show the decrease of T_c for other values of Γ_d and $U_{eff}\chi(0)$. The value of the anisotropy was taken to be $\langle a^2\rangle=0.011$, which is the value for pure aluminum superconductors.

ductor and to describe the impurity states formed by dissolving Mn impurities in Al. Figure 1 shows the decrease in T_c of an isotropic superconductor due to LSF's for several different values of $U_{eff}\chi(0)$. Looking at the figure, we find that the curve for $U_{eff}\chi(0)=1.8$ nearly duplicates the decrease in T_c of the *Al*Mn superconductors as measured by Huber and Maple.[15] The significance of this fit is not very important since Eq. (36), of which the curves on the figure are representative, is obtained on the basis of an isotropic picture of the host superconductor. Most evidences[10,16] point to Al being an anisotropic superconductor with $\langle a^2\rangle=0.011$. To see how the presence of an anisotropy in the host superconductor affects the influence of the LSF's, we have plotted in Fig. 2, the decrease in T_C predicted by Eq. (36) for several values of $\langle a^2\rangle$. The attractive electron-phonon potential $V_{e\text{-ph}}$ and repulsive Coulomb potential V_C were adjusted so that

$$N(0)V_{e\text{-ph}}(1+\langle a^2\rangle)-N(0)V_C$$

was always equal to 0.17022 so as to produce a T_{c0} of 1.18 K, the transition temperature of pure aluminum aluminum superconductors. The value of $U_{eff}\chi(0)$ was arbitrary picked to be 0.2. As we see, the presence of the anisotropy in the superconductor leads to a greater influence of the LSF's on the superconductor, i.e., the decrease in T_c becomes greater as the mean square of the anisotropy increases.

To obtain the curves on Fig. 2, we have assumed that the half-width of the Mn impurity state Γ_d is 0.01 eV. This value of Γ_d is the one used by Machida and Nakanishi[17] to obtain a fit of their expression for the nuclear spin-lattice relaxation rate to the observed ^{27}Al relaxation rate in the dilute *Al*Mn superconductors. The value is of the same magnitude as the half-width determined by Aoki and Ohtsuka,[18] from their analysis of the initial decrease in T_c of the *Al*Mn superconductors and is close to the value of 80 meV obtained from the normal-state magnetic susceptibility measurements.[19] Parvin and MacLanghlin[10] and Zuckermann[20] argue that the value of 0.01 eV is unreasonably small in light of the fact that the half-widths of the impurity states formed when other 3*d* TM impurities are dissolved in Al are of order 1–2 eV. Zuckermann[20] argued that a proper analysis of the initial decrease in T_c of the *Al*Mn superconductors gives a Γ_d of 1.5 eV. To see what would happen if the half-widths were changed, we have plotted the decrease in T_c for different values of Γ_d on Figs. 3 and 4. The anisotropy of the pure aluminum superconductor was taken to be $\langle a^2\rangle=0.011$. Figure 3 shows the decrease in T_c for different values of $U_{eff}\chi(0)$ and Γ_d. Figure 4 shows the decrease as the half-width increases. Reasonable decreases of T_c in the presence of LSF's appears to be achieved with Γ_d of order 0.01 eV. For Eq. (36) with $\Gamma_d=1.5$ eV, to predict behaviors similar to the decrease in T_c observed, the LSF parameter $U_{eff}\chi(0)$ would have to be very small.

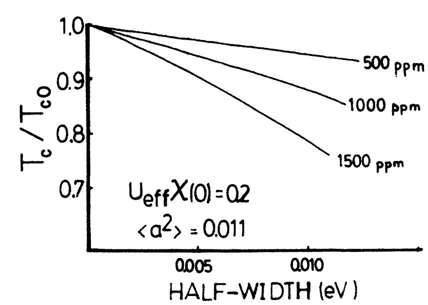

FIG. 4. Decrease of T_c as a function of the half-width of the impurity states. The curves show the decrease in T_c as the half-width Γ_d increases for several impurity concentrations. The rapid decrease of T_c as Γ_d increases clearly indicates that in the presence of strong local spin fluctuations [high values of $U_{eff}\chi(0)$], values of γ_d of order 1–2 eV would not produce the types of decreases seen in *Al*Mn superconductors. Equation (36) would only predict decreases similar to those observed in *Al*Mn if Γ_d is of order 0.01 eV.

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