Dielectric interpretation of Lei-Ting nonlinear force-momentum—balance transport equation for isothermal resistivity

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(Received 18 November 1985)

A dielectric interpretation of the nonlinear Lei-Ting force-momentum—balance transport equation for steady-state dc current flow is developed here in correspondence with standard techniques for calculating fast-particle energy loss to a plasmalike medium. In conjunction with this we interpret the result to be an *isothermal* resistivity calculated to lowest order in the impurity scattering potentials, isothermal in the sense that all energy dissipated is removed from the system, essentially instantaneously as it is generated, by a heat bath in contact with the system which maintains it at constant temperature throughout the nonlinear dc conduction process. On the basis of its isothermal character, we argue that the Lei-Ting dc resistivity calculated to lowest order in the impurity scattering potentials—whose linear limit is significantly different from the corresponding linear resistivity of an adiabatic character (for a system admitting no drainoff of dissipated energy, developing under a purely mechanical Hamiltonian)—is immune to serious critical objections of the type brought by Argyres and Sigel against similar lowest-order adiabatic linear resistivity calculations some time ago. Moreover, we also show that a dielectric Lei-Ting type formulation of linearized ac resistivity leads to the standard high-frequency linear resistivity formula, and that its zero-frequency limit naturally yields the isothermal dc linear Lei-Ting resistivity.

I. INTRODUCTION

The Lei-Ting force-momentum—balance equation[1] has proven to be a powerful mechanism for studying nonlinear hot-electron transport in semiconductors, and it continues to provide penetrating and successful analyses of steady-state dc current flow in a wide variety of circumstances of interest, particularly in heterostructures,[2] where the nonlinearity of the conductance occurs at such a low field that it strongly influences the common operation of semiconductor heterostructure devices. One central feature of this approach is the separation of the center-of-mass motion from "relative electron" coordinates in a uniform electric field which accelerates only the center of mass, with coupling to the "relative electrons" provided by impurity scattering potentials which are treated to leading order in an averaged Langevin-type force-momentum—balance equation, and with a corresponding treatment for energy transfer. (We shall not at this time treat the phonon scattering mechanisms, which also couple the electric-field-driven center-of-mass coordinate to the relative electrons.) Our object here is to report on an alternative interpretation of the Lei-Ting balance equation involving a purely dielectric interpretation of its formulation, which clearly exhibits the fact that the only involvement of quantum mechanics in this theory is confined to the structure of the dielectric response properties of the conducting medium. Moreover, our formulation will make it clear that temperature is constant throughout the history of the dynamics as described here, so that the Lei-Ting resistivity must be understood as an isothermal resistivity with all heat removed from the system as it is generated by an appropriate bath which maintains the system at constant temperature throughout the nonlinear conduction process.

The Hamiltonian for a system of N interacting electrons driven by a uniform applied electric field $\mathbf{E}$ and slowed by impurity scattering potentials $u(\mathbf{r}_i-\mathbf{R}_a)$ centered at $\mathbf{R}_a$ is given by ($\hbar\to 1$)

$$H=\sum_{i=1}^{N}(p_i^2/2m)+\sum_{\substack{i,j=1\\ i<j}}^{N}e^2/(4\pi\,|\,\mathbf{r}_i-\mathbf{r}_j\,|\,)+\sum_{i=1}^{N}\sum_{a}u(\mathbf{r}_i-\mathbf{R}_a)-e\sum_{i=1}^{N}\mathbf{r}_i\cdot\mathbf{E}\;. \tag{1}$$

Here $\mathbf{r}_i$ and $\mathbf{p}_i=-i\nabla_i$ are the coordinate and momentum of the ith electron with effective mass m and charge e (measuring the strength of the electron-electron interaction as well as the external $\mathbf{E}$ field interaction). Introducing center-of-mass (CM) momentum $\mathbf{P}$ and coordinate $\mathbf{R}$ variables as well as relative electron momentum $\mathbf{p}_i'$ and coordinate $\mathbf{r}_i'$ variables, we have

$$\mathbf{P}=\sum_{i=1}^{N}\mathbf{p}_i,\quad \mathbf{R}=\sum_{i=1}^{N}\mathbf{r}_i/N\;, \tag{2}$$

$$\mathbf{p}_i'=\mathbf{p}_i-\mathbf{P}/N,\quad \mathbf{r}_i'=\mathbf{r}_i-\mathbf{R}\;. \tag{3}$$

The point has already been made that the CM momentum-coordinate variables and the relative electron momentum-coordinate variables obey canonical commutation relations to within a negligible error or order $1/N$. In terms of these variables the Hamiltonian has the form $H=H_{\mathrm{CM}}+H_e+H_{ei}$, where

$$H_{\mathrm{CM}}=P^2/2Nm-Ne\mathbf{E}\cdot\mathbf{R}\;, \tag{4}$$

$$H_e=\sum_{i=1}^{N}(p_i')^2/2m+\sum_{\substack{i,j=1\\ i<j}}^{N}e^2/(4\pi\,|\,\mathbf{r}_i'-\mathbf{r}_j'\,|\,)\;, \tag{5}$$

$$H_{ei}=\sum_{i=1}^{N}\sum_{a}u(\mathbf{r}_i'+\mathbf{R}-\mathbf{R}_a)\ , \tag{6}$$

and it is at once evident that only the center-of-mass motion is driven by an electric field of strength $N\mathbf{E}$, having mass $M=Nm$, so large that only its classical motion is significant, and that the relative electrons of H_e (which are not directly driven by the electric field) constitute an interacting many-body system which may be treated by standard quantum field-theoretic techniques, and that the resistive mechanism in the driven CM motion is provided by a Langevin-type force associated with the impurity scattering potential H_{ei} for random scattering centers $\mathbf{R}_a$ which will be averaged over in due course.

II. DIELECTRIC FORMULATION OF RESISTIVE FORCE

Focusing attention on the resistive force which impedes the acceleration of the center of mass in steady-state dc current flow, we follow the Lei-Ting assertion of a constant c-number drift velocity $\mathbf{V}_d$ for the center of mass, such that the current is given by $\mathbf{J}=Ne\mathbf{V}_d$ (no magnetic field, $\mathbf{J}\|\mathbf{E}$)

$$\mathbf{R}(t)-\mathbf{R}(t')=\mathbf{V}_d(t-t')\ . \tag{7}$$

The basic framework of this theory is to calculate the resistive force on the CM particle to lowest order in its interaction with the relative electrons.[1] From a dielectric point of view we may do so by considering that the CM particle impresses on the relative electron system the impurity potential given by $(2=\mathbf{r}_2',t_2)$

$$U(2)=\sum_a u(\mathbf{r}_2'+\mathbf{R}(t_2)-\mathbf{R}_a)\ . \tag{8}$$

However, this time-dependent impressed potential is dynamically screened by the inverse dielectric function $K(1,2)$ to produce the effective potential $V(1)$ as

$$V(1)=\int d^42\,K(1,2)U(2)\ , \tag{9}$$

and the associated electric field acting on relative electrons at (1) is $\mathbf{E}(1)=-\nabla_1V(1)$. On the other hand, a density perturbation of relative electrons is also induced at (1), and this is given by

$$\begin{aligned}\rho(1)&=\int d^43\,R(1,3)V(3)\\&=\int d^44\int d^43\,R(1,3)K(3,4)U(4)\ ,\end{aligned} \tag{10}$$

where $R(1,3)=\delta\rho(1)/\delta V(3)$ is the density perturbation response function.[3] It is to be noted that the calculation of $K(1,2)$ or $R(1,2)$ in the presence of the time-dependent potential $U(2)$ would be severely complicated. However, within the framework of linear response theory as it relates to evaluation of the resistive force to lowest order in $U(2)$, $U(2)$ is to be neglected in the calculation of $K(1,2)$ and $R(1,2)$. With this, and employing the random phase approximation (RPA) for relative electron dynamics, the density perturbation response function may be eliminated from explicit appearance by using the RPA integral equation in the form[3]

$$K(1,2)-\delta^4(1,2)=\int d^44\int d^43\,v_C(1-3)\times R(3,4)K(4,2)\ , \tag{11}$$

where v_C is the Coulomb interaction. Applying ∇_1^2 we find $\nabla_1^2v_C(1-3)=\delta^4(1-3)$ [note that $e^2\rightarrow1$ is suppressed here and will be restored later: A time δ function arises from the instantaneous nature of $v_C(1-3)$], and therefore

$$\nabla_1^2[K(1,2)-\delta^4(1,2)]=\int d^44\,R(1,4)K(4,2)\ , \tag{12}$$

whence

$$\rho(1)=\int d^44\,\nabla_1^2[K(1,4)-\delta(1,4)]U(4)\ . \tag{13}$$

Taken together, Eqs. (9) and (13) yield the force per unit volume on the relative electrons due to the CM particle as $\mathbf{F}(1)=-\rho(1)\nabla_1V(1)$, and by the law of action and reaction the force on the CM particle due to the relative electron concentration at (1) is just $\rho(1)\nabla_1V(1)$, so that if we add contributions from all space points $\mathbf{r}_1'$ we find the total resistive force on the CM particle to be $\mathbf{f}$ $(1=\mathbf{r}_1',t_1)$:

$$\mathbf{f}=\int d^3r_1'\rho(1)\nabla_1V(1)=\int d^3r_1'\int d^42\int d^44[\nabla_1K(1,2)U(2)]\{\nabla_1^2[K(1,4)-\delta(1,4)]U(4)\}\ . \tag{14}$$

We remark again that to leading order in the impurity scattering potentials $[U(2)U(4)]$ the inverse dielectric function $K(1,2)$ of the conducting medium is independent of U. For a homogeneous bulk infinite medium conductor at a fixed temperature this yields $K(1,2)=K(1-2)$, and it is convenient to Fourier analyze $K(1-2)\rightarrow K(p)$ in space-time obtaining $[p=\mathbf{p},\ p_0(\omega)]$

$$\nabla_1V(1)=\int[d^4p/(2\pi)^4]i\mathbf{p}\exp(ip\cdot1)K(p)U(p)\ , \tag{15}$$

$$\rho(1)=-\int[d^4q/(2\pi)^4]\,|\mathbf{q}|^2\exp(iq\cdot1)[K(q)-1]U(q)\ . \tag{16}$$

The impressed potential by the CM particle on the relative electrons is given by Eq. (8), and recalling that for the steady state $\mathbf{R}(t)=\mathbf{V}_dt$ we have the Fourier representation

$$U(p)=2\pi\delta(p_0+\mathbf{p}\cdot\mathbf{V}_d)u(\mathbf{p})\sum_a\exp(-i\mathbf{R}_a\cdot\mathbf{p})\ , \tag{17}$$

whence, setting $\omega_p=p_0=-\mathbf{p}\cdot\mathbf{V}_d$ and $\omega_q=q_0=-\mathbf{q}\cdot\mathbf{V}_d$, we have

$$\nabla_1V(1)=\int[d^3p/(2\pi)^3]i\mathbf{p}\exp(i\mathbf{p}\cdot\mathbf{r}_1'+i\omega_pt)K(\mathbf{p},\omega_p)u(\mathbf{p})\sum_a\exp(-i\mathbf{R}_a\cdot\mathbf{p})\ , \tag{18}$$

$$\rho(1) = -\int (d^3q/(2\pi)^3)\,|\mathbf{q}|^2 \exp(i\mathbf{q}\cdot\mathbf{r}_1' + i\omega_q t)[K(\mathbf{q},\omega_q)-1]u(\mathbf{q}) \sum_b \exp(-i\mathbf{R}_b\cdot\mathbf{q}) \, , \tag{19}$$

and forming $\mathbf{f}$ by Eq. (14) yields

$$\mathbf{f} = -\int [d^3p/(2\pi)^3] i\mathbf{p}\,|\mathbf{p}|^2 K(\mathbf{p},\omega_p)[K(-\mathbf{p},-\omega_p)-1]u(\mathbf{p})u(-\mathbf{p}) \sum_{a,b} \exp[-i\mathbf{p}\cdot(\mathbf{R}_a-\mathbf{R}_b)] \, . \tag{20}$$

Averaging the resistive force $\langle \mathbf{f} \rangle$ over impurity site locations[4(a),4(b)]

$$\left\langle \sum_{a,b} \exp[-i\mathbf{p}\cdot(\mathbf{R}_a-\mathbf{R}_b)] \right\rangle = n\mathscr{V} \tag{21}$$

(n is the volume density of impurity sites and $\mathscr{V}$ designates volume), we have

$$\langle \mathbf{f} \rangle/\mathscr{V} = -n \int [d^3p/(2\pi)^3] i\mathbf{p}\,|\mathbf{p}|^2 K(\mathbf{p},\omega_p)[K(-\mathbf{p},-\omega_p)-1]u(\mathbf{p})u(-\mathbf{p}) \, . \tag{22}$$

Of the two terms of $[K-1]$ involved here, the first one ($\mathbf{f}_{KK}$) has the form

$$\langle \mathbf{f}_{KK} \rangle/\mathscr{V} = -n \int [d^3p/(2\pi)^3] i\mathbf{p}\,|\mathbf{p}|^2 K(\mathbf{p},\omega_p) \times K(-\mathbf{p},-\omega_p)u(\mathbf{p})u(-\mathbf{p}) \, , \tag{23}$$

and note that under the transformation $\mathbf{p}\to-\mathbf{p}$ it becomes its own negative, so it vanishes identically. Hence,

$$\langle \mathbf{f} \rangle/\mathscr{V} = n \int [d^3p/(2\pi)^3] i\mathbf{p}\,|\mathbf{p}|^2 K(\mathbf{p},\omega_p)u(\mathbf{p})u(-\mathbf{p}) \, . \tag{24}$$

Separating the real and imaginary parts of $K=K_1+iK_2$, we recall the properties[5]

$$K_{1,2}(\mathbf{p},\omega) = K_{1,2}(-\mathbf{p},\omega)$$

and

$$K_1(\mathbf{p},\omega) = K_1(\mathbf{p},-\omega) \, ,$$

whereas

$$K_2(\mathbf{p},\omega) = -K_2(\mathbf{p},-\omega) \, .$$

Hence $K_1(\mathbf{p},-\mathbf{p}\cdot\mathbf{V}_d)$ is even under $\mathbf{p}\to-\mathbf{p}$, and consequently its contribution to $\langle \mathbf{f} \rangle$ vanishes. Thus only K_2 contributes to Eq. (24):

$$\langle \mathbf{f} \rangle/\mathscr{V} = n \int [d^3p/(2\pi)^3] \mathbf{p}\,|\mathbf{p}|^2 \times K_2(\mathbf{p},\mathbf{p}\cdot\mathbf{V}_d)u(\mathbf{p})u(-\mathbf{p}) \, . \tag{25}$$

Expressing K_2 in terms of the bulk density-density correlation function $\hat{\Pi} = \hat{\Pi}_1 + i\hat{\Pi}_2$, we have

$$|\mathbf{p}|^2 K_2(\mathbf{p},\mathbf{p}\cdot\mathbf{V}_d) = \hat{\Pi}_2(\mathbf{p},\mathbf{p}\cdot\mathbf{V}_d) \, ,$$

and thus we obtain the Lei-Ting resistive force due to impurity scatterings as

$$\langle \mathbf{f} \rangle/\mathscr{V} = n \int [d^3p/(2\pi)^3] \mathbf{p}\hat{\Pi}_2(\mathbf{p},\mathbf{p}\cdot\mathbf{V}_d)u(\mathbf{p})u(-\mathbf{p}) \, . \tag{26}$$

The purely dielectric interpretation of the resistive force due to impurity scatterings is being extended to phonon scatterings, which are somewhat more complicated, and adjustments must be made for the fact that phonon interactions are not instantaneous and so the use of the law of action and reaction must be modified. Nonetheless this dielectric formulation of the resistive force offers considerable insight, even if considered for impurity scatterings alone. It is manifestly of a purely classical electrostatic nature, save for the fact that the dielectric response properties of the conductive medium, as reflected in $K(1,2)$, may bear quantum effects. This gives a clear perspective on the original fully quantum mechanical derivation. Furthermore, although the dielectric derivation of $\langle \mathbf{f} \rangle$ is exhibited here only for the bulk infinite medium case, it is in fact easily applied to a medium of any geometrical configuration whose constraint forces have no component in the direction of current flow and $\mathbf{E}$ field, and we have already applied this technique to conduction constrained to the quasi-two-dimensional planar sheets of a type I superlattice (interacting) array of quantum wells.

It is to be noted that the presumption of constant temperature of the (relative) electron gas is implicit in the assumed time translational invariance of the linear response function $K(1,2)=K(1-2)$, with linearity of the response function referring to the impressed potential $U(2)$ which the relative electron system experiences in consequence of impurity scattering [and meaning that $K(1-2)$ is independent of $U(2)$]. Nevertheless,

$$K(1,2) \to K(\mathbf{p},\omega=\mathbf{p}\cdot\mathbf{V}_d)$$

clearly depends on $\mathbf{V}_d$ and hence $\mathbf{E}$ in a nonlinear manner, and therefore the steady-state dc constant uniform field balance equation

$$-Ne\mathbf{E}/\mathscr{V} = \langle \mathbf{f} \rangle/\mathscr{V} = n \int [d^3p/(2\pi)^3] \mathbf{p}\,|\mathbf{p}|^2 \times K_2(\mathbf{p},\mathbf{p}\cdot\mathbf{V}_d)u(\mathbf{p})u(-\mathbf{p}) \tag{27}$$

describes nonlinear conduction at constant temperature. This means that the system thusly described must be understood to be in contact with a heat bath which removes all heat from the system essentially instantaneously as it is generated (by energy provided through work done by the electric field) and maintains the system at constant temperature throughout its dynamical history. In this context the Lei-Ting resistivity must be understood as an isothermal resistivity, albeit nonlinear.

III. CORRESPONDENCE WITH THE ENERGY-LOSS PROBLEM AND ISOTHERMAL INTERPRETATION

In order to clarify the interpretation of the conduction process described by the Lei-Ting balance equation as an isothermal one, we may consider the classic many-body quantum-theoretic calculation of fast particle energy loss to a solid-state plasma described by a Hamiltonian

$$H_e=\sum_{i=1}^{N} p_i^2/2m+\sum_{i<j} e^2/(4\pi\,|\mathbf{r}_i-\mathbf{r}_j|)\ ,$$

with an interaction term

$$H_I=\sum_{i=1}^{N} U_0(\mathbf{r}_i+\mathbf{R}(t))\ ,$$

where $\mathbf{R}(t)=\mathbf{V}_0 t$. Formulating it from a dielectric point of view, one has the passing particle impressing a potential $U_0(2)$ on the solid-state plasma (at a fixed temperature) which responds linearly to $U_0(2)$ and dynamically screens $U_0(2)$ with an inverse dielectric function $K(1,2)=K(1-2)$ at a fixed temperature. The effective potential is given as in Eq. (9), and the density perturbation of the solid-state plasma is as given in Eq. (10), both with $U(2)\rightarrow U_0(2)$. Again employing the RPA as in Eq. (11), and using Newton's third law, the resistive force is obtained as in Eq. (14) with $U(2)\rightarrow U_0(2)$. These dynamics of fast-particle energy loss to a solid-state plasma at a fixed temperature (in contact with a heat bath) are in fact isomorphic with the transport dynamics of the CM particle moving with uniform drift velocity while losing energy to the relative electron plasma except for the replacement $U(2)\rightarrow U_0(2)$, and for a Coulomb potential the Fourier transform of this is

$$U_0(p)=2\pi\delta(p_0+\mathbf{p}\cdot\mathbf{V}_0)u_0(\mathbf{p})\ ,$$

where (28)

$$u_0(\mathbf{p})=e^2/|\mathbf{p}|^2$$

instead of Eq. (17), which would correspond to this if $\mathbf{V}_d\rightarrow\mathbf{V}_0$, $\sum_a\rightarrow 1$, and $\mathbf{R}_a\rightarrow 0$. Considering this and forming the resistive force as above we obtain

$$\mathbf{f}=-\int[d^3p/(2\pi)^3]i\mathbf{p}\,|\mathbf{p}|^2K(\mathbf{p},-\mathbf{p}\cdot\mathbf{V}_0)[K(-\mathbf{p},\mathbf{p}\cdot\mathbf{V}_0)-1]u_0(\mathbf{p})u_0(-\mathbf{p})\ . \qquad (29)$$

Again employing the even/odd properties of $K=K_1+iK_2$, we obtain

$$\mathbf{f}=\int[d^3p/(2\pi)^3]\mathbf{p}\,|\mathbf{p}|^2K_2(\mathbf{p},\mathbf{p}\cdot\mathbf{V}_0)u_0(\mathbf{p})u_0(-\mathbf{p})\ . \qquad (30)$$

The energy lost per unit time by the passing particle is

$$dE/dt=\mathbf{f}\cdot\mathbf{V}_0=\int[d^3p/(2\pi)^3]\mathbf{p}\cdot\mathbf{V}_0\,|\mathbf{p}|^2 \times K_2(\mathbf{p},\mathbf{p}\cdot\mathbf{V}_0)u_0(\mathbf{p})u_0(-\mathbf{p})\ , \qquad (31)$$

which matches the well-known formula for energy loss to solid-state plasma excitations, including plasmons and electron-hole pairs. Notwithstanding the nonlinear dependence of dE/dt on $\mathbf{V}_0$ this theory is predicated on the understanding that the plasma is kept at a fixed temperature (by a heat bath) and its predictions of resonances when V_0 matches the plasmon phase velocity have been thoroughly studied.[6] It should be noted that while casuality of the linear response function $K=K_1+iK_2$ is germane to our argument, no assumption of an adiabatic nature is made: Indeed, the fact that each particle of the succession forming the current finds the medium response function K to be the same as did all its predecessors (reflecting time translational invariance of K for the steady state to lowest order in the impurity scattering potentials) is a clear indication that the temperature and average energy of the medium are constant in time, notwithstanding the nonlinear character of this theory. This is to say that the energy transferred to the medium by preceding particles (originating in work done on them by the external electric field) must be understood to be instantaneously drained off by a heat bath, so that the medium appears unchanged to succeeding particles in the current train, and in this situation an isothermal characterization is appropriate rather than an adiabatic one. The isomorphism of the dynamics of fast particle energy loss with the transport dynamics of the CM particle thus confirms the view that the relative electron plasma temperature must be understood as being a fixed constant, and thus the Lei-Ting resistivity must be understood as being isothermal in the sense that all heat is removed as it is generated by a heat bath in contact with the system.

IV. LINEAR PREDICTIONS FOR BOTH CONSTANT AND HIGH-FREQUENCY FIELDS

It is of course of interest to review the linear resistivity prediction of the Lei-Ting theory. Expanding Eq. (25) to linear order in $\mathbf{V}_d=V_d\hat{\mathbf{i}}$ [note that $K_2(\mathbf{p},0)=0$],

$$Ne\mathbf{E}/\mathscr{V}=n\int[d^3p/(2\pi)^3]\mathbf{p}\,|\mathbf{p}|^2 \times\left[\frac{\partial K_2(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=0}\mathbf{p}\cdot\mathbf{V}_d u(\mathbf{p})u(-\mathbf{p})\ . \qquad (32)$$

Using the notation $N/\mathscr{V}\rightarrow N$ equals the volume density of carriers, we obtain the linear Lei-Ting dc resistivity ρ

as ($J_x = NeV_d = E_x/\rho$)

$$\rho = \frac{n}{N^2e^2}\int [d^3p/(2\pi)^3]p_x^2\,|\,\mathbf{p}\,|^2 u(\mathbf{p})u(-\mathbf{p}) \times \left[\frac{\partial K_2(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=0} . \tag{33}$$

While the theory considered here explicitly exhibits the dynamical origin and role of screening (and nonlinear descreening) of the impurity scattering potential through $K(\mathbf{p},\omega)$, it is of interest to point out the connection with unscreened theories. The neglect of screening may be achieved by the replacement

$$K(\mathbf{p},\omega) = 1/\epsilon(\mathbf{p},\omega) = 1/[1+\alpha(\mathbf{p},\omega)] \to 1-\alpha(\mathbf{p},\omega) , \tag{34}$$

where $\alpha(\mathbf{p},\omega)$ is the free-electron polarizability. Nevertheless, we shall maintain the presence of screening in comparing with other work. Using the even/odd properties of $K_{1,2}(\mathbf{p},\omega)$ as a function of frequency, we have

$$\left[\frac{\partial K_2(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=0} = -\frac{1}{\epsilon_1^2(\mathbf{p},0)}\left[\frac{\partial\alpha_2(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=0} , \tag{35}$$

where $\epsilon_1(\mathbf{p},0)$ is the static screening RPA dielectric function (real part) and $\alpha_2(\mathbf{p},\omega)$ is the imaginary part of the free-electron polarizability given by

$$\alpha_2(\mathbf{p},\omega) = 2\frac{(2\pi e)^2}{p^2}\int\frac{d^3q}{(2\pi)^3}f_0(q^2/2m)\delta\left[\frac{(\mathbf{q}-\mathbf{p})^2}{2m}-\frac{q^2}{2m}-\omega\right] - (\omega\to-\omega) \tag{36}$$

[$(\omega\to-\omega)$ indicates a term identical in structure with its predecessor except for the replacement $\omega\to-\omega$; $f_0(x)$ is the Fermi distribution]. Taken jointly with Eqs. (33) and (35), this yields the linear Lei-Ting dc resisitivity due to impurity scattering as

$$\rho = \frac{m}{Ne^2}\left\langle\frac{1}{\tau}\right\rangle_{\text{avg}} , \tag{37a}$$

where the indicated average is that of the inverse energy-dependent relaxation time $[1/\tau(q)]$, averaged in accordance with $[f_0'(x) = df_0/dx]$

$$\left\langle\frac{1}{\tau}\right\rangle_{\text{avg}} = -1/(3\pi mN)\int d^3q f_0'(q^2/2m)q^2\frac{1}{\tau(q)} , \tag{37b}$$

where

$$1/\tau(q) = \int d\Omega'\omega(\phi)(1-\cos\phi) \tag{38a}$$

is a solid angle integration and

$$\omega(\phi) = mnq\,|\,\overline{U[2q\sin(\phi/2)]}\,|^2/(2\pi)^2 , \tag{38b}$$

with $\overline{U(p)} = U(p)/\epsilon_1(\mathbf{p},0)$ as the statically screened impurity scattering potential. It is to be noted that this linear dc result differs from the more traditional analyses of linear dc conductivity based on the Kubo formula and its diagrammatic approximations[4(a),4(b)] as well as Boltzmann-type transport equations, which yield a corresponding and different result, the simplest expression of which is

$$\tilde{\rho} = \frac{m}{Ne^2\langle\tau\rangle_{\text{avg}}} \tag{39}$$

in terms of the *direct* average of the energy-dependent relaxation time $\tau(q)$ [Eqs. (37) and (38)] (here shielding has been installed *ad hoc*). Although the Lei-Ting result is coincident with this at zero temperature, it differs at finite temperatures. While the difference is small in the degenerate case, it is substantial for nondegeneracy. We shall address this important matter below.

Although our attention heretofore has been mainly directed at constant uniform electric fields and associated dc currents, it is of interest to also explore the current response to time-oscillatory electric fields to linear order $\mathbf{E}\to\lambda\mathbf{E}_0e^{i\omega t}$ ($\lambda\to 1$ will be taken to measure electric field strength). Such a linear theory has it that the CM velocity $\mathbf{V}(t)$ and CM position $\mathbf{R}(t)$ are both small and linear in λ, and for times large enough that transients damp out, both oscillate at the driving frequency ω, whence

$$\begin{aligned}\mathbf{R}(t_2) &\to \lambda\mathbf{R}\exp(i\omega t_2+i\alpha) ,\\ \mathbf{V}(t_2) &\to \lambda\mathbf{V}\exp(i\omega t_2+i\beta)\end{aligned} \tag{40}$$

(α and β are phases). As these quantities are both small in such a high-frequency analysis, one may expand the impressed potential to linear order as

$$\begin{aligned}U(2) &= \sum_a u(\mathbf{r}_2'+\mathbf{R}(t_2)-\mathbf{R}_a)\\ &\to \sum_a u(\mathbf{r}_2'-\mathbf{R}_a)\\ &\quad -\lambda\mathbf{R}\exp(i\omega t_2+i\alpha)\cdot\sum_a\nabla_{\mathbf{R}_a}u(\mathbf{r}_2'-\mathbf{R}_a) ,\end{aligned} \tag{41}$$

or Fourier transforming

$$\begin{aligned}U(p) &= 2\pi\delta(p_0)\sum_a e^{-i\mathbf{p}\cdot\mathbf{R}_a}u(\mathbf{p})\\ &\quad -2\pi\lambda\mathbf{R}e^{i\alpha}\delta(\omega+p_0)\cdot\sum_a\nabla_{\mathbf{R}_a}e^{-i\mathbf{p}\cdot\mathbf{R}_a}u(\mathbf{p}) .\end{aligned} \tag{42}$$

Forming $\langle U(p)U(q)\rangle$, where $\langle\cdots\rangle$ denotes averaging over random impurity site locations, and noting that for such averages[4(a),4(b)]

$$\left\langle\sum_a\sum_b e^{-i(\mathbf{p}\cdot\mathbf{R}_a+\mathbf{q}\cdot\mathbf{R}_b)}\right\rangle = (2\pi)^3 n\delta^{(3)}(\mathbf{p}+\mathbf{q}) , \tag{43}$$

we obtain to linear order in λ

$$\langle U(p)U(q)\rangle=(2\pi)^5u(\mathbf{p})u(\mathbf{q})\delta^{(3)}(\mathbf{p}+\mathbf{q})n\{\delta(p_0)\delta(q_0)+\lambda\mathbf{R}e^{i\alpha}\cdot[\partial(p_0)\delta(\omega+q_0)i\mathbf{q}+\delta(q_0)\delta(\omega+p_0)i\mathbf{p}]\} \ . \quad (44)$$

Employing this in conjunction with Eqs. (14)—(16), we find $\langle \mathbf{f}\rangle=\langle \mathbf{f}\rangle_{\mathrm{I}}+\langle \mathbf{f}\rangle_{\mathrm{II}}$ where

$$\langle \mathbf{f}\rangle_{\mathrm{I}}=-n(\mathscr{V})\int \frac{d^3p}{(2\pi)^3}i\mathbf{p}\,|\,\mathbf{p}\,|^2K(\mathbf{p},0)[K(-\mathbf{p},0)-1]u(\mathbf{p})u(-\mathbf{p})=0 \ , \quad (45)$$

which vanishes since the integrand is an odd function of $\mathbf{p}$ for zero frequency [note that $K_2(\mathbf{p},0)=0$]. Hence

$$\langle \mathbf{f}\rangle=\langle \mathbf{f}\rangle_{\mathrm{II}}=-\lambda(\mathscr{V})ne^{(i\omega t_1+i\alpha)}\mathbf{R}\cdot\int \frac{d^3p}{(2\pi)^3}\mathbf{p}\mathbf{p}\,|\,\mathbf{p}\,|^2u(\mathbf{p})u(-\mathbf{p})[K(\mathbf{p},\omega)-K(\mathbf{p},0)] \ . \quad (46)$$

Here $K(\mathbf{p},\omega)=K_1(\mathbf{p},\omega)+iK_2(\mathbf{p},\omega)$ and $K(\mathbf{p},0)=K_1(\mathbf{p},0)$. This is equivalent to the well-known high-frequency resistivity result of Ron and Tzoar[7] for degenerate semiconductors, based on corresponding work by Dawson and Oberman[8] for classical gas plasmas. Of course the equation of motion for the CM particle now involves the acceleration term as

$$Nm\frac{d\mathbf{V}}{dt}=Ne\mathbf{E}_0e^{i\omega t}+\langle \mathbf{f}\rangle/(\mathscr{V}) \ , \quad (47)$$

and the corresponding oscillatory current is given by $\mathbf{J}=Ne\mathbf{V}$. The resulting expression for linear high-frequency resistivity is

$$\rho(\omega)=(im/Ne^2)[\omega+M(\omega)] \ , \quad (48)$$

where

$$M(\omega)=\frac{-n}{Nm\omega}\int\frac{d^3p}{(2\pi)^3}p_x^2p^2u(\mathbf{p})u(-\mathbf{p})\times[K_1(\mathbf{p},\omega)-K_1(\mathbf{p},0)+iK_2(\mathbf{p},\omega)] \ . \quad (49)$$

It is noteworthy that the zero frequency limit of this $\rho(\omega\rightarrow 0)$ identically reproduces the linear Lei-Ting dc resistivity ρ given by Eq. (33).

V. CONCLUSIONS

In its essential features, the nonlinear Lei-Ting theory is similar in its focus on the direct expansion of the resistivity to lowest order in the scattering interactions (impurity scattering only here) to an analogous linear theory advanced some time ago by Kenkre and Dresden and others[9] (but without many-electron screening interactions). In fact, the Kenkre-Dresden linear dc resistivity just matches the Lei-Ting linear dc resistivity precisely if shielding is neglected, and both are in disagreement with the result cited above based on the Kubo linear conductivity formula and its traditional diagrammatic approximations, and attendent integral transport equations whose simplest solution is given by Eq. (39) above. Argyres and Sigel[10] have duly acknowledged that the Kenkre-Dresden (and consequently Lei-Ting) technique of directly expanding the resistivity to lowest order in the scattering interactions would be of paramount importance if correct, since one could then bypass the often difficult task of solving integral transport equations. However, in a deeply penetrating analysis, Argyres and Sigel[10] argue that this procedure is in fact incorrect, since higher-order terms in the expansion of the resistivity in powers of the scattering interactions *diverge*, and if properly accounted for in summing their infinite series in a procedure equivalent to the "λ^2t limit" of Van Hove, then the lowest-order dc result is identical to the more traditional result obtained through the integral transport equation given by Eq. (39).

It is our conviction—as explained above—that the nonlinear (as well as linear) Lei-Ting result [Eqs. (27) and (37)] is an *isothermal* resistivity, for a system in contact with a heat bath that drains off heat instantaneously as it is generated, maintaining the system at constant temperature. As such the Lei-Ting theory is not subject to the critical objections of the Argyres-Sigel analysis which is rooted in the dynamics of a system driven out of equilibrium by a purely mechanical Hamiltonian having no coupling to a heat bath, describing conduction for an adiabatically sealed system which heats in accordance with considerations first discussed by Kohn and Luttinger.[11] This is to say that the Argyres-Sigel analysis, while very insightful and meaningful for *adiabatic* resistivity, is *not* in fact applicable to the *isothermal* resistivity of the Lei-Ting theory. Moreover, the Kenkre-Dresden result, while incorrect as stated for adiabatic conditions, could also find validity under isothermal conditions, neglecting screening. With this recognition the substantial benefits of the greater simplicity of the Lei-Ting resistivity expansion (to lowest order in the scattering interactions) over the integral transport equation formulation are seen to be legitimately available for use in the theoretical analysis of isothermal conduction.

Ultimately the criterion determining the correctness of any theory lies in its correspondence with experimental data. In this connection, we offer two figures[12] showing nonlinear Lei-Ting resistivity predictions (embodying both impurity scattering and phonon scattering mechanisms) for a GaAs-AlGaAs system in comparison with experimental data. Figure 1 shows the Lei-Ting nonlinear mobility normalized to its ohmic limit plotted as a function of electric field in solid curves 1 and 2 for $T=10$ K and 77 K, respectively, and also experimental data points as well as a different theoretical calculation represented by the dashed curve. Figure 2 shows the normalized Lei-Ting mobility (solid curves) and electron temperature T_e (Ref. 16) (dashed curves) over a much wider range of electric field strengths at lattice temperature $T=4.2$ K for two different sets of parameters representing mobile carrier density, impurity density (alternatively linear mobility) and remote impurity sheet location, as well as experi-

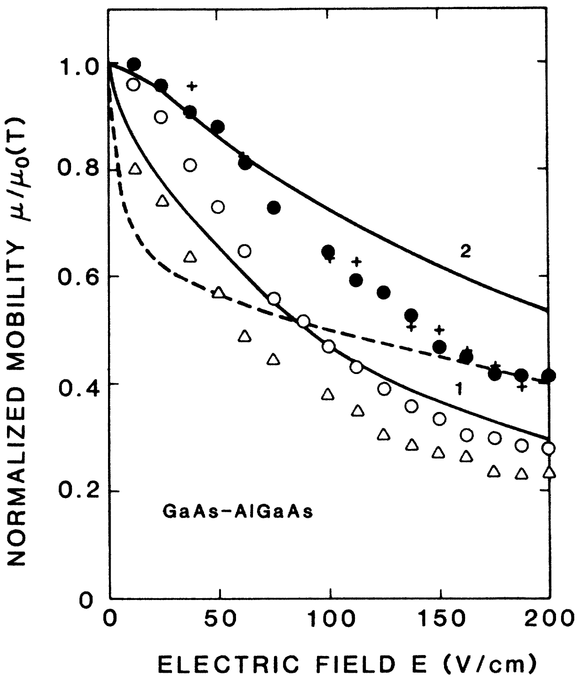

FIG. 1. Normalized mobility μ/μ_0 versus electric field E. Solid curves 1 and 2 are the present theoretical results for $T=10$ K and 77 K, respectively. The dashed curve is the theoretical calculation given in Ref. 13 at $T=10$ K. The experimental data are taken from Ref. 13 on sample 367/1 (○ represents 10 K, + represents 77 K) and sample 367/2 (△ represents 10 K, ● represents 77 K).

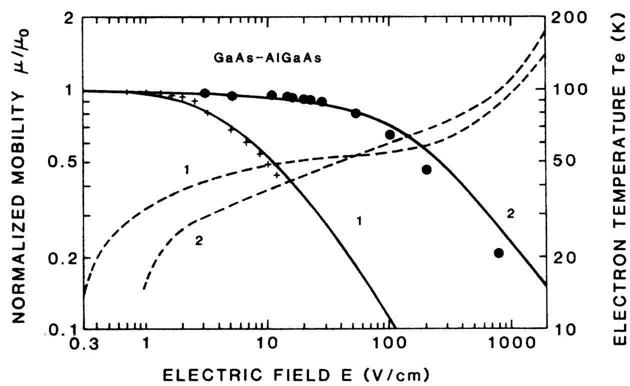

FIG. 2. Calculated normalized mobility μ/μ_0 (solid curves) and electron temperature T_e (dashed curves) versus electric field E at lattice temperature $T=4.2$ K for two systems: (1) $N=2.5\times10^{11}$ cm^{-2}, $\mu_0=1\times10^6$ cm^2/Vs, $S=250$ Å and (2) $N=3.9\times10^{11}$ cm^{-2}, $\mu_0=7.9\times10^4$ cm^2/Vs, $S=125$ Å. The experimental data are taken from Ref. 14 (crosses) and Ref. 15 (dots). (S is the separation of a remote impurity sheet from its neighboring quantum well.)

mental data points. Doubtlessly many other experimental results could and indeed should be brought forward to be compared with the isothermal dc resistivity predicted by the Lei-Ting theory for both linear and nonlinear regimes. In this connection it would be of special value to carefully control both isothermal and adiabatic dc resistivity measurements in order to clearly distinguish between the two experimental conditions and to facilitate their comparison with the corresponding theoretical predictions.

ACKNOWLEDGMENT

This work was partially supported by the U.S. Army (Electronic Materials Research Division) at the Electronics Technology and Devices Laboratory, Fort Monmouth, NJ.

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