Effects of interface charge on the quantum Hall effect

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The dependence of the quantum Hall effect on systematic changes in the electronic scattering has been studied in experiments on the inversion layer electrons in Si metal-oxide-semiconductor field-effect transistors which have driftable Na^+ ions in the oxide. The widths and positions of the quantum Hall plateaus corresponding to filled valley levels, filled spin levels, and filled Landau levels were measured for oxide charges up to 10^{12} cm^{-2} at a temperature of 1.35 K and a magnetic field of 13 T. The widths of the plateaus were observed to increase with increasing oxide charge, which is consistent with a picture in which the number of localized states at band edges increases with increased electronic scattering. With increasing interface charge, however, the positions of the plateaus corresponding to filled valley and filled spin levels exhibited systematic shifts with respect to those for filled Landau levels. In addition, the rate of increase of the widths of the plateaus corresponding to filled spin levels for increasing interface charge was less than that for filled Landau levels. Additional experiments in which the direction of the magnetic field was changed in order to change the relative splittings of the spin and valley states as compared to those for the Landau levels were performed, and shifts in the positions of the plateaus corresponding to filled Landau levels then were observed. The results observed here for shifts in the positions of the quantum Hall plateaus are interpreted in terms of asymmetrical overlaps of the corresponding bands in the density of states. It is argued that these overlaps arise from the effects of the positive interface charge on the localized states at band edges.

I. INTRODUCTION

Since its discovery five years ago[1] the quantum Hall effect has attracted a great deal of experimental and theoretical interest.[2,3] At low temperatures and high perpendicular magnetic fields the quasi-two-dimensional electron gas (2D EG) formed in the inversion layer of a Si metal-oxide-semiconductor field-effect transistor (MOSFET) is found to exhibit flat plateaus in its Hall resistivity as a function of electron density which is controlled by the gate voltage. The plateaus occur at densities corresponding to the filling of complete Landau levels, and the values of the Hall resistivity at these plateaus are multiples of the universal constant h/e^2 to a very high degree of accuracy. This effect also has been observed[4] in the 2D EG at the interface of a modulation doped $GaAs/Ga_{1-x}Al_xAs$ heterojunction; in this case the 2D EG, generally, is at a fixed density, and the Hall resistivity is studied for varying magnetic field. In later work[5,6] quantum Hall plateaus also have been observed at densities corresponding to the filling of simple fractions of the Landau levels. This phenomenon is referred to as the fractional quantum Hall effect and is believed to arise from many-body effects unlike the case for the integral quantum Hall effect which is studied here.

The basic physical origin of the integral quantum Hall effect has now come to be fairly well understood.[7–13] The system considered is a 2D EG in a perpendicular magnetic field. The picture generally used is that the magnetic field splits the states of the 2D EG into fully quantized Landau levels which are separated by an energy $\hbar\omega_c$, where the cyclotron frequency $\omega_c = eB/m^*c$, and m^* is the effective mass. For a density such that a Landau level is completely filled there is effectively a gap of $\hbar\omega_c$ in the excitation spectrum, and thus there are no energetically accessible states into which the carriers can scatter. The transport therefore is dissipationless, and the diagonal resistivity ρ_{xx} vanishes (as does the corresponding conductivity component σ_{xx}). The Hall resistivity then becomes

$$\rho_{xy} = B/en \, , \tag{1}$$

where n is the 2D EG density. For a filled Landau level the density n is an integral multiple of the degeneracy Be/h, and the Hall resistivity becomes

$$\rho_{xy} = h/e^2 i, \quad i = 1, 2, \ldots \, . \tag{2}$$

Here i is called the plateau index, and it is the number of levels of equal degeneracy Be/h which are filled. The energy-level structure for the Si MOSFET system studied here is shown in more detail in Fig. 1. Each Landau level is split into two spin levels, and each spin level is split further into two valley levels which arise from the orbital degeneracy of the Si(100) surface. Each of these levels has the same degeneracy Be/h and therefore gives rise to a separate quantum Hall plateau.

In order to understand the occurrence of finite plateau widths at these values of the Hall resistivity, localized states due to impurities and disorder generally are invoked.[8,9,13] These localized states are supposed to exist at the edges of well-separated levels. As the density increases the Fermi level moves into the localized states and is pinned there. The localized states do not contribute to the conductivity, and thus the resistivity does not change

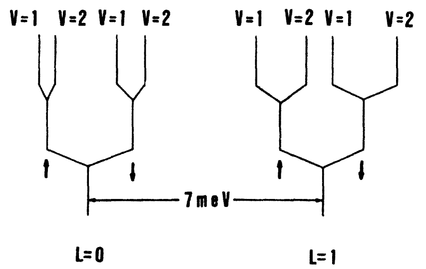

FIG. 1. Sketch of the energy-level diagram for the 2D EG at 13 T on the (100) surface of Si indicating the Landau, spin (arrows), and valley splittings. The spin and valley splittings are dependent on filling; typical values are 2.5 and 0.5 meV for the spin and valley splittings, respectively.

during their filling. The plateau width in density is equal to the number of such localized states at the top of the last band to be filled plus the number at the bottom of the next band. A number of model calculations have been performed to show that the existence of such impurity states at Landau band edges should not affect the values of the resistivity at the plateaus.[10,11,14] Other possibilities for the origin of the plateau widths, such as the pinning of the Fermi level by the chemical potential of a reservoir for electrons, also have been suggested, particularly for the case of the $GaAs/Ga_{1-x}Al_xAs$ heterojunctions.[15]

The quantum Hall effect raises two different but related issues. The first is the high degree of accuracy with which the Hall resistivity at the plateaus is given by simple multiples of h/e^2, and the second is the physical origin of the plateau widths. The former question now is relatively well understood based on pictures of the physical system which involve well-separated bands with localized states at the edges. The origin of the plateau widths in terms of the microscopic physics of the interface, on the other hand, is less well understood and to date has attracted relatively less attention. An important matter relating to the systems on which experiments are performed, especially the Si MOSFET system, concerns the situation in which the adjacent Landau bands overlap substantially. Recently, heat capacity[16] and de Haas–van Alphen[17] studies of the $GaAs/Ga_{1-x}Al_xAs$ heterostructure system suggest that the bands may overlap substantially even in this very high mobility system where one might have expected the Landau bands to be relatively sharp and well separated.[7] The possibility of substantial band overlap appears to raise questions about the applicability of simple models based on well-separated bands for the quantum Hall effect. The present work addresses the question of the microscopic origin of the plateaus, and the results obtained here are discussed in terms of the form of the density of states.

The present studies were made using specially fabricated Si MOSFET's which had intentional doping of the oxide with Na^+ ions. The Na^+ ions in these systems have been found to be quite mobile at high temperatures, and the amount of fixed charge at the interface at low temperatures can be controlled by drifting the ions toward or away from the interface.[18] At low densities Na^+ ions at the interface produce bound states for the inversion layer electrons which have been studied theoretically[19–21] and experimentally.[22–25] The added interface charge from the Na^+ ions causes increased scattering from these charges and from potential fluctuations associated with them. This results in a decrease in the channel mobility,[18,25] in the broadening of the Landau levels,[23] and in an increase in the number of localized states at the band edges.[11,18,26] The added positive charge at the $Si\text{-}SiO_2$ interface also causes the conductivity threshold to shift. The determination of the conductivity threshold in these systems with Na^+ ions in the oxide has proven to be difficult, and discrepancies between results using methods based on extrapolations of the transconductance and extrapolations of Shubnikov–de Haas structures have been reported.[26] The origin of this discrepancy is addressed and resolved in the present work.

The objectives of the present work were to study the effects of controllable and systematic changes in the interface scattering on the plateau widths in the quantum Hall effect. Increases in the plateau widths with increasing oxide charge were observed as expected. Several other novel and unexpected features also were observed. The rate of increase of the plateau widths for levels associated with filled spin levels was found to be different from those associated with filled Landau levels. In addition, the positions (in electron density) of the plateaus associated with the filled spin and filled valley levels were found to shift with increasing oxide charge, but those associated with filled Landau levels did not shift. The latter feature appears to raise an important question with respect to the quantum Hall effect because the plateaus are supposed to occur at densities corresponding to the filled levels and therefore to be at constant density independent of oxide charge. We interpret these effects in terms of the overlap of the density of states corresponding to the adjacent bands.

The experimental results are presented next in Sec. II. These results are discussed in Sec. III, and some brief concluding remarks are made in Sec. IV. A brief version of some of the present results has been given in Ref. 27.

II. EXPERIMENTAL RESULTS

For these studies Si MOSFET's in the form of Hall bars 0.25 mm wide and 1.3 mm long, as shown in Fig. 2, with Na^+ doping in the gate oxide were used. These samples fall into two categories: (1) high mobility samples which have μ_{max} at 4.2 K greater than 2.5 $m^2/V\,s$ and which have maximum driftable oxide charge (ΔN_{ox}) less than 10^{11} cm^{-2}, and (2) good mobility samples which have μ_{max} at 4.2 K approximately 1 $m^2/V\,s$ and which have maximum driftable oxide charge greater than 3×10^{12} cm^{-2}. The gate oxide thicknesses are between 100 and 200 nm.

A number of measurements were made on these samples. First, transconductance ($d\rho/dV_g$) at 77 K and conductivity at 4.2 K were measured to characterize them. Then magnetoresistivity measurements at 1.35 K and 13 T were made. Both components of the resistivity tensor, the diagonal magnetoresistivity ρ_{xx}, and the Hall resistivi-

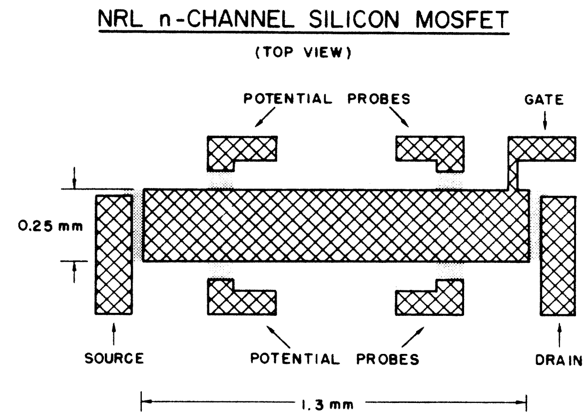

FIG. 2. Schematic drawing of the MOSFET's used in this study.

ty ρ_{xy}, along with their derivatives with respect to gate voltage or equivalently with respect to electron density were measured. ac techniques were used for all these measurements. For the transconductance and derivative measurements a gate modulation of 10 mV at 1 kHz was used. The dc source-drain voltage was less than 0.1 V for the transconductance measurements, and the source-drain current was 100 nA for the derivative measurements. The components of the magnetoresistivity also were measured using ac techniques. In this case a sine-wave source-drain current of 100 nA at 1 kHz with a source impedance of 10 MΩ was used for excitation, and both ρ_{xx} and ρ_{xy} were measured simultaneously. The lock-in amplifiers and current source were calibrated several times each day with a known resistance to ensure consistent measurements.

After a complete set of measurements had been taken for a given Na^+ concentration at the oxide-semiconductor interface, the Na^+ concentration was changed by heating the sample to 373 K and applying a small ($\sim$0.1 V) positive gate bias for a time of about 10 s to increase the Na^+ concentration at the interface. A large negative bias of about 50 V for 5–10 min was used to reduce the Na^+ concentration at the interface back to its minimum value which was reproducible in a given sample to within 10^{10} cm^{-2}.

In the present work the results generally will be displayed as functions of electron density rather than the measured gate voltage V_g, because this makes the physics clearer and also because it permits straightforward comparison of different samples having different oxide thicknesses. When the gate voltage is above the threshold value for conduction, V_{tc}, a MOSFET may be considered to be a capacitor with some capacity C due to the gate oxide. We define V_{tc} in the usual manner for low-temperature measurements as the gate voltage where the transconductance at 77 K extrapolates linearly to zero.[2,26] It then follows that the electron density is given by

$$n = C(V_g - V_{tc})/e \ , \tag{3}$$

where n is the electron density per unit area. A useful measure of electron density for quantum Hall effect studies is the filling factor η which is the measure of the density in units of the Landau degeneracy ξ:

$$\eta = n/\xi \ , \tag{4a}$$

$$\xi = eB/h \ . \tag{4b}$$

It has been found[18,24] that Na^+ ions in MOSFET's have two stable locations in the gate oxide: one near the metal and one near the Si-SiO_2 interface. When the Na^+ ions are drifted to the Si-SiO_2 interface their concentration can be determined by the shift in V_{tc}. We use this method to determine ΔN_{ox}, the change in interface charge in the gate oxide. The position of the conductivity threshold for a given sample with the minimum oxide charge N_{ox} indicates that in this case the total N_{ox} is less than 2×10^{10} cm^{-2}, and therefore the values of ΔN_{ox} quoted here are quite close to the total N_{ox}. We have an indication that the added Na^+ ions are distributed uniformly in those samples which have substantial Na^+ ($N_{ox} > 5\times10^{11}$ cm^{-2}) because a well-defined impurity band like that reported previously[18] has been observed for them.

Figures 3 and 4 show the effects of changing oxide charge on ρ_{xx} and ρ_{xy} at 13 T and 1.35 K. Quantum Hall plateaus in ρ_{xy} and corresponding broad minima in ρ_{xx} are observed near $\eta=2$ and 4. The ρ_{xx} minima near $\eta=1$ correspond to the filling of the first valley level of the spin-up state (see Fig. 1). Those near $\eta=2$ correspond to the filling of the second valley level of the spin-up state

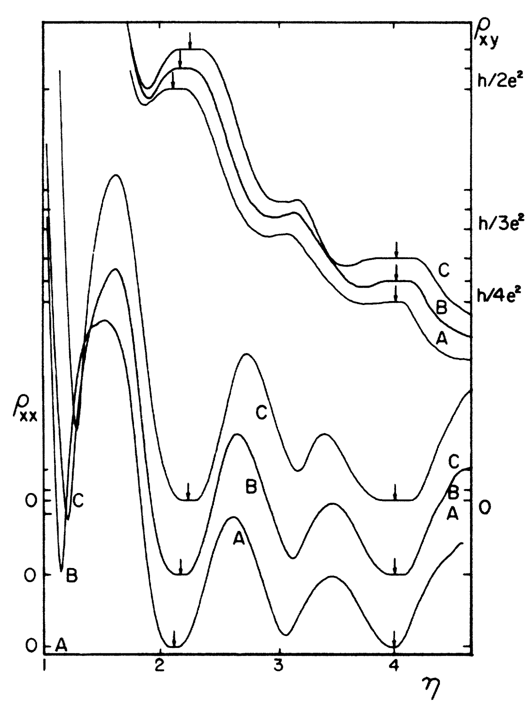

FIG. 3. Diagonal resistivity ρ_{xx} and Hall resistivity ρ_{xy} as a function of gate voltage in units of level filling factor η (see text) at 13 T and 1.35 K for small Na^+ concentrations in the highest mobility sample. Curve A, $\Delta N_{ox}=0$ and $\mu_{max}=2.5$ $m^2/V\,s$; curve B, $\Delta N_{ox}=1.4\times10^{10}$ cm^{-2} and $\mu_{max}=2$ $m^2/V\,s$; curve C, $\Delta N_{ox}=4.5\times10^{10}$ cm^{-2} and $\mu_{max}=1.2$ $m^2/V\,s$. The curves are offset vertically from one another for clarity, and the arrows indicate the positions of the quantum Hall plateaus.

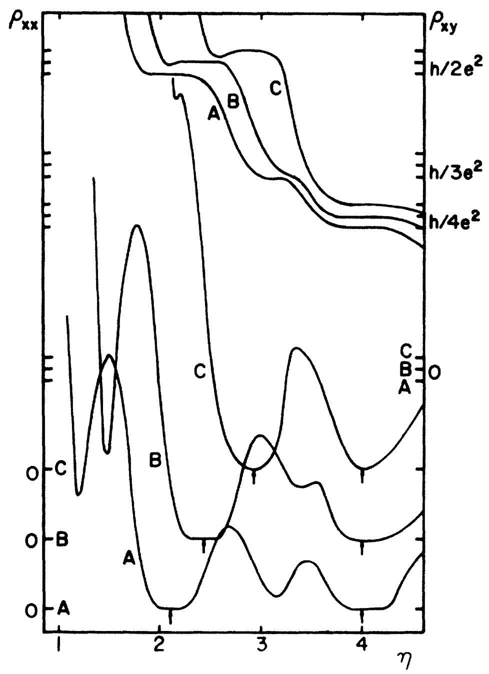

FIG. 4. ρ_{xx} and ρ_{xy} as functions of gate voltage in units of filling factor η at 13 T and 1.35 K for larger Na^+ concentrations in a typical good mobility sample. Curve A, $\Delta N_{ox}=0$ and $\mu_{max}=1$ $m^2/V\,s$; curve B, $\Delta N_{ox}=1.7\times10^{11}$ cm^{-2} and $\mu_{max}=0.54$ $m^2/V\,s$; curve C, $\Delta N_{ox}=3.5\times10^{11}$ cm^{-2} and $\mu_{max}=0.6$ $m^2/V\,s$. The curves are offset vertically from one another for clarity, and the arrows indicate the positions of the quantum Hall plateaus.

giving a filled spin-up state and is referred to as the spin plateau. The plateaus near $\eta=3$ correspond to filling the first valley level of the spin-down state. Those near $\eta=1$ and 3 are referred to as valley plateaus. The plateaus at $\eta=4$ correspond to the complete filling of the first Landau level, and are referred to as the Landau plateaus. Figure 3 shows the effects of small ΔN_{ox} on a very high mobility sample, and Fig. 4 shows the effects of greater amounts of ΔN_{ox} on a good mobility sample.

All of the basic features of the changes in the quantum Hall effect with changing oxide charge which we will discuss are shown in Figs. 3 and 4. In Fig. 3 the Hall plateaus are observed to become wider as small amounts of Na^+ are added. This can be seen in that the plateaus in ρ_{xy} and the corresponding minima in ρ_{xx} near $\eta=2$ and 4 become wider as the oxide charge increases in going from curve A to curve B and to curve C. In addition, it can be seen that the plateaus and minima at $\eta=4$ show more change than do those near $\eta=2$. The minima in ρ_{xx} near $\eta=1$, 2, and 3 are seen to move upward in density toward the minimum corresponding to $\eta=4$ as N_{ox} is increased. In Fig. 4, on the other hand, the Hall plateaus become narrower as increasing amounts of Na^+ are added. The effects of these larger Na^+ concentrations can be seen most easily by examining the ρ_{xx} curves. The minimum in ρ_{xx} at $\eta=4$ becomes considerably narrower for increasing Na^+ concentration, and in curve C, ρ_{xx} ceases to have a zero value. There is much less effect on the minimum corresponding to the filled spin level near $\eta=2$; the minimum becomes broader in going from curve A to curve B, and it still has a zero value in curve C. The valley minima almost cease to exist at the oxide charge densities in Fig. 4, curve C; the minimum and plateau near $\eta=3$ are gone, and the minimum near $\eta=1$ is very weak. The movement of the spin and valley minima with respect to the first Landau minimum can be seen quite dramatically here. At the oxide charge in Fig. 4, curve C, the first spin plateau and first valley plateau in fact are nearer to $\eta=3$ and $\eta=2$, respectively, than to $\eta=2$ and $\eta=1$.

It should be noted that within the accuracy of these experiments ($\sim0.1\%$) the Hall resistivity at the quantum Hall plateaus always is given by the usual formula, Eq. (2), with the plateau index i having the integral values $i=2,6,\ldots$ for the spin plateaus and $i=4,8,\ldots$ for the Landau plateaus. That is, the values of the Hall resistivity at the plateaus correspond to the exact filling of levels of equal degeneracy. Furthermore, within the accuracy of the experiments the Hall resistivity at the plateaus is independent of changes in the oxide charge.

The widths of the quantum Hall plateaus were measured in the usual way[27–29] in which the width of a given plateau is taken to be the region in V_g or n for which $\rho_{xy}=h/ie^2\pm\epsilon$, where ϵ is a small quantity associated with the experimental resolution. The widths at 13 T and 1.35 K, for example, are those for which $d\rho_{xy}/dn<3.5\times10^{10}$ $\Omega\,cm^2$, where n is given by Eq. (3), and the corresponding ϵ is ~1 Ω. Because $d\rho_{xy}/dn$ changes rapidly with n at the edges of the Hall plateaus the widths determined in this way are relatively insensitive to the value of ϵ. The dependence of the Hall plateau widths on the concentration of interface charge is shown in Figs. 5 and 6. In Fig. 5 it is seen that the widths increase with increasing N_{ox} for the highest mobility sample. Here the interesting fact that the rate of increase in the plateau width for the full

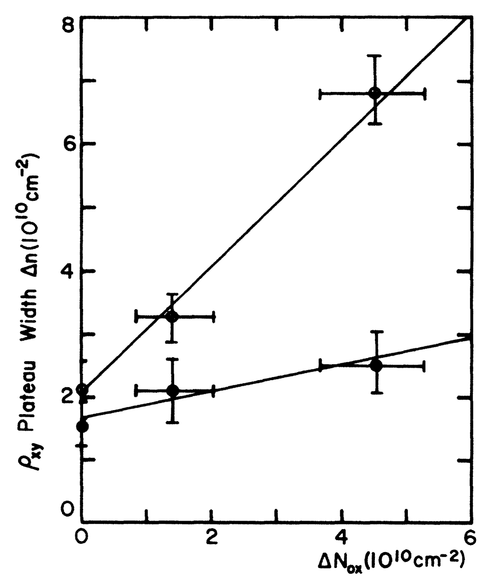

FIG. 5. Quantum Hall plateau widths in units of density as a function of added oxide charge ΔN_{ox} for the highest mobility sample. The open symbols are for the full Landau plateau at $\eta=4$, and the solid symbols are for the spin plateau near $\eta=2$.

Landau level at $\eta = 4$ measured in units of density is roughly equal to the number of added Na^+ ions at the interface can be seen. It is also seen that the spin plateau widths near $\eta = 2$ increase only about one-fifth as rapidly as do the Landau plateau widths. Figure 6 shows the effects of added interface charge for all the samples plotted on a common set of axes. In order to make comparisons between different samples the widths are plotted against the mobility measured at the gate voltage of the given Hall step and at 4.2 K and zero magnetic field. The reason for the usefulness of this kind of plot is that $1/\mu$ is a simple measure of interface potential fluctuations.[28] Again there are systematic differences between the behavior of the plateaus corresponding to filled Landau levels as compared to those for filled spin levels. The widths of the Landau plateaus increase rapidly to a maximum value at a mobility of about 0.9 $m^2/V\,s$, where N_{ox} is about 1.5×10^{11} cm^{-2}, and then decrease with increasing $1/\mu B$. The spin plateaus, on the other hand, continue to increase in width down to the lowest mobilities for which data are available. In both cases the initial increase in width is consistent with the width being linearly proportional to $1/\mu$. Recent studies of these samples[6,30] have shown that the mobility actually increases about 10% for the highest mobility samples ($\mu > 2$ $m^2/V\,s$) between 4.2 and 1.3 K, making the agreement better. This behavior also is consistent with the data of Störmer *et al.*[28] In Fig. 6 the different symbols refer to different substrate biases which give different mobilities.

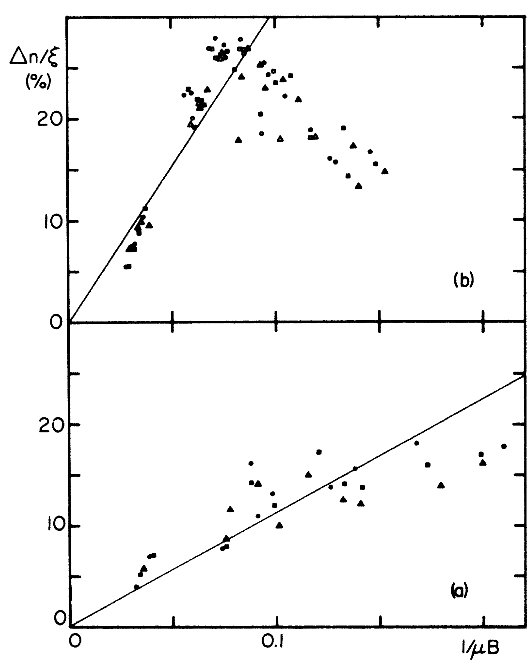

FIG. 6. Quantum Hall plateau widths in units of level degeneracy ξ as a function of inverse mobility. (a) Widths of the spin plateau near $\eta = 2$; (b) widths of the Landau plateau for $\eta = 4$. The mobility in all cases is the mobility at the gate voltage of the corresponding Hall plateau at zero magnetic field and 4.2 K. The different symbols indicate different substrate biases: ○, + 1 V; □, 0 V; △, −1 V; ●, −2 V; ■, −4 V; △, −8 V. The lines are a guide to the eye.

The changes in the positions of the plateaus corresponding to filled spin levels ($i = 2,6,10,\ldots$) with respect to those for filled Landau levels ($i = 4,8,12,\ldots$) are shown systematically in Fig. 7. Here the plateau indices for the spin and Landau plateaus are plotted against the gate voltages at which each occurs. The data in Fig. 7 are from a typical sample which has an intermediate value of oxide charge, $N_{ox} \sim 1.7 \times 10^{11}$ cm^{-2}. Here the gate voltage which gives the position of a given plateau is taken to be that at the corresponding ρ_{xx} minimum. In those cases where the ρ_{xx} minimum is broad, the position is obtained from a well-defined minimum which occurs at an increased temperature (always less than 4.2 K). The shifts of the spin plateaus with respect to the Landau plateaus occur systematically out to the highest gate voltages studied. This is seen in Fig. 7 for the first three Landau plateaus. The positions of the spin plateaus are plotted only for these levels because for the higher-lying plateaus the valley splitting becomes larger than the spin splitting, making the determination of the plateau position and index difficult. The positions of the plateaus for filled valley levels, which have been omitted from the figure for clarity, show a behavior similar to those for filled spin levels. Also, it can be seen that for a given Na^+ concentration the shifts are relatively insensitive to magnetic field; the data for different fields extrapolate to nearly the same gate voltage at zero plateau index. The position for this extrapolation is different for the spin plateaus and is compared to the Landau plateaus. We refer to the position of this extrapolation as the effective threshold for the given manifold of plateaus: V_{tv} for the valley levels, V_{ts} for the spin levels, and V_{tl} for the full Landau levels. The shifts of these effective thresholds with respect to the measured conductivity threshold at 77 K are plotted in Fig. 8 in units of density using Eq. (3) as a function of added interface charge. Here it can be seen that the effective thresholds obtained from the extrapolation of the positions of plateaus corresponding to filled spin and filled valley levels shift roughly linearly with Na^+ concentra-

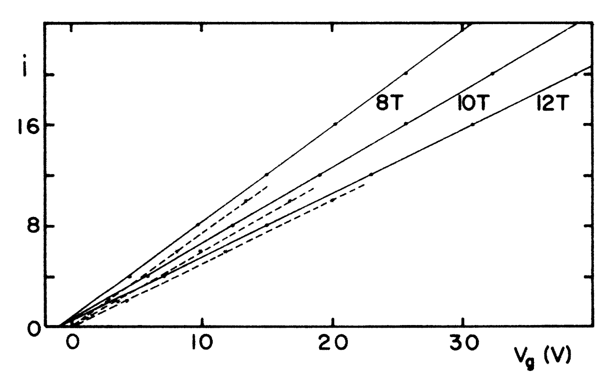

FIG. 7. Extrapolations of the quantum Hall plateau indices as functions of gate voltage at zero gate voltage to determine the Landau and spin threshold voltages V_{tl} and V_{ts} as discussed in the text. The dashed lines with asterisks correspond to the spin plateaus and the solid lines with dots correspond to the Landau plateaus.

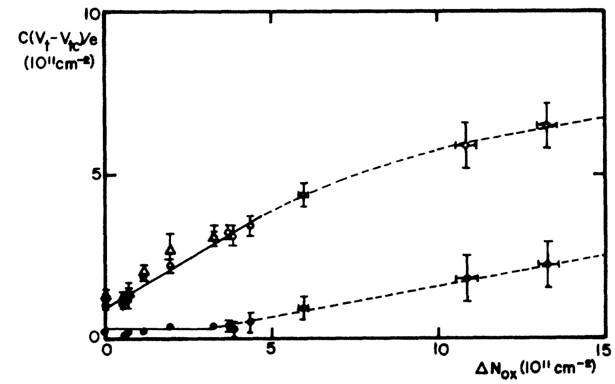

FIG. 8. Shifts of the effective threshold positions for spin and valley plateaus (upper curve) and Landau plateaus (lower curve) from the conductivity threshold V_{tc} measured using the transconductance at 77 K as a function of ΔN_{ox}. Voltages are measured in units of density. The different symbols correspond to the different apparent thresholds as follows: ●, V_{tl}; ○, V_{ts}; △, V_{tv}. The lines are a guide to the eye.

tion as small to moderate numbers of Na^+ ions are moved to the Si-SiO_2 interface ($\Delta N_{ox} < 5\times10^{11}$ cm^{-2}). The positions of the effective thresholds from the plateaus corresponding to filled Landau levels, on the other hand, show very little if any shift from the conductivity threshold until $\Delta N_{ox} > 3\times10^{11}$ cm^{-2}, after which the effective thresholds move with respect to the conductivity thresholds (see the dashed lines in Fig. 8).

In order to study further the observed shifts of the positions of the spin and valley plateaus with respect to the Landau plateaus, additional experiments were performed for varying magnetic fields perpendicular to the interface. There are two different ways of changing the perpendicular magnetic field: first by changing its magnitude and keeping its direction constant, and second by changing its direction and keeping its magnitude constant. The spin and valley energy splittings depend on the total magnetic field, but the Landau splitting depends only on the magnetic field component normal to the plane of the 2D EG. Therefore varying the direction of a constant magnetic field changes the splittings of the valley and spin levels as compared to the Landau level splittings. This effect has been used to measure the electronic g factor[31] and the valley splitting[32] in these systems. In addition, changing the perpendicular component of the magnetic field changes the level degeneracy $B_\perp e/h$. Therefore varying the direction of the magnetic field is useful in studying the widths and the positions of quantum Hall plateaus.

The effects of varying the direction of the magnetic field are shown in Fig. 9. Here ρ_{xx} is given as a function of filling factor for different magnetic field directions with a total field of 13 T. The vertical dashed lines show the positions in density of spin minima derived from the gate voltage using Eqs. (3) and (4), and the vertical solid lines show the positions in density of the minima corresponding to filled Landau levels. At low angles the minima in ρ_{xx} corresponding to filled Landau levels coincide with the integral Landau level filling factors, and the other minima in ρ_{xx} corresponding to filled spin levels occur

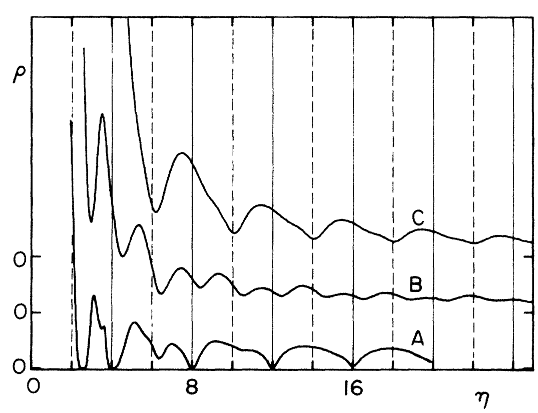

FIG. 9. ρ_{xx} as a function of gate voltage in units of filling factor for a total magnetic field of 13 T at 1.35 K with $\Delta N_{ox} = 1.6\times10^{11}$ cm^{-2}. $B_\perp$ is changed by rotating the sample in the magnetic field. Curve A, $B_\perp = 13$ T; curve B, $B_\perp = 7.4$ T; curve C, $B_\perp = 5.6$ T. The solid lines show the ideal degeneracy of the full Landau levels, and the dashed lines show the ideal degeneracy of the spin levels.

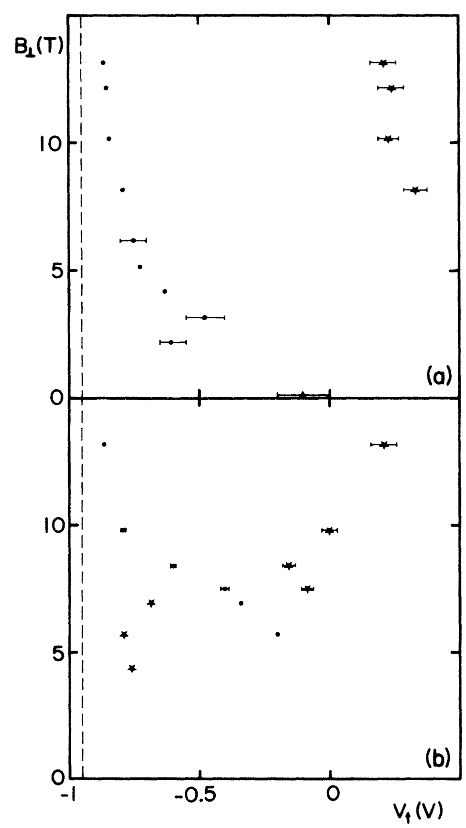

FIG. 10. Threshold positions for different $B_\perp$. The dashed lines represents V_{tc} measured using the transconductance at 77 K. The dots and stars represent the effective Landau and spin thresholds V_{tl} and V_{ts} measured by the extrapolation of plateau indices in Fig. 7. The triangle at zero magnetic field in (a) is obtained using the traditional fan construction as discussed in the text. (a) $B_\perp$ is varied by changing the total magnetic field which is always normal to the 2D EG. (b) $B_\perp$ is changed by rotating a constant 13-T magnetic field with respect to the 2D EG.

at slightly higher filling factors than the dashed lines. At the largest angles this behavior is reversed. The minima in ρ_{xx} which correspond to filled Landau levels are at slightly higher filling factors than the solid lines, and the other minima in ρ_{xx} which correspond to filled spin levels occur at the spin level filling factors. The effective thresholds, V_{ts} and V_{tl}, are plotted as a function of the perpendicular magnetic field component in Fig. 10(b). For comparison, the effect of changing the magnitude of the magnetic field with its direction always perpendicular to the interface is shown in Fig. 10(a).

III. DISCUSSION

The properties of the systems studied here are somewhat simpler in the region of low oxide charge, $N_{\rm ox} < 3\times10^{11}$ cm^{-2}, than in the region of high oxide charge $N_{\rm ox} > 3\times10^{11}$ cm^{-2}. This can be seen, for example, for the plateau shifts in Fig. 8. Therefore we will discuss the low and high oxide charge regions separately. Finally, we have found that based on the present analysis we are able to resolve the discrepancy between previous threshold determinations which were made based on transconductance, as compared to those made by extrapolations of Shubnikov–de Haas features for Si MOSFET's with Na^+ in the oxide, and we discuss this issue at the end of the present section.

A. The low oxide charge region

We begin by considering the fact that all of the plateaus tend to increase in width with increasing oxide charge as seen in Figs. 3 and 5. These increases with increasing oxide charge can be understood straightforwardly in terms of a picture in which the plateau widths arise from the pinning of the Fermi level by localized states at the band edges. When the oxide charge is increased the scattering of the inversion layer electrons increases. The scattering of the electrons may arise directly from the charges of the Na^+ ions or it may arise from longer-range potential fluctuations associated with the oxide charge distribution and from other localized electrons. The increased scattering gives rise to broadening of the levels, to an increased number of localized states at the band edges, and, in the standard Mott picture of localization, to movement of the mobility edges inward toward the centers of the bands. The plateau widths (in density) are given by the number of localized electrons above the upper mobility edge in one band plus the number below the lower mobility edge in the next band. Therefore the widths are expected to increase with increasing oxide charge as observed.

The relative shifts in the positions of the plateaus associated with the filled spin and valley levels, with respect to those for filled Landau levels as well as the markedly different rates of increase of the plateau widths for filled spin, as compared to those for filled Landau levels with increasing Na^+ concentration are more difficult to understand however. We begin by discussing the positions of the plateaus. From the data for ρ_{xy} and ρ_{xx} the quantum Hall plateaus at $B=13$ T are displayed in Figs. 3 and 4 as functions of the filling factor. It is seen that the plateau corresponding to the filling of the first complete Landau level occurs at the expected density which is given by a filling factor of 4, and that this position does not change with Na^+ concentration. The plateau corresponding to the first filled spin level, on the other hand, occurs at a higher density than the expected filling factor of 2, and its position moves higher in density with increasing Na^+ concentration. We might note that this displacement of the position of the plateau for the filled spin level from its ideal position at a filling factor of 2 occurs in the original data of von Klitzing *et al.*,[1] but was not discussed there. These shifts occur not only for the first Landau level but also consistently for higher-lying spin and Landau plateaus (see Fig. 7). The plateaus corresponding to filled valley levels behave in a manner similar to those for filled spin levels.

These results for plateau positions are displayed systematically in Fig. 7, where the measured voltages at which plateaus occur are plotted against the plateau indices. The plateau indices represent the ideal degeneracy of the levels whose filling gives the plateaus. Note that for a given magnetic field the shifts of the spin plateaus with respect to the Landau plateaus are uniform, giving straight lines displaced from one another. Here it is seen that at each magnetic field the line formed by the voltages corresponding to the spin plateaus is shifted to higher values than expected by the ideal degeneracy which is represented by the line for Landau plateaus. As seen in the figure, this feature can be represented by saying that the effective threshold at zero degeneracy has been shifted to higher voltages. It should be emphasized, however, that this shift in the threshold merely represents the fact that the density at which the plateau for each filled spin level occurs is more that half of the density for the plateau for the completely filled Landau level as seen in Figs. 3 and 4. These shifts in plateau position increase systematically with increasing Na^+ concentration as shown in Fig. 8.

These results for shifting plateau positions cannot be understood on the basis of a picture of separated levels of constant degeneracy because in that case adjacent minima should be separated by the same density, and from Eq. (3), by the same voltage. A density of states having separated bands is sketched in Fig. 11(a) where it is seen that no changes in the shape of the bands will give shifts in the density at which the plateaus occur, provided the localized states occur at the band edges. We argue that in order for the observed shifts to occur the band edges must overlap so that electrons from the higher band move down into the upper edge of the lower band and vice versa. Moreover, for a plateau to move up in density from the expected position the overlaps of the adjacent bands must be asymmetrical in such a way that more electrons move into the lower band than move into the upper one. Such a possibility is shown schematically in Fig. 11(b). In order to account for the increases of the shifts with increasing Na^+ concentration the asymmetrical overlaps must increase with increasing concentration; we will return to this point shortly. We note here that in the present experiments the positions of some of the broader plateaus were

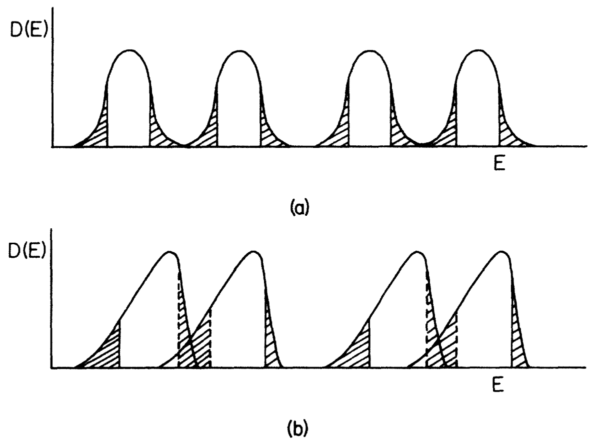

FIG. 11. Sketches of possible forms of the density of states. Hatched regions correspond to localized states beyond the mobility edges at band edges. (a) corresponds to bands with symmetrical shapes. (b) corresponds to bands with asymmetrical shapes and overlaps between the bands as discussed in the text.

determined by raising the temperature in such a way that ρ_{xx} exhibits a distinct minimum. This is consistent with the present interpretation in which the position of the plateau is associated with the region of lowest density of states which contains the most strongly localized states.

From the absence of shifts for the plateaus corresponding to filled Landau levels at $B=13$ T, we argue that the corresponding bands do not overlap; the observed shifts of the plateaus corresponding to filled spin and valley levels, on the other hand, suggest that these bands do overlap. These features are consistent with the magnitudes of the energy splittings between these levels. The Landau level splitting at $B=13$ T is $\hbar\omega_c=7$ meV, the spin splitting $g\mu_B B$ is smaller on the order of 2.5 meV, and the valley splitting is very small on the order of 0.5 meV (the spin and valley splittings depend on occupancy). Estimates[2] of the bandwidths for systems of this kind are in the range 1.0—1.5 meV, which is consistent with the occurrence of overlap at the spin and valley splittings.

The data in Figs. 9 and 10 for the effects of changing the total magnetic field and of changing its direction with respect to the 2D EG system provide a detailed confirmation of the present interpretation of the origin of the shifts of the plateau positions. The spin splittings are proportional to the total magnetic field; the Landau level splitting and the degeneracy of each level, however, are proportional to the component of the magnetic field perpendicular to the 2D EG. When the total magnetic field is changed both the spin and Landau level splittings decrease in proportion to the field. The degeneracy of the levels and the widths of the levels also decrease however. The relatively weak dependence of the positions of the plateaus for filled spin and filled Landau levels seen in Fig. 10(a) can be understood by noting that although both the spin and Landau splittings decrease the level widths also decrease, and therefore there is relatively little net change in the band overlaps.

For the case in which the direction of the magnetic field is varied, on the other hand, the relative magnitudes of the spin and Landau level splittings are changed. In this case the thresholds for the Landau plateaus are observed to move upward and those for the spin plateaus move downward. We interpret these data in the following way. The spin splittings remain constant while the bandwidths decrease, leading to a decreased band overlap and to the observed downward shift in the positions of the plateaus for filled spin levels. The splitting between the Landau levels decreases while the spin splittings remain constant, and therefore the upper spin state from one Landau level begins to overlap the lower spin state from the next Landau level, and the positions of the plateaus for the filled Landau levels begin to shift as observed.

We now turn to the different rates at which the widths of the plateaus corresponding to filled spin levels increase with Na^+ concentration as compared to those for filled Landau levels as seen in Figs. 5 and 6. We argue that this feature also has a reasonable interpretation in the present picture. The interpretation of the shifts in the plateau positions for filled valley and filled spin levels have required substantial band overlaps which increase with Na^+ concentration. As the band overlap increases the localized states in the tails of one band tend to move into the region of extended states of the adjacent band and thus tend to become delocalized. Therefore the rate of increase of the number of localized states giving the plateaus for filled spin states for which the overlap is greatest will be less than that giving the plateau widths for filled Landau states for which the overlap is least. This behavior corresponds to the experimental results shown in Figs. 5 and 6. We note that the interesting fact, that the widths of the plateaus for filled Landau levels when measured in units of density is approximately equal to the number of added Na^+ ions (see Fig. 5), suggests that in this case each added Na^+ ion gives rise to approximately one localized electronic state. We note that such a situation is plausible but that its quantitative understanding lies in the microscopic physics of the formation of the localized states at the band edges.

We argue here that the dominant physical effect giving rise to the asymmetrical band shapes discussed above is straightforward and easy to understand. If a positive charge with a sufficiently strong potential is introduced at the interface a localized electronic state appears at the low-energy edge of the band. In general, when a distribution of positive charges is introduced at the interface a distribution of localized states will appear at the low-energy edges of the bands. The net effect is that the band will acquire a low-energy tail as indicated schematically in Fig. 11(b). These features, which are expected on the above general grounds, have been observed both in numerical studies[33] and in analytical calculations[34] carried out on model systems consisting of attractive short-ranged scatterers having densities roughly comparable to those in the present experiments.[35] Even in the absence of intentionally doped Na^+ ions the oxide charge in a Si MOSFET generally is positive. The resulting band asymmetry and overlapping bands then would account for the shifts of the spin plateaus in the absence of intentional Na^+ doping observed in both the present experiments and in the original experiments of v. Klitzing *et al.*[1] The introduction of additional Na^+ ions gives rise to an increase of the low-energy tails of the bands and to increases of the shifts of the corresponding plateau positions as observed experimentally here.

Although we feel that the dominant physical mechanism giving rise to the shifts of the plateau positions is that described above, there may be additional effects which contribute to a lesser degree. We mention three such possibilities here. The effect of screening is known to increase as the band fills,[2,36] and therefore it will affect the states at the tops of the bands most and those at the bottoms least. This would tend to leave a higher density of localized states at the lower edges of the bands which would tend to increase the asymmetrical overlap. In addition, some novel many-body effects might be exhibited by this confined, fully quantized system. An example is that the lowest-lying localized impuritylike state associated with the Na^+ ions has a preferred spin orientation in the magnetic field, and therefore when the lowest-lying spin state which has parallel spin is filled, the electrons will be required to avoid the region of the localized electron wave function and thereby will have their energies raised. This will not be the case for the higher spin state which has opposite spin. More generally, all of the spin and valley states of a complete Landau level except the highest spin state will have either a spin or valley function which is the same as that of the lowest-lying localized state. These effects will tend to increase the overlaps between the levels, except for those corresponding to the completely filled Landau levels. Finally, we might note that theories which treat the two-dimensional localization in a more sophisticated way[37] also may give some insight into the physical origin of the plateau shifts discussed here.

B. The high oxide charge region

Although the scattering at the interface for this region of high oxide charge becomes quite large, the general features of the quantum Hall effect observed in the present experiments for this region appear to be consistent with the picture developed here for the region of lower oxide charge. It is seen in Fig. 8 that the positions of the plateaus corresponding to both filled spin levels and filled Landau levels now shift with increasing oxide charge, but that the magnitude of the shifts of the spin plateaus remain greater than those of the Landau plateaus for each value of N_{ox} in the region studied. This behavior is consistent with there being (asymmetrical) overlaps of all bands due to increased scattering for these values of N_{ox}. The bands corresponding to the Landau splitting now also begin to overlap for these values of N_{ox}, and as a result the corresponding plateau positions begin to shift as seen in Fig. 8. The shifts of the spin and valley plateaus remain somewhat larger than those of the Landau plateaus, and thus the overlaps corresponding to the spin and valley plateaus appear to remain comparable to or greater than those for the Landau levels.

The behavior of the plateau widths shown in Fig. 6 for the highest values of interface scattering is less straightforward. The plateau widths for filled Landau levels at first increase more rapidly with increasing interface scattering than do those for filled spin levels, but at the highest values of the interface scattering they begin to decrease with increased interface scattering. The widths of the plateaus corresponding to filled spin levels, on the other hand, continue to increase with increasing scattering to the highest values of oxide charge used. It should be noted, however, that the widths of the Landau plateaus remain greater than or comparable to those for the spin plateaus for all values of the oxide charge. The interpretation of the behavior of the Landau plateau widths may be that they increase to the greatest values observed at a point at which substantial band overlap begins to occur. After that their widths begin to decrease with increasing interface scattering due to the fact that the localized states from the tail of one band overlap the extended states of the adjacent band and become extended states. This occurs until the widths of the Landau plateaus decrease to values comparable to those for the spin plateaus, which apparently are typical of the widths of plateaus corresponding to bands which have substantial overlap. We should point out, however, that at very high oxide charges the Landau plateaus cease to exist, and the present picture of quantum Hall plateaus breaks down. In addition, in this region of high oxide charge other descriptions of the behavior of the quantum Hall plateaus may be appropriate.[3,37]

C. Determination of conductivity threshold

The gate voltage at which the inversion layer first begins to fill with electrons is called the conductivity threshold, and it has been a central issue in studies of the 2D EG in MOSFET's.[2] The usual method[2,26] for determining the threshold voltage is the linear extrapolation of the transconductance as a function of gate voltage to zero transconductance at 77 K. This method was used to determine V_{tc} in the present work. It is shown by the dashed line in Figs. 10(a) and 10(b). This method is simple to perform and is reliable. A second method which is suitable for threshold determinations at liquid-helium temperatures was developed in some of the initial studies[38] on MOSFET's. In this method the extrapolation to zero magnetic field of the positions in gate voltage of Shubnikov–de Haas structures gives the conductivity threshold voltage, because at zero magnetic field the Landau levels coalesce at the band bottom of the 2D EG. This procedure is the traditional fan diagram construction. The determination of threshold voltages in this way is usually made based on the low magnetic field structure of the Shubnikov–de Haas oscillations where the structure is sinusoidal as a function of gate voltage and where it does not have complications due to spin or valley splittings. These two methods have been found to give consistent results for the threshold voltage in MOSFET's which have relatively thick oxides (greater than 50 nm) and which have a low density of interface states.[2,26] For samples in which the gate oxide contains an appreciable number of Na^+ ions, however, the fan diagram method gives consistently higher threshold voltages than does the extrapolation of the 77-K transconductance.[26] We believe that much of this inconsistency, especially in cases which $N_{ox} < 5\times10^{11}$ cm^{-2}, can be explained by the results presented here.

As we have discussed above and have shown in Fig. 7, we have introduced a different method to determine what we have called effective thresholds by using the extrapolation to zero index of the positions in gate voltage of the quantum Hall plateaus plotted versus plateau index. From Eqs. (1) and (2) it can be seen that this method nominally gives threshold positions. These effective thresholds were used to characterize the shifts in plateau position with Na^+ concentration as shown in Fig. 8. We have found that there are different threshold positions for the different types of plateaus (spin, valley, and full Landau level), and we have accounted for the shifts of the plateaus on the basis of asymmetrical overlaps of the corresponding densities of states. The points at finite magnetic field in Fig. 10(a) were obtained using this method for the plateaus corresponding to filled Landau levels. We now relate the results for these effective thresholds to those obtained from the other methods.

First we have used the data from which the effective thresholds in Fig. 10(a) were obtained to obtain the fan diagram result. When this is done we find that the threshold obtained by the traditional fan diagram method depends strongly on the plateau index used and moves to higher voltages with higher indices. The zero-field point in Fig. 10(a) was obtained by averaging these fan diagram results for plateau indices 16, 20, 24, 28, 32, 36, and 40. Smaller indices were excluded because they included distortions due to the spin and valley levels. Higher indices were excluded because sufficient data were not available to give a reliable extrapolation to zero field.

In Fig. 10(a) we can see the differences between the three methods for determining threshold for a typical Na^+ drifted sample. The difference between the threshold determined using the fan diagrams [the zero-field point in Fig. 10(a)] and the threshold determined using the extrapolation of the transconductance at 77 K [dashed line in Fig. 10(a)] is 1.3×10^{11} cm^{-2}, which is similar to the difference obtained in Ref. 26 using these two methods. In Fig. 10(a) it also can be seen that the fan diagram result appears to be the appropriate limiting point for the finite magnetic field values which were obtained using the plateau index extrapolation. In this sense one can interpret the difference, between the threshold obtained from the transconductance and that from the fan diagrams, as arising from the same origin as the shifts in positions of the ρ_{xx} minima associated with the filled Landau plateaus in the plateau index extrapolation as the magnetic field is lowered. From Fig. 8 it is seen that for $N_{ox} < 5\times10^{11}$ cm^{-2} the high-field plateau index extrapolation for the filled Landau plateaus gives results consistent with the thresholds obtained from the extrapolation of the transconductance at 77 K.

These results for the thresholds can be understood in the following way. At sufficiently high magnetic field the Landau levels are well separated, and an extrapolation based on their indices produces the physically correct band bottom for the 2D EG system. We note that for high magnetic fields these thresholds approach that obtained from the transconductance at zero magnetic field. The small difference between these two results at high magnetic field is $\sim3\times10^{10}$ cm^{-2}, which we believe is a real threshold shift between 77 and 4.2 K due to interface states. As the magnetic field is lowered the present method based on plateau index extrapolations gives thresholds which begin to shift due to the asymmetric overlaps of the bands. These overlaps increase as the magnetic field is lowered. In the limit of very low fields these thresholds go over to the fan diagram result. The band overlaps give rise to an apparent threshold which is shifted from the correct band bottom for the 2D EG. We therefore believe that both the present method and the method based on the transconductance are reliable measures of the gate voltage at which the Fermi level reaches the bottom of the first subband in the inversion layer for MOSFET's with $N_{ox} < 5\times10^{11}$ cm^{-2}. This is true if the magnetic field used for the present method is high enough to give a threshold which is independent of magnetic field.

The present method of extrapolation based on plateau indices is particularly well suited for the analysis of data obtained at magnetic fields which are sufficiently high that the plateau index can be determined easily. The traditional fan diagrams do not depend on knowing the plateau index, and thus they are more appropriate in analyzing low magnetic field data. For a sample with a low concentration of interface charge the plateau index extrapolation is equivalent to the traditional fan diagram method, and they both give reliable results for the determination of the conductivity threshold. On the other hand, as noted above, for systems with substantial oxide charge the traditional fan diagram approach can lead to substantial errors in determining the band bottom due to the overlaps in the densities of states at low magnetic fields, whereas the present approach gives the most reliable results for these systems.

IV. CONCLUDING REMARKS

We have presented the results of a series of experiments on the quantum Hall effect in Si MOSFET's in which the oxide charge was varied systematically by drifting mobile Na^+ ions in the oxide to the interface. This work represents the first systematic investigation of the microscopic origin of the plateau widths in these systems. The widths were observed to increase with increasing Na^+ concentration, as would be expected on the basis of an increased number of localized states at the band edges due to the increased disorder. In addition, two kinds of novel features were observed. The plateaus corresponding to filled spin and filled valley levels were observed to occur at higher densities than expected, compared to those for filled Landau levels on the basis of filling levels of equal degeneracy, and secondly the rates of increase of the plateau widths for filled spin levels and filled Landau levels were observed to be different. We have interpreted these novel phenomena in terms of substantial asymmetrical overlaps in the densities of states corresponding to the levels for which the positions shift. Experiments in which the magnetic field was tilted with respect to the plane of the electronic system cause even the plateaus corresponding to filled Landau levels to shift, which is consistent with the present interpretation. We have argued that scattering and localization by the positive charges added at the interface gives rise to these asymmetrical band broadenings and overlaps.

We conclude by remarking that the present work may have some interesting implications for our understanding of the quantum Hall effect. The two fundamental issues arising in the quantum Hall effect are the accuracy with which the Hall resistivity at the plateaus is given by multiples of h/e^2 and the microscopic origin of plateaus. To date these features have been understood in terms of a picture involving the filling of well-separated bands of equal degeneracy which have localized states at the band edges. The present work has shown the important role played by substantial band overlap in understanding the origin of the plateau widths in the Si MOSFET system. Other experiments involving heat capacity[16] and de Haas—van Alphen[17] experiments suggest that there may be substantial band overlaps even in the $GaAs/Ga_{1-x}Al_xAs$ heterojunction system. When such large overlaps occur it is no longer entirely clear how to understand the meaning of the occupancy of a band or how to identify the origin of a given electron state with a particular band. These issues may affect our understanding of the values of the Hall resistivity observed at the plateaus, because that understanding is based on a picture of well-separated bands of equal degeneracy. We note that in the present work shifts of plateau positions which suggest band overlaps of up to the entire band degeneracy have been observed, but that nevertheless the corresponding values of the Hall resistivity at the plateaus are given by their ideal values independent of the magnitude of the shift in plateau position or of the magnitude of the band overlap. It is not clear on the basis of the present picture of the origin of the quantum Hall effect how these accurate values of the Hall resistance can be reconciled quantitatively with such large band overlaps. In addition, the present results concerning the effects of interface scattering and overlaps from different bands in the density of states may have implications for the understanding of the fractional quantum Hall effect; in this case the gap in the excitation spectrum originates from many-body effects but is not yet fully understood.

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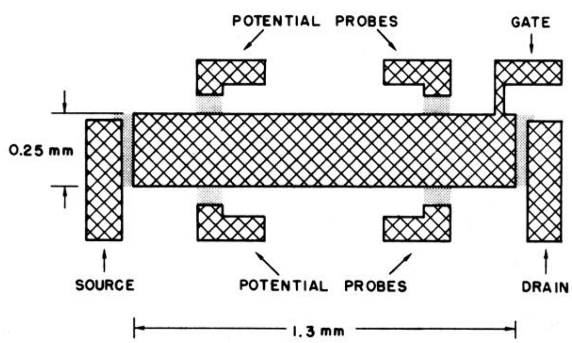

FIG. 2. Schematic drawing of the MOSFET's used in this study.