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Anisotropic magnetoresistance in a Fermi glass

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Insulating thin films of indium oxide exhibit negative, anisotropic magnetoresistance. The systematics of these results imply that the magnetoresistance mechanism may give different weight to the distribution of the localization lengths than that given by the hopping conductivity.

One of the characteristic features of a Fermi glass is the localized nature of the electronic states at the Fermi energy. Namely, the envelope of these states is believed to have the general form $\psi(\mathbf{R}) \sim \exp(-|\mathbf{R} - \mathbf{R}_i|/\xi)$, where \mathbf{R}_i is a particular site and ξ is the localization length. ξ is usually conceived to be a natural length scale for the quasiparticle spatial phase coherence. In particular, ξ may play a decisive role¹ in determining the effective dimensionality of the system under study.

In this Rapid Communication some transport measurements performed on insulating films of indium oxide are described. Special emphasis is given to the magnetoresistance (MR) results at liquid-He temperatures where some intriguing features are observed. The relevancy of these findings to the issue of phase coherence in a Fermi glass is stressed.

The In_2O_{3-x} films used in this study were prepared, characterized, and heat treated as described elsewhere.² A standard four-probe dc technique was employed for resistance measurements. A technical point that should be explicitly mentioned here concerns the sensitivity of the film resistance to the magnitude of the electric fields used in the measurements. It was found necessary to reduce the fields to values that were typically smaller than 0.5 V/cm in order to be in the Ohmic regime. This precaution somewhat compromised the ultimate resolution in terms of the fractional change in resistance, $\Delta R/R$, but it is evidently the only way to get electric-field-independent MR results. The electric field dependence and the Hall constant as a function of temperature as well as a fuller account of experimental techniques will be given elsewhere. In the following, attention is focused on the resistance versus temperature, R(T), and the MR results as a function of magnetic field, all obtained in the Ohmic regime.

Figure 1 depicts R(T) for several In_2O_{3-x} samples having the same thickness (d = 100 Å). At low temperatures, all these curves are fairly consistent with two-dimensional (2D) variable-range hopping³ $R(T) \sim \exp[(T_0/T)^{1/3}]$ with³

$$T_0 = \frac{3}{k_B N(0)\xi^2 d} \quad . \tag{1}$$

The values of ξ for each sample, as marked in Fig. 1, were calculated [using Eq. (1)], assuming a constant density of states, $N(0) = 10^{32} \text{ erg}^{-1} \text{ cm}^{-3}$. Note that these values for ξ are quite plausible for a material with an electron concentration of $\sim 4 \times 10^{19} \text{ cm}^{-3}$ (based on the Hall-effect measurement). If allowance is made for a possible² reduction in

N(0) with disorder, somewhat higher values for ξ will be obtained. Note that N(0) used here is the thermodynamic density of states which is finite for a Fermi glass.²

Applying a magnetic field causes the resistance to drop by a temperature-dependent amount (Fig. 1). A negative MR in this range of temperatures and fields has been observed in the weakly localized (WL) regime in 2D and 3D indium oxide samples⁴ and was interpreted as reflecting quantuminterference⁵ effects. There are some evident resemblences between the MR results found in the WL regime and those observed in the present, strongly localized (SL) case. Most notably, there is a pronounced anisotropy (Fig. 2), a fact that points to a "nonlocal" mechanism.⁶ Before any use is made of these similarities, let me mention some of the



FIG. 1. Sheet resistance as a function of temperature for five 100-Å-thick samples. Indicated are the values of the sheet resistances at 4.2 K, for identification, and the respective values of ξ calculated through Eq. (1). Full circles designate the R(T) curve in the presence of a perpendicular field, H = 5.6 kOe.



FIG. 2. The (negative) fractional change of resistance as a function of angle between the field direction and sample's plane for three samples identified by their R_{\Box} (4.2 K) and thickness. Note that for the 100-Å samples H = 5.6 kOe, whereas H = 2.4 kOe for the 250-Å film. (The anisotropy was found to decrease with field, a behavior similar to that found before in Ref. 4.) Measurements were taken at 4.2 K.

more obvious differences. It is observed (Fig. 3), in agreement with the preliminary study of Ref. 1, that $\Delta R/R$ varies *faster than* H^2 in small fields $(\Delta R/R \propto H^2)$ is the WL result⁵). Except for the 33-k Ω sample, the present study probes the regime where $l_H = (c\hbar/2eh)^{1/2} > \xi$ or $H << c\hbar/2e\xi 2$. This is a *necessary* condition¹ for the results to be characteristic of the insulating (or SL) phase. Clearly, a power-law dependence is not obeyed even for small fields. This observation seems to rule out MR mechanisms that involve magnetic field modifications⁷ of ξ . (In fact, $\Delta R/R \propto H^x$ with $0.5 \le x \le 1$ for small H is a common



FIG. 3. Negative MR at 4.2 K for two of the samples of Fig. 1. Note that the 33-k Ω sample has still the qualitative features expected of the WL regime (Ref. 5), whereas the 140-k Ω sample does not show a power-law dependence at any range of fields.

feature of most known MR mechanisms and clearly the data are not compatible with it.) A functional dependence that does seem to agree with the MR data is illustrated in Fig. 4 for three of the samples of Fig. 1. It will become clear below that this dependence may have some merits that transcend the tentative fitting agreement exhibited in Fig. 4. To make any progress with the analysis of the data some assumptions will have to be made.

It may be possible to give a consistent interpretation of these MR results if it is assumed that there is a long-tailed distribution of coherent "loops" in the system on which the MR mechanism operates. The latter may be, e.g., the quantum-interference mechanism recently proposed by Nguyen, Spivak, and Shkolovskii.⁸ In fact, if it is assumed that $P(\xi)$, the probability of finding a "loop" of lateral size ξ , goes like $\exp(-\xi)$ then the macroscopic MR may resemble the experimentally observed one. A simple way to see that is to note that the basic condition for a "loop" of size ξ to contribute is $H\xi^2 \sim \phi_0$. Namely, the weighting factor for the macroscopic MR at a given field is essentially $\sim \exp(-H^{-1/2})$.

A possible scenario for $P(\xi) \sim \exp(-\xi)$ to arise was discussed by Thouless⁹ in connection with the consistency of localized states. States that are a distance L apart may be strongly mixed if the energy associated with their spatial overlap is of the order of their energy difference. Since the energy difference decreases as a power law of L while the overlap decreases exponentially with it, such spread-out



FIG. 4. Negative MR of several In_2O_{3-x} insulating samples [identified by R_{\Box} (4.2 K) and thickness], measured at 4.2 K. Note the markedly different slope for the thicker sample.

states will occur with exponential probability.

When a film of a thickness d is sliced off the bulk insulator the number of states available for forming linear combinations with a given site may be drastically reduced thus making $P(\xi)$ a function of d (as well as of disorder). This may qualitatively account for the parallelism of the 100-Å sample curves in Fig. 4. Indeed, the MR results for a 250-Å sample, whose study was motivated by the above considerations, exhibit quite a different slope (Fig. 4).

Without a detailed model, this point cannot be further elucidated. However, it is evident that the following features of the observed MR may be qualitatively explained by such a scenario: (1) the functional dependence of $\Delta R/R$ on H, (2) the fact that $\Delta R/R$ at a given field and temperature decreases with growing disorder (Fig. 4), and (3) the MR anisotropy and its dependence on disorder and thickness (Fig. 2).

It should be emphasized that the existence of "loops" with $\xi > d$ must be assumed from the outset to account for the anisotropic MR. The samples studied are composed of randomly oriented crystallites with bcc symmetry.² Properties related to *bulk* conductivity should then be quite isotropic. It is therefore necessary to invoke the assumption of *some* L > d as the most plausible explanation for the anisotropy.

That states with $\xi > d$ are important for the MR while the fact that the "hopping-conductivity-derived" ξ is less than d

may merely reflect the different weighting given by either process to the ξ distribution: The hopping conductivity gives more weight to ξ values that percolate in the system, whereas the low-field MR is presumably sensitive to the high end of the distribution. (When H increases from zero, "loops" satisfying $H\xi^2 \sim \phi_0$ may contribute whether they percolate or not.)

In summary, we have presented results of MR measurements performed on insulating In_2O_{3-x} films. The MR is consistently found to be negative and exhibits an anisotropy the degree of which is thickness and disorder dependent. Most notably, the MR has an unusual field dependence which cannot be reconciled with current theoretical models. A heuristic conjecture is then raised that it may be necessary to take into account the shape of the localization-length distribution to explain the data.

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