Zener tunneling and dissipation in small loops

Rolf Landauer
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
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Small and one-dimensional metallic loops, without leads, and only with elastic scattering, have been shown to have Josephson-like behavior, and should not exhibit a resistance, when excited by a time-dependent magnetic flux. Electrons in this loop behave like electrons in a superlattice whose potential variation, within a superlattice period, matches that found in the circuit around the ring. The earlier work neglected the Zener tunneling in this effective superlattice band structure. It is shown that transitions to higher-lying bands, arising from this Zener tunneling, can be undone, and represent energy storage, not dissipation.

Two earlier papers[1,2] discussed the current flow in small, normal, and one-dimensional loops of metals, induced by a magnetic flux. A somewhat separate series[3,4] treated the transmission behavior and resistance of loops with leads, eventually going beyond the one-dimensional case.[5,6] Subsequently, these results were given a striking experimental confirmation by Webb, Washburn, Umbach, and Laibowitz,[7] elaborated by other investigations.[8] The present article, once again, specializes to the case of a loop without leads and, again, limits the discussion to a strictly one-dimensional ring. Thus, we examine the case where the wave function has a variation only along the ring and need not discuss its transverse variation. This is, therefore, a paper of conceptual significance, and not directly comparable to experiment. However, just as Ref. 5 extended the discussion of Ref. 4 from the idealized one-dimensional case to a more realistic case, this present discussion may be subject to further extension. Here, we will provide an extension of Ref. 1, and first summarize the results of Ref. 1.

Reference 1 pointed out that the single-electron states of a ring can be obtained from the band structure of a crystal with $\mathscr{V}(x)=\mathscr{V}(x+L)$, where $\mathscr{V}(x)$ is the potential around the loop with circumference L. The electronic states of the loop are obtained from the bands, $U_n(k)$, of this periodic loop potential via the rule $k=-(2\pi/L)\Phi/\Phi_0$, where $\Phi_0=hc/e$ is the single-electron flux quantum. For each flux Φ, there is a ladder of states, $U_n(\Phi)$, shown in Fig. 1. The electronic states are periodic in flux, with period Φ_0. For a time-independent flux Φ, we find a zero-temperature current $j=-(e/L)\sum_n v_n = c\sum_n \partial U_n/\partial\Phi$. The summation includes all occupied states, up to the Fermi energy. Thus, Ref. 1 predicts a persistent current, which is a periodic function of the flux, with period Φ_0. In the presence of a flux which increases linearly in time the induced electromotive force $E=(1/cL)d\Phi/dt$ drives the ladder of states through the Brillouin zone, according to $\hbar\dot{k}=-eE$. If we assume that E is small enough so that Zener tunneling between "bands" is negligible, then the field produces an oscillating current with frequency $\omega=eV/\hbar$, i.e., a Josephson frequency with a single electronic charge. Here we have taken $V=EL$. The time-average current vanishes.

Reference 2 extended the discussion of Ref. 1 to allow for inelastic scattering, and an alternative approach to that was also provided in Ref. 9. In this note we will, instead, extend Ref. 1 in a different direction. Reference 1 assumed that the applied acceleration force, acting on the electrons, was small enough to make Zener tunneling between the bands of Fig. 1 unlikely. Here, we discuss Zener tunneling more explicitly. In particular, we take issue with an analysis provided by Lenstra and van Haeringen[10–12] who interpret Zener tunneling as a source of energy dissipation. It is true, of course, that tunneling permits electrons to go into higher-lying bands, requiring an energy input from the source of the time-dependent flux. We maintain, however, that this increase in electronic energy is energy storage, not dissipation, and that by a suitable choice of the time dependence of the applied flux, the energy can be recovered. That is the principal point of this note. Matrix elements interconnecting the bands of Fig. 1 also appear in a recent analysis by Imry and Shiren,[13] dealing with the behavior of a ring in the presence of a small *oscillatory* flux. (The oscillatory flux problem is also treated in Ref. 3 with conclusions which are at variance with Ref. 13.) The analysis of Ref. 13 does not, at least in any explicit way, emphasize Zener tunneling. In contrast to Ref. 13, Refs. 10–12 carefully choose

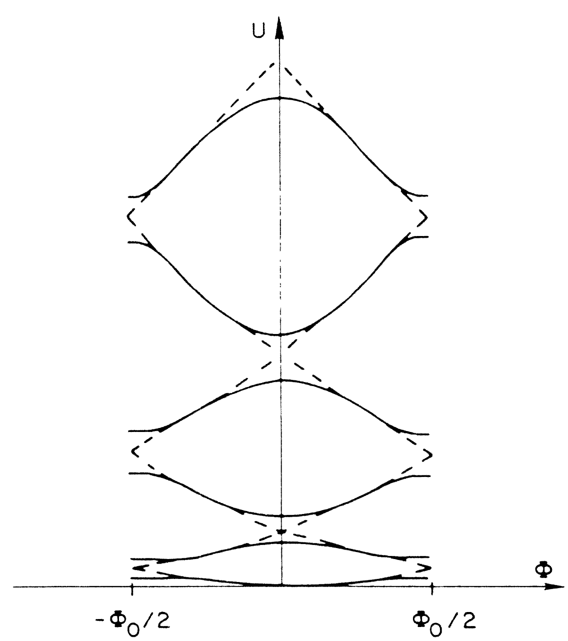

FIG. 1. One-electron energies of the ring as a function of flux. The dashed lines are free electrons without elastic scattering.

their representation so that off-diagonal matrix elements do not occur from the simple time evolution of the wave function which occurs as a result of the motion of k, along the bands of Fig. 1. Our view is essentially identical to that of Refs. 10–12, with a difference only in the final physical interpretation.

References 10–12 were not motivated by a physical ring structure, but introduce periodic boundary conditions only as a mathematical convenience. (Whether it is a justified convenience in the representation of a length of conductor attached, through leads, to a power source, is a question we will not, again, take up here. The distinction between a closed ring and a length of conductor connected to reservoirs was, after all, our key point in Ref. 1.) We have little argument with most of the analysis of Refs. 10–12, which mirrors a number of other classic conductivity papers, cited by Lenstra and van Haeringen, particularly that of Greenwood.[14] It is only Lenstra and van Haeringen's physical interpretation of Zener tunneling as a source of dissipation, which we question. Indeed, Ref. 10 anticipated the results presented in Ref. 1, but without our conclusion that a closed Hamiltonian loop cannot exhibit a resistance.

For our subsequent argument we will need an auxiliary result: Under a given applied field magnitude the probability of Zener tunneling between *two* adjacent bands is independent of the choice of initial band. This result is established, for special cases, in Eqs. (43) and (45) of Ref. 10. It can also be deduced with complete generality from Eq. (30) of Ref. 10. Here, instead, we provide a simple alternative physical argument. Figure 2 shows two adjacent bands, in a field, coupled by Zener tunneling. Readers who dislike an unlimited range of motion may want to picture an additional impenetrable barrier at the far right of the diagram. (In that case, however, let the barrier be inelastic, so that it does not induce quantization and Stark ladders.) Consider the narrow energy range ΔU, shown in Fig. 2. To avoid exclusion principle considerations, assume that the energy range is in the sparsely occupied tail of the Fermi distribution. In thermal equilibrium, as many electrons within this range, must tunnel from the lower band, to the higher, as in the other direction. The electrons in the lower band, moving to the right at x_1, and toward the zone boundary, transmit a current across the gap proportional to the density of states, the velocity, and the transmission probability. As usual, the product of the first two factors is a constant, independent of the details of the situation. Thus the equality of the two opposing tunneling currents, in the steady state, requires an equality of transmission coefficients.

Now consider an electron initially in the lower band, at A, in Fig. 3, and accelerated to the right, in k space. After passing the zone boundary, we can expect a field-dependent Zener tunneling probability, T, for appearance in the upper band, say at D, in Fig. 3. Thus, the probability that we are left in the lower band at B will be $1-T$. Clearly the tunneling event has increased the expectation value of the electronic energy.

Now, after reaching B and D, let us maintain a constant flux through the ring, i.e., cease the acceleration. We furthermore assume that at the value of k corresponding to B and D the tunneling is essentially complete. B and D have differing energies, and if we let the electrons stay at that value of k, the relative phase of the respective contribution to the wave function, from the two bands, can be adjusted as desired by a suitable choice of "inactive" time at that particular value of k. Then reverse the field, driving k toward the left, with the same magnitude of field as was used originally in driving k to the right. Now let us follow, separately, the wave function contributions from B and D, starting with B. This component, of amplitude $\sqrt{1-T}$, will tunnel to the upper band, during the return trip with probability T. Thus, the total probability that we stayed in the lower band on the original forward trip and tunneled into the upper band during the return is $T(1-T)$. Now consider the component at D, of relative amplitude $\sqrt{T}$. On the return trip it will stay in the upper band with probability $(1-T)$, thus leading to a contribution of relative probability $T(1-T)$ in the upper band after the return trip. Therefore, we are left with two contributions arriving at C, one via D and one via B, which are of the same magnitude. By suitable choice of the relative phase of these two contributions, they can be made to cancel. The required phase control is, in turn, obtained by correctly selecting the "inactive" time. We have, therefore, shown that Zener tunneling can be reversed, and the electron brought back to its original band and original energy. This shows that the energy put into the electron in Zener tunneling is energy storage. Indeed, as we let the energy gaps in Fig. 1 become narrower, and Zener tunneling more likely, we approach free-electron behavior. In that limit it is very clear that electron acceleration is energy storage, reflected in an increase in the loop's effective self-inductance. Our disagreement with Ref. 12 arises because we have invoked a viewpoint allowing for quantum mechanical coherence, whereas Ref. 12 uses Zener tunneling probabilities, as if successive tunneling events were incoherent statistical events.

We have, of course, only analyzed the case of a single

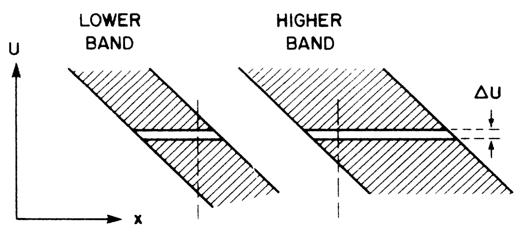

FIG. 2. Two adjacent energy bands, tilted by applied field. ΔU is a small range in which we consider the net electron exchange between the two bands.

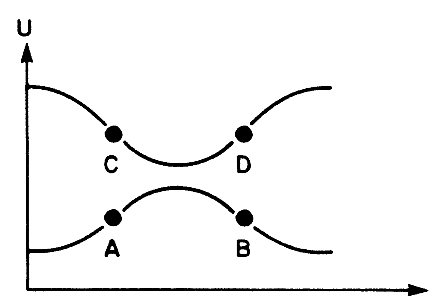

FIG. 3. Adjacent bands, with applied field initially causing k to move to the right, and later back to the left. A, B, C, and D are outside of the range in which Zener tunneling occurs.

electron, tunneling into the next band. We have not addressed the case of a number of successive Zener tunneling events, nor the case where the electron tunneling out of a band may be partially replaced by tunneling from the next lower band. We believe, however, that when we deal with a closed Hamiltonian system, uncoupled to a reservoir, the presumption must be that this is a conservative system. The burden of demonstrating resistive effects, clearly and unambiguously, is upon those who claim such behavior. The fact that energy can be put into such a system, as in the case of a reactance in an electrical circuit, is hardly proof of dissipation. This comment applies not only to Refs. 10–12, but also to other treatments of the one-dimensional closed loop, without leads.[15,16]

The treatment of a closed loop with a real cross section does not yet exist. We would expect, however, that its qualitative behavior would mirror that found in Ref. 1 for the one-dimensional case. In the complete absence of inelastic effects we would have no reason to expect resistive behavior. In the presence of a flux through the loop, increasing linearly with time, we would expect no dc current. As in Ref. 2 it would be expected that such a current would appear in a continuous way, as the probability for inelastic events is allowed to increase.

Our discussion, as in the case of all treatments of Zener tunneling, has ignored self-consistency. The interband matrix elements involved in Zener tunneling, however, represent charge rearrangements within the unit cell. (Here, we use ordinary solid-state terminology, rather than that appropriate to our loops.) The possibility of such charge rearrangements is very likely affected by electrostatic energy considerations, and this raises questions related to the duration of the tunneling event[17] and its comparison to other time scales related to screening and to the dielectric constant.

M. Büttiker has been an indispensable partner, whose contributions throughout this subject have been hard to delineate and separate from my own. I am also indebted to Y. Gefen, who has been concerned with the role of inelastic events in Zener tunneling. Zener tunneling arises in a number of connections, other than the classical solid-state case, and the small loops of Ref. 1. It also arises, for example, in the quantum oscillations in small tunnel junctions discussed by Ben-Jacob and Gefen,[18] as well as in "Bloch oscillations" in Josephson junctions.[19]

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