Reply to "Comment on 'Critical relaxation of the one-dimensional Blume-Emery-Griffiths model' "

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(Received 20 June 1985)

A previous value of the dynamic exponent z, which characterizes the critical relaxation of the one-dimensional Blume-Emery-Griffiths model, was found by Weir and Kosterlitz to be incorrect. The correct value, $z=2$, is found with use of a renormalization-group transformation in a different parameter space. In this parameter space, which is coupled to the three components of the $S=1$ spin, the master equation is invariant under an exact renormalization-group transformation.

Weir and Kosterlitz found an error in Equation (3.10) of my paper. That equation describes the transformation of the σ variable at the tricritical point. They concluded that the parameter space in which the renormalization group transformation was performed is not a complete one. Hence, they speculated that z_c should be 2, as in the Potts model, and that Equation (4.3) should be omitted. I would like to point out that indeed $z_c=2$, that the parameter space is a complete one, and that this result is an exact one and can be obtained from the calculations in the paper.[1]

We pointed out in the paper[1] that although (μ,σ) has four possible values, only three of them correspond to the physical model. The (μ,σ) space is reduced to the physical one by using a projection operator, $(\mu,\frac{1}{2}(1+\mu)\sigma)\equiv(\mu,\bar{\sigma})$. Equation (2.5) (Ref. 1) is already written in the projected space. A short calculation reveals that σ and $\mu\sigma$ have the same scaling behavior. Thus, the rest of the calculation is applied to the (μ,σ) as well as to the projected $(\mu,\bar{\sigma})$ space. The only difference is at point C. Starting with $\Phi=1+h\sigma$ one obtains after the renormalization group transformation $\Phi'=1+h'(1+\mu)\sigma$. That shows that σ is not an eigenvector of the renormalization group transformation. However, in the projected space, $\bar{\Phi}=1+h\bar{\sigma}$ is transformed into $\bar{\Phi}'=1+\lambda h(1+\mu)\bar{\sigma}$, where $\lambda=\frac{1}{2}$. This exact result is consistent with the general behavior of systems with $T_c=0$, in which the magnetization scales as the dimensionality.[2] Equation (3.18) describes the scaling of the right-hand side in both spaces. It shows that $h\sigma$ as well as $h\bar{\sigma}$ have the same scale factor, $\frac{1}{2}$ [the third line of (3.18)], which leads to $(\omega_{\bar{\sigma}})^c=\frac{1}{2}$ [the value 1 in Equation (3.19) is a misprint]. These values of λ^c and ω^c which correspond to the eigenvector $\bar{\sigma}$ lead to $z^c=2$.

This result is an exact one. It agrees with the ϵ expansion. It exhibits the same slowing down as other one-dimensional Ising models. Similar to the dynamic behavior near the critical points of the Blume-Emery-Griffiths model, the tricritical point has another faster time scale for the other degree of freedom.

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[1]Y. Achiam, Phys. Rev. B **31**, 260 (1985).

[2]B. Nienhuis and M. Nauenberg, Phys. Rev. Lett. **35**, 477 (1975); A. N. Berker and M. E. Fisher, Phys. Rev. B **26**, 2507 (1982).

