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Comment on "Critical relaxation of the one-dimensional Blume-Emery-Griffiths model"

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In a recent paper Achiam [Phys. Rev. B 31, 260 (1985)] analyzes the dynamics of the one-dimensional, Blume-Emery-Griffiths model using real-space renormalization-group (RG) techniques. We find that his results for the dynamical critical exponent are incorrect at the tricritical fixed point of the RG procedure.

In a recent article[1] Achiam uses the real-space renormalization-group (RG) method to study the dynamics of the Blume-Emery-Griffiths (BEG) model in one dimension and finds the values $z=1$ and 2 for the dynamical critical exponent. He uses a generalization of the Glauber master equation[2] to analyze the time-dependent properties. The three states of the system are described using two coupled Ising variables, and the dynamics is expressed in terms of flips of the Ising spins. One expects this procedure to give a good description of the dynamics, since Ising dynamics is known to be well treated by this method. Achiam explains why one of the Ising spin flips does not correspond to a physical transition in the original BEG model: "These transitions are between two states having the same energy. Inclusion of these transitions does not effect the long-time dynamics of the model." We, however, are unable to reproduce his results near the fixed point C of his model. In fact, considering the special case of the Potts model, we find different values for the parameters needed to calculate z.

In the RG transformation of the left-hand side of the master equation, one needs to calculate

$$\underline{M}\cdot\underline{\sigma}_{2n+1}\cdot\underline{M}\ ,$$

where we use the same notation as Ref. 1. We find

$$\underline{M}\cdot\underline{\sigma}_{2n+1}\cdot\underline{M}=[A\underline{M}(K')\cdot(\underline{\sigma}_n'+\underline{\sigma}_{n+1}')/2]\delta_{\mu_n'1}\delta_{\mu_{n+1}'1}\ ,$$

rather than the result zero of Ref. 1.

The matrix in the above equation has $\pm x^2y^2w^2$ as its first two diagonal elements, which are certainly not of low order. This seems to imply that new terms are generated in the RG procedure, or in other words that the subspace of operators being considered is not closed under the RG operation. The above equation is intuitively quite reasonable, since it says that only when all the μ variables are 1, that is, the σ spin flip really corresponds to a physical transition, does one recover the simple Ising result.

We further consider the special case of the BEG model which gives the three-state Potts model. We choose $K=3$ J and $\Delta=4$ J or $y=x^{3/4}$ and $w=\sqrt{2}/\sqrt{x}$, again in the notation of Ref. 1. In this system, there is only one fixed point which is tricritical (point C in Ref. 1). We find at this fixed point that $\omega_\sigma^C=1$ and $\omega_\mu^C=\frac{1}{2}$, which appear in the renormalization of the right-hand side of the master equation. This would imply that $\lambda_\mu^C/\omega_\mu^C=4$, which gives $z_\mu^C=2$. The λ_σ^C factor is not defined, since we generate extra terms in the RG transformation of the left-hand side as mentioned above.

We conclude that the result $z=1$ obtained in Ref. 1, at the fixed point C, is suspect, since even if the problem that seems to occur in the definition of λ_σ^C could be overcome, the variable μ relaxes with $z=2$. One could thus never obtain $z=1$, since this will always correspond to a faster relaxation mechanism. In fact, in a recent paper,[3] we speculate that $z=2$ for any one-dimensional spin model, provided that the dynamics can be described in terms of Ising spin flips.

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[1]Y. Achiam, Phys. Rev. B 31, 260 (1985).
[2]R. J. Glauber, J. Math. Phys. 4, 297 (1963).
[3]P. O. Weir and J. M. Kosterlitz, Phys. Rev. B (to be published).

