Enhancement of the spin susceptibility in disordered interacting electrons and the metal-insulator transition

C. Castellani
Istituto di Fisica, Università degli Studi dell'Aquila, I-67100 l'Aquila, Italy
and Dipartimento di Fisica, Università di Roma "La Sapienza," I-00185 Roma, Italy

C. DiCastro
Dipartimento di Fisica, Università di Roma "La Sapienza," I-00185 Roma, Italy

P. A. Lee and M. Ma
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

S. Sorella
Istituto di Fisica, Università degli Studi dell'Aquila, I-67100 l'Aquila, Italy

E. Tabet
Laboratorio di Fisica, Istituto Superiore di Sanità, I-00161 Roma, Italy
(Received 11 July 1985)

The response of a disordered interacting electron gas to a time and spatially varying magnetic field is discussed. Local spin conservation leads to a generalized Ward identity, which together with global spin conservation implies that the dynamic magnetic susceptibility $\chi(\mathbf{q},\Omega)$ must obey a simple diffusive form. The same identity, when combined with the general perturbative structure of $\chi(\mathbf{q},\Omega)$, also relates the renormalization of static susceptibility χ^{st} and the spin diffusion constant D_s to the renormalization of the charge diffusion constant and the Fermi-liquid interaction amplitudes. These relations are shown to be consistent with perturbations to first order in $t\ \{=1/[(2\pi)^2N_0D]\}$ but only after nontrivial cancellations. Thus the Ward identity allows both easy derivation of $\chi(\mathbf{q},\Omega)$ from the renormalized theory and a consistency check on the scaling equations. By using the renormalization-group equations for these parameters, it is shown that there is strong enhancement of χ^{st} and decrease in D_s with lowering temperature. The significance of this with respect to the metal-insulator transition is discussed.

I. INTRODUCTION

The investigation of the metal-insulator transition[1] in disordered interacting electrons has made important progress after the work of Finkel'shtein[2] who showed how to map the problem into an appropriate nonlinear σ model. Once this effective-field theoretical model is obtained one can go through the canonical scheme of the renormalization group and generate the equations which govern the flow of the effective couplings and the scaling behavior of the physical quantities.

While the analogous approach[3–8] for noninteracting electrons succeeded in describing the scaling theories of the localization problem, there are still relevant open problems concerning localization in the presence of electron-electron interaction. A well-behaved metal-insulator transition in $2+\epsilon$ dimensions is obtained only when the effect of spin fluctuations is at least partially suppressed as in the case of spin-flip impurity scattering,[9–11] external magnetic field[10,11] with spin Zeeman splitting, and spin-orbit coupling.[9,12]

A rather unexpected situation is obtained instead[2,10,13,14] when a weak magnetic field is turned on (simply to avoid further complications due to maximally crossed diagrams) and the Zeeman splitting is not considered. This "particle-hole channel only" model has recently been shown[14] to share physical features with the more general case of nonmagnetic impurities. In two dimensions and at one-loop expansion, it was found[2] that while the resistance was scaling to zero, Γ and Γ_2, the effective interaction amplitudes at small and large angles, respectively, and the energy renormalization parameter z were flowing to infinity as the rescaling parameter λ of the renormalization group (RG), was approaching zero.

The original group equations, however, had to be modified.[13,14] The resulting physical picture appears more promising although not yet completely settled. One-loop perturbative calculations once again give a diverging Γ, Γ_2, and z but at a finite value λ_c of the rescaling parameter λ of the renormalization group. Due to the dominance of Γ_2 in the conductivity equation, the corresponding β function changes sign and forbids the metal-insulator transition in $2+\epsilon$ dimensions, the resistance remaining finite at λ_c.

Although the group equations themselves lose their validity near λ_c, the nature of this instability has been interpreted[13] as a tendency to form clusters of aligned localized spins, the size of spin clusters being connected to λ_c. This interpretation was based on the fact that the static spin susceptibility χ^{st} diverges at λ_c, while the spin diffusion constant D_s goes to zero. The strong divergence of χ^{st} is connected to the divergences of the interaction scattering amplitude via the relation $\chi^{st}=N_0(z+\Gamma_2)$. As we shall see, this relation is a generalization of the standard Fermi liquid result in terms of the scattering amplitudes; and it stems from the spin conservation and the diagrammatic structure of χ, just as the relation $z-2\Gamma+\Gamma_2=0$ among the renormalized scattering amplitudes themselves stems from particle conservation.[2,10,15]

The total spin conservation implies a diffusive form for $\chi(\mathbf{q},\Omega)$ in terms of the true static spin susceptibility and of the true spin diffusion constant D_s. The local spin conservation is implemented by a Ward identity. The renormalized form of χ^{st} in terms of the renormalized effective couplings is then generally identified. Such general constraints of the theory have been verified by the one-loop expansion.

An enhancement of spin fluctuations in disordered systems at low temperature has found some experimental evidence.[16] However, whether it is derived from the same disorder-induced enhancement of the Stoner factor here discussed is still an open question.

In the "particle-hole channel only" model the divergence of χ^{st} which follows from the group equations has to be considered only as an indication of a tendency to form local ordered structures. Higher-order terms in the group equations or a self-consistent analysis could in fact stabilize the system, either giving a new fixed point or driving the system toward one of the universality classes associated with the suppression of the effect of Γ_2 (the spin-flip, spin-orbit, or magnetic field cases).

The paper is organized as follows. We shall first discuss the RG equations for the simple case when the Cooper channel is suppressed. The general structure of the spin magnetic susceptibility will be given, together with some general relations (Ward identities) stemming from spin-current conservation. In this way a general relation between χ^{st} and the "coupling constants" z and Γ_2 will be derived, as a natural generalization of the Landau-Fermi liquid theory for translation invariant systems.

The renormalization-group equations will then allow us to obtain the behavior of the susceptibility. In the conclusion possible physical scenarios which could result for the metal-insulator problem will be indicated.

II. MODEL

The system we shall consider is an interacting electron gas in the presence of a random impurity potential $u(\mathbf{r})$ subject to the usual conditions:

$$\overline{u(\mathbf{r})}=0,\overline{u(\mathbf{r})u(\mathbf{r}')}=\frac{1}{2\pi N_0\tau_0}\delta(\mathbf{r}-\mathbf{r}') , \tag{2.1}$$

where N_0 is the (free) single-particle density of states and τ_0 is the scattering time in the Born approximation.[17] The diffusive effects introduced by disorder are described in terms of two types of propagators: the "particle-hole channel" diffusion propagator $L_d^{(0)}(\mathbf{q},\Omega)$ represented by the scattering processes shown in Fig. 1:

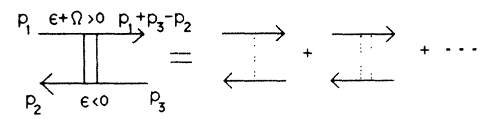

FIG. 1. "Particle-hole channel" diffusive propagator $L_d^{(0)}(\mathbf{q},\Omega)$. This is obtained by treating the impurity scattering with the "ladder approximation."

$$L_d^{(0)}(\mathbf{q},\Omega)=\frac{1}{2\pi N_0\tau_0}\frac{1}{-i\Omega+D_0q^2};\quad \mathbf{q}=\mathbf{p}_1-\mathbf{p}_2 , \tag{2.2}$$

and the "particle-particle channel" propagator shown in Fig. 2:

$$L_c^{(0)}(\mathbf{q},\Omega)=\frac{1}{2\pi N_0\tau_0}\frac{1}{-i\Omega+D_0q^2};\quad \mathbf{q}=\mathbf{p}_1+\mathbf{p}_3 . \tag{2.3}$$

$D_0=v_F^2\tau_0/d$ is the bare diffusion constant. In Figs. 1 and 2 continuous lines represent the fermion Green's functions with energy on opposite sides of the Fermi surface $[(\epsilon+\Omega)\epsilon<0]$, which from now on will be indicated with a "+" and a "−," respectively. The dashed lines indicate the random potential correlation given in Eq. (2.1).

In the following ("particle-hole channel only" model) we assume that the time-reversal invariance is broken by the presence of an external magnetic field. However, the spin Zeeman splitting will not be considered. Under these conditions a masslike term $\tau_H^{-1}\sim eHD/c$ appears[18] in the denominator of Eq. (2.3) so that $L_c^{(0)}(\mathbf{q},\Omega)$ stays finite in the limit $q,\Omega\to 0$. It is therefore negligible when compared with $L_d^{(0)}$.

Interaction among the electrons is initially introduced by means of a spin-independent two-body potential $v(\mathbf{q})$. From perturbation theory[1] we learn that the combined effects of disorder and interaction are relevant only when the energy of the electrons is confined in a shell $|\epsilon|<\tau_0^{-1}$ around the Fermi surface. This is so because the leading corrections (which are logarithmic in two dimensions) originate from the presence of the diffusive propagators.

All the remaining many-body effects are included by substituting the initial bare potential $v(\mathbf{q})$ with the standard Landau-Fermi liquid amplitudes. Three energy-momentum regions appear to be relevant for the electron-electron interaction (Fig. 3) in the presence of disorder.

We shall call $\tilde{\Gamma}_1^{(0)}$ the interaction amplitude for values of the energies and momenta such that $\epsilon_1\sim\epsilon_3$ and $\mathbf{p}_1\sim\mathbf{p}_3$;

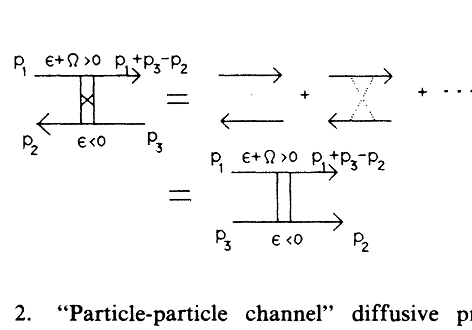

FIG. 2. "Particle-particle channel" diffusive propagator $L_c^{(0)}(\mathbf{q},\Omega)$. This includes the set of "maximally crossed" graphs.

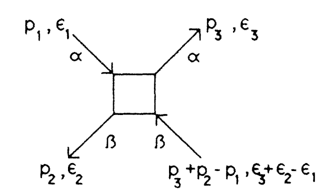

FIG. 3. Interaction amplitudes; α,β are the spin indices.

$\tilde\Gamma_2^{(0)}$ for $\epsilon_1 \sim \epsilon_2$ and $\mathbf{p}_1 \sim \mathbf{p}_2$; and $\tilde\Gamma_3^{(0)}$ for $\epsilon_2 \sim -\epsilon_3$ and $\mathbf{p}_2 \sim -\mathbf{p}_3$. $\tilde\Gamma_1^{(0)}$ corresponds to the small-angle interaction scattering, $\tilde\Gamma_2^{(0)}$ is obtained by averaging over events where the transferred momenta are $\lesssim 2p_F$, and $\tilde\Gamma_3^{(0)}$ is related to Cooper fluctuations. Since in this paper we shall not deal explicitly with the presence of L_c, $\tilde\Gamma_3^{(0)}$ will be neglected. However, this approach should be generalizable to those situations.

When long-range forces are present, we shall call $\tilde\Gamma_0^{(0)}$ the statically screened part of the Coulomb potential and $\tilde\Gamma^{(0)} = \tilde\Gamma_0^{(0)} + \tilde\Gamma_1^{(0)}$, the total small-angle interaction amplitude. Moreover, for reasons which will become apparent later, it is convenient to introduce the familiar singlet and triplet scattering amplitudes as defined in the Fermi liquid theory:

$$\tilde\Gamma_s^{(0)} = \tilde\Gamma^{(0)} - \tfrac{1}{2}\tilde\Gamma_2^{(0)}; \quad \tilde\Gamma_t^{(0)} = \frac{-\tilde\Gamma_2^{(0)}}{2} . \tag{2.4}$$

III. DENSITY-DENSITY CORRELATION FUNCTION AND THE GENERAL STRUCTURE OF THE THEORY

In this section we review earlier work on the density-density correlation function.[2,10,15] This review prepares one for the calculation of the spin-spin correlation function which is the main subject of this paper.

In the model we are considering, corrections[1] to the normal Landau-Fermi liquid derive from integrals over the momentum appearing in the diffusive pole of the particle-hole propagators [Eq. (2.2)]. The expansion parameter is $t_0 = 1/(2\pi)^2 N_0 D_0$, which is a small quantity if the disorder scattering is assumed to be weak.

Already at zeroth order in t_0, dynamical corrections due to the disorder are obtained for the interaction amplitudes by noticing that if $L_d^{(0)}$ is inserted between two $\tilde\Gamma_2^{(0)}$ as shown in Fig. 4, no additional integration is required on the internal momenta which appear in the diffusion propagators.

Denoting the dynamically corrected amplitude corresponding to $\tilde\Gamma_2$ by $\tilde U_2$, we then have

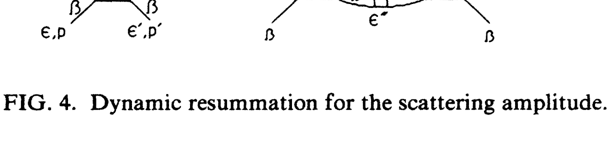

FIG. 4. Dynamic resummation for the scattering amplitude.

$$\tilde U_2^{(0)}(\mathbf{q},\Omega) = \tilde\Gamma_2^{(0)} - \tilde\Gamma_2^{(0)}(K_{00}^{(0)})^{+-}(\mathbf{q},\Omega)\tilde U_2^{(0)}(\mathbf{q},\Omega) = \frac{\tilde\Gamma_2^{(0)}(-i\Omega + D_0 q^2)}{-i\Omega(1+N_0\tilde\Gamma_2^{(0)}) + D_0 q^2} , \tag{3.1}$$

where we have used Eq. (2.2) to obtain the zero-order noninteracting density-density correlation bubble $(K_{00}^{(0)})^{+-} = -i\Omega N_0/(-i\Omega + D_0 q^2)$ defined as the part of $K_{00}^{(0)}$ which is associated with $[\epsilon''(\epsilon''+\Omega)<0]$ by integrating out of the four Green's functions which appear in the repeated structure of the infinite resummation. Analogous steps lead[2,10] to the dynamical amplitudes $\tilde U_0^{(0)}(\mathbf{q},\Omega)$ and $\tilde U_1^{(0)}(\mathbf{q},\Omega)$.

When evaluating physical quantities at a given order in t_0, the infinite order in the Γ's is obtained just by using the corresponding dynamical amplitudes instead of the static ones. No weak-coupling assumption is therefore made on the interaction.

Limiting ourselves to first order t_0 and any order in the Γ's, it can be shown[10] that a dressed diffusion propagator $L_d(\mathbf{q},\Omega)$ appears as a consequence of the interaction

$$L_d(\mathbf{q},\Omega) = \frac{\zeta^2}{-iz\Omega + Dq^2} , \tag{3.2}$$

where we have dropped the factor $(2\pi N_0 \tau_0)^{-1}$ for convenience. ζ plays the role of a wave-function renormalization and z allows for an independent energy renormalization. D is the dressed diffusion constant at the same order. Cross correction terms lead also to dressed scattering amplitudes $\tilde\Gamma$ and $\tilde\Gamma_2$. By using Eq. (3.2) instead of Eq. (2.2), the singlet and triplet dynamical amplitudes now read

$$\tilde U_{s,t}(\mathbf{q},\Omega) = \frac{\tilde\Gamma_{s,t}(-iz\Omega + Dq^2)}{-i\Omega(z - 2N_0\zeta^2\tilde\Gamma_{s,t}) + Dq^2} , \tag{3.3}$$

the various dynamical scattering amplitudes being rescaled differently in frequency. This and the following equations similar to Eq. (3.3), which, strictly speaking, are integral equations, will be considered in the spirit of the scaling analysis as algebraic equations.

$\tilde\Gamma$, $\tilde\Gamma_2$, ζ, z, and D, which are logarithmically divergent in two dimensions, are expressed in terms of Ω at zero temperature, in terms of T itself at finite temperature, and of the rescaling parameter λ when the corresponding group equations are considered. The identification of the renormalization parameters in terms of physical quantities is performed by considering the density-density response function K_{00}.

The general structure of $K_{00}(\mathbf{q},\Omega)$ can be made explicit by dividing it into the static and the dynamic part and repeating the infinite summation as for the interaction amplitudes,[10]

$$K_{00}(\mathbf{q},\Omega) = K_{00}^{\rm st} + \frac{-i\Omega N_0 \zeta^2 \Lambda_0^2}{-iz_1\Omega + Dq^2} , \tag{3.4}$$

where z_1 is given by

$$z_1 = z - 2N_0\zeta^2\tilde\Gamma_s^{\rm SR}, \quad \tilde\Gamma_s^{\rm SR} = \tilde\Gamma_1 - \frac{\tilde\Gamma_2}{2} = \tilde\Gamma_s - \tilde\Gamma_0 . \tag{3.5}$$

Only the short-range part of the singlet amplitude appears in Eq. (3.4) since K_{00} is irreducible for cutting a Coulomb line. Λ_0 is the vertex which when multiplied by the advanced and retarded part of the density-density response function $(K_{00})^{+-}$,

$$(K_{00})^{+-}(\mathbf{q},\Omega)=\frac{-i\Omega N_0\zeta^2\Lambda_0}{-iz_1\Omega+Dq^2}\ , \tag{3.6}$$

gives the total dynamic part of K_{00} by including the final $++$ and $--$ contributions. It should be noted that all renormalization constants are absorbed in L_d, Λ_0, and the Γ's, leaving all the single-particle Green's functions unrenormalized in Fig. 5. This implies that in evaluating Fig. 5, the Green's function integrals are over noninteracting Green's functions and are therefore trivial.

Λ_0 appears also in the static screened Coulomb interaction

$$\tilde{\Gamma}_0=\lim_{q\to 0}\frac{v_c(q)\Lambda_0^2}{1-2v_c(q)K_{00}^{\text{st}}}=\frac{-\Lambda_0^2}{2K_{00}^{\text{st}}}\ , \tag{3.7}$$

K_{00}^{st} being the static part of the density-density response function coincides with $-\partial n/\partial\mu$. $\partial n/\partial\mu$ is the thermodynamic density of states and remains finite.[19,20,2] The resummation for K_{00}^{st} is completed by the insertion of the bare $\tilde{\Gamma}_s^{\text{SR}}$:

$$K_{00}^{\text{st}}=-\frac{\partial n}{\partial\mu}=-N_0[1-2N_0(\tilde{\Gamma}_s^{(0)})^{\text{SR}}]\ . \tag{3.8}$$

The total particle conservation condition,

$$K_{00}(q=0,\Omega)=0\ , \tag{3.9}$$

when imposed on Eq. (3.4) determines $\zeta^2\Lambda_0^2$ in terms of K_{00}^{st} and z_1 as

$$\zeta^2\Lambda_0^2=-z_1\frac{K_{00}^{\text{st}}}{N_0}\ , \tag{3.10}$$

K_{00} assumes then the standard diffusive form

$$K_{00}(\mathbf{q},\Omega)=\frac{\partial n}{\partial\mu}\frac{D'q^2}{-i\Omega+D'q^2},\quad D'=\frac{D}{z_1}\ . \tag{3.11}$$

D' is the diffusion constant related to the conductivity by the Einstein relation $\sigma=(\partial n/\partial\mu)D'$.

z_1 is the rescaling parameter of the frequency mode associated to the density fluctuations. As it was also shown in perturbation theory,[2,10] it remains finite just as $\partial n/\partial\mu$ does and equal to its bare value given by equation

$$z_1=\frac{-K_{00}^{\text{st}}}{N_0}=\frac{1}{N_0}\frac{\partial n}{\partial\mu}\ . \tag{3.12}$$

In addition to z and z_1, there are the frequency rescaling parameters which control the dynamical behavior of the singlet (long-range) and triplet scattering amplitudes according to Eq. (3.3):

$$z_s=z-2N_0\zeta^2\tilde{\Gamma}_s=z_1-2N_0\zeta^2\tilde{\Gamma}_0,\quad z_2=z+N_0\zeta^2\tilde{\Gamma}_2\ . \tag{3.13}$$

As shown in the next section, z_2 is associated to the spin fluctuation. Due to the expression (3.7) for $\tilde{\Gamma}_0$ and to Eq. (3.10) for $\zeta\Lambda_0$, z_s instead is identically zero leading to the general relation

$$z=2N_0\zeta^2\tilde{\Gamma}_s\ , \tag{3.14}$$

which was first derived in Ref. 2.

As was carried out in Ref. 15, the identification of the theory is completed by determining ζ in terms of the true single-particle density of states $N(\Omega)$:

$$N(\Omega)=-\frac{1}{\pi}\int d^dp\ \text{Im}G(\mathbf{p},\Omega),\quad \Omega>0\ , \tag{3.15}$$

where $G(\mathbf{p},\Omega)$ is now the full interacting renormalized Green's function. The requirement of local gauge invariance implies a Ward identity for the electromagnetic vertices of the theory, which permits one to relate[19] the single-particle density of states N to $(K_{00})^{+-}$:

$$N(\Omega)=(K_{00})^{+-}(q=0,\Omega),\quad \Omega\to 0\ . \tag{3.16}$$

Expression (3.6), for $(K_{00})^{+-}$, then implies

$$N(\Omega)=\frac{N_0\zeta^2\Lambda_0}{z_1},\quad \Omega\to 0\ . \tag{3.17}$$

The condition (3.10) on Λ_0 following from the global particle conservation together with Eq. (3.17) determines ζ to be

$$\zeta=\frac{N}{N_0}\ , \tag{3.18}$$

as it has been verified by perturbative analysis.[10] Actually it was shown that the renormalized interaction amplitude Γ's (to be distinguished from $\tilde{\Gamma}$'s), which appeared in Ref. 2, already included the "wave-function renormalization" parameter ζ^2 from the beginning. For the rest of this paper, renormalized amplitudes

$$\Gamma_i=\tilde{\Gamma}_i\zeta^2N_0 \tag{3.19}$$

will be used where appropriate. When expressed in terms of these renormalized couplings, the group equations do not contain the single-particle density of states explicitly.[10]

IV. RENORMALIZED SPIN SUSCEPTIBILITY

We now proceed to identify the role of the parameter z_2. The Fourier transform $\chi^{ii}(\mathbf{q},\Omega)$ of the response function to an external magnetic field along a given direction $i=1,\ldots,d$ for $\Omega>0$, is related to the time-ordered function

$$\chi^{ii}(\mathbf{q},\Omega)=\mathcal{F}\left\{\frac{i}{2}\langle T_t[S^i(\mathbf{r},t)S^i(\mathbf{r}',t')]\rangle\right\}\ , \tag{4.1}$$

where the i component of the spin density is given in terms of the Pauli matrices σ^i as

$$S^i(\mathbf{r},t)=\psi_\alpha^\dagger(\mathbf{r},t)\sigma^i_{\alpha\beta}\psi_\beta(\mathbf{r},t)\ , \tag{4.2}$$

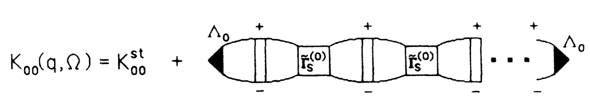

FIG. 5. General structure of K_{00}. $\tilde{\Gamma}_s^{(0)}$ denotes $(\tilde{\Gamma}_s^{(0)})^{\text{SR}}$.

with summations over repeated indices α,β understood.

According to its perturbative structure (see the Appendix), $\mathcal{X}(\mathbf{q},\Omega)$ can be divided into its static part $\mathcal{X}^{\rm st}$ and into its dynamical one which has the general representation shown in Fig. 6. $\mathcal{X}$ must be irreducible for cutting a Γ line since otherwise the spin indices at the end vertices are independent and sum to zero. Therefore only $\tilde{\Gamma}_2$ (and not $\tilde{\Gamma}_s$ as for K_{00}) enters in its skeleton resummation. When all the terms depicted in Fig. 6 are summarized up, taking into account Eq. (3.2) for the dressed L_d, one obtains

$$\frac{i\Omega N_0\zeta^2\Lambda_s^2}{-i\Omega z_2+Dq^2} . \tag{4.3}$$

The vertex Λ_s plays the same role as the vertex Λ_0 for the polarization bubble K_{00} and completes the $+-$ part at both ends to include the $++$ and $--$ contributions so that

$$\mathcal{X}(\mathbf{q},\Omega)=\mathcal{X}^{\rm st}+\frac{i\Omega N_0\zeta^2\Lambda_s^2}{-iz_2\Omega+Dq^2} . \tag{4.4}$$

We introduce the quantity

$$\mathcal{X}^{+-}(\mathbf{q},\Omega)=\frac{i\Omega N_0\zeta^2\Lambda_s}{-iz_2\Omega+Dq^2} , \tag{4.5}$$

which is simply the part of the $\mathcal{X}(\mathbf{q},\Omega)$ starting with a $+$ line and a $-$ line. Just as in the case of K_{00}, Eqs. (4.4) and (4.5) are simplified versions of true integral equations.

We now proceed as we did in Eq. (3.9) and take advantage of the total spin conservation. We impose on Eq. (4.4) that

$$\mathcal{X}(q=0,\Omega)=0 . \tag{4.6}$$

Equation (4.6) determines $\zeta\Lambda_s$ in terms of z_2 and $\mathcal{X}^{\rm st}$,

$$\zeta^2\Lambda_s^2=\frac{z_2}{N_0}\mathcal{X}^{\rm st} . \tag{4.7}$$

The full $\mathcal{X}(\mathbf{q},\Omega)$ can now be written in the very simple form

$$\mathcal{X}(\mathbf{q},\Omega)=\mathcal{X}^{\rm st}\frac{D_sq^2}{-i\Omega+D_sq^2},\quad D_s=\frac{D}{z_2} , \tag{4.8}$$

where D_s, in strict analogy with Eq. (3.11), plays the role of the dressed spin-diffusion constant.

In order to complete the identification of our renormalized theory, we have to relate $\mathcal{X}^{\rm st}$ to z_2 by means of a Ward identity. In addition to the spin density $S^i(\mathbf{r},t)$ given by Eq. (4.2), we introduce a "spin-current" operator

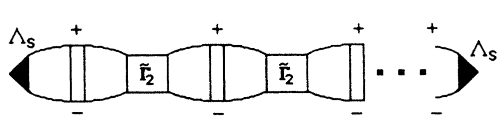

FIG. 6. General perturbative structure of the dynamical part of $\mathcal{X}(\mathbf{q},\Omega)$.

$$\mathbf{I}^i(\mathbf{r},t)=-i/2\{\psi_\alpha^\dagger(\mathbf{r},t)\sigma^i_{\alpha\beta}\nabla\psi_\beta(\mathbf{r},t) -[\nabla\psi_\alpha^\dagger(\mathbf{r},t)]\sigma^i_{\alpha\beta}\psi_\beta(\mathbf{r},t)\} . \tag{4.9}$$

In the case of nonmagnetic impurities with spin-independent electron interaction, the spin is a conserved quantity. In the present model we have suppressed the Cooper channel contribution by hand. If this is due to a small magnetic field, we assume that its presence does not introduce other effects.

The equation of motion leads in this case to the continuity equation

$$\frac{\partial S^i}{\partial t}+\nabla\cdot\mathbf{I}^i(\mathbf{r},t)=0,\quad i=1,2,\ldots,d . \tag{4.10}$$

Quite generally we define the vertex

$$\Lambda^i_{\mu,\alpha\beta}(x;x',x'')=\langle T_tI^i_\mu(x)\psi_\alpha(x')\psi^\dagger_\beta(x'')\rangle , \tag{4.11}$$

where for simplicity we have used the notation $I^i_\mu\equiv(S^i,\mathbf{I}^i)$ and $x_\mu\equiv(t,\mathbf{r})$, with $\mu=0,1,\ldots,d$.

The time-space divergence $\partial/\partial x_\mu$ of $\Lambda^i_{\mu,\alpha\beta}$, by virtue of the continuity equation (4.10), acts on the right-hand side of Eq. (4.11) only via the time derivative of the time-ordering operator. Therefore only $I_0^i=S^i$ enters in its evaluation. We then have

$$\frac{\partial}{\partial x_\mu}\Lambda^i_{\mu,\alpha\beta}(x;x',x'')=i\delta(x-x'')G_{\alpha\gamma}(x',x)\sigma^i_{\gamma\beta} -i\delta(x-x')\sigma^i_{\alpha\gamma}G_{\gamma\beta}(x,x'') . \tag{4.12}$$

Upon impurity averaging, we have translational invariance, and we can Fourier transform Eq. (4.12) to read

$$q_\mu\Lambda^i_{\mu,\alpha\beta}(p+q,p)=\sigma^i_{\alpha\gamma}G_{\gamma\beta}(p+q)-G_{\alpha\gamma}(p)\sigma^i_{\gamma\beta} , \tag{4.12$'$}$$

where

$$q_\mu\Lambda^i_\mu=\Omega\Lambda^i_0-\sum_{j=1}^{d}q_j\Lambda^i_j .$$

If we send the external momentum $\mathbf{q}$ to zero, we are left with the Ward identity for the vertex Λ_0^i

$$\Omega\Lambda^i_{0,\alpha\beta}(\mathbf{p},\epsilon+\Omega;\mathbf{p},\epsilon)=\sigma^i_{\alpha\gamma}G_{\gamma\beta}(\mathbf{p},\epsilon+\Omega)-G_{\alpha\gamma}(\mathbf{p},\epsilon)\sigma^i_{\gamma\beta} . \tag{4.13}$$

The dynamic susceptibility is related to Λ_0^i by

$$\mathcal{X}(\mathbf{q},\Omega)=\frac{-i}{2}\int\frac{d\epsilon}{2\pi}\int\frac{d\mathbf{p}}{(2\pi)^d}\sigma^i_{\alpha\beta}\Lambda^i_{0\beta\alpha}(\epsilon+\Omega,\mathbf{p}+\mathbf{q};\epsilon,\mathbf{p}) . \tag{4.14}$$

When we consider only the part $\mathcal{X}^{+-}$ of $\mathcal{X}(\mathbf{q},\Omega)$, we have

$$\mathcal{X}^{+-}(\mathbf{q},\Omega)=\frac{-i}{2}\int\frac{d\epsilon}{2\pi}\int\frac{d\mathbf{p}}{(2\pi)^d}\sigma^i_{\alpha\beta}\Lambda^i_{0\beta\alpha}\Theta(-\epsilon(\epsilon+\Omega)) . \tag{4.15}$$

As $\Omega\to0$, the integration over ϵ only gives a factor Ω. Considering the fact that $G_{\alpha\beta}=G\delta_{\alpha\beta}$ and the Ward identity (4.13), we have

$$\lim_{\Omega\to 0}\lim_{q\to 0}\chi^{+-}(\mathbf{q},\Omega)=-\frac{i}{2(2\pi)}\int\frac{d^dp}{(2\pi)^d}\Omega\sigma^i_{\alpha\beta}\Lambda^i_{0\beta\alpha}$$
$$=-\frac{i}{2\pi}\int\frac{d^dp}{(2\pi)^d}[G(\mathbf{p},0^+)-G(\mathbf{p},0^-)]. \quad (4.16)$$

Since the term in the square bracket coincides with $2i\,\mathrm{Im}G(p,0^+)$, Eq. (4.16) together with the definition (3.15) of the single-particle density of states $N(\Omega)$ and the expression (4.5) for χ^{+-} leads to

$$N(\Omega)=-\chi^{+-}(q=0,\Omega)=\frac{N_0\zeta^2\Lambda_s}{z_2},\quad \Omega\to 0 . \quad (4.17)$$

By means of Eqs. (4.17) and (3.18) for ζ, Λ_s and ζ can be eliminated from Eq. (4.7) to provide the required expression for $\chi^{\rm st}$ in terms of z_2:

$$\chi^{\rm st}=\frac{N_0\zeta^2\Lambda_s^2}{z_2}=N_0z_2\equiv N_0(z+\Gamma_2) , \quad (4.18)$$

which was independently derived in Refs. 13 and 14. As mentioned before, $\Gamma_2=\tilde{\Gamma}_2\zeta^2N_0$.

It is important to stress that Eq. (4.18) has been derived here on a quite general basis exploiting the skeleton structure of the theory and the symmetry contraints coming both from local and global particle and spin conservation. The simultaneous use of these conditions [Eqs. (3.9) and (3.16) for K_{00} and Eqs. (4.6) and (4.16) for $\chi(\mathbf{q},\Omega)$] permits us to identify all the renormalization parameters of the theory. Usually only the global conditions (3.9) and (4.6) have been considered in the literature.

V. GROUP EQUATIONS

The group equations were derived at first order in t by means of the Wilson procedure. Fast degrees of freedom are consequently integrated out up to a lower energy and momentum cutoff, $\lambda\Lambda^2\equiv\lambda(\tau_0D_0)^{-1}$ in the following region:[21]

$$0<\frac{z\omega}{D\Lambda^2}<\lambda,\quad \lambda<\frac{q^2}{\Lambda^2}<1 ,$$
$$\lambda<\frac{z\omega}{D\Lambda^2}<1,\quad 0<\frac{q^2}{\Lambda^2}<1 . \quad (5.1)$$

The parameter $\lambda\to 0$ while iterating the transformation.

A set of RG equations was originally derived by Finkel'shtein[2] using a field theoretical formulation. Since then, it was "rederived" via a diagrammatic perturbation theory approach.[10] Recently it was, however, realized that the contributions listed in Ref. 10 for diagrams 10(b), 10(c), 11(b), and 11(c) are overly small by a factor of 2. The RG equations in Refs. 2 and 10 are thus incorrect. Taking into account the correct contributions to those diagrams and defining the scaling variable $\xi=-\ln\lambda$, one obtains the following flow equations of t, Γ, and Γ_2:

$$\frac{dt}{d\xi}=t^2\left[4-3\frac{z+\Gamma_2}{\Gamma_2}\ln\left[\frac{z+\Gamma_2}{z}\right]\right]$$
$$\frac{d\Gamma_2}{d\xi}=t\left[\Gamma+2\frac{\Gamma_2^2}{z}\right],\quad \frac{d\Gamma_1}{d\xi}=t\left[\Gamma_2+\frac{\Gamma_2^2}{z}\right] , \quad (5.2)$$

for $d=2$. Since Γ_2 and Γ, instead of $\tilde{\Gamma}_2$ and $\tilde{\Gamma}$ are used, the RG equations do not contain the density of states renormalization. The scaling equation for z is readily obtained from Eq. (5.2) using Eq. (3.14).

Defining $w=\Gamma_2/\Gamma$, $\gamma_2=\Gamma_2/z$; from the above equations and Eq. (3.14), one gets

$$\frac{dw}{d\xi}=t,\quad \frac{d\gamma_2}{d\xi}=\frac{t}{2}(\gamma_2+1)^2 . \quad (5.3)$$

Both w and γ_2 increase as ξ increases as long as t is finite. On the other hand from Eq. (3.14) we have

$$\frac{w-2}{2}=-\frac{1}{1+\gamma_2} .$$

w has to be upperly limited by 2, where γ_2 diverges, as the condition $z>0$ has to be satisfied in order to ensure the stability of the Fermi liquid. It is easy to see now that γ_2 diverges as a finite ξ_c, scaling toward strong coupling:

$$\gamma_2\sim|\xi-\xi_c|^{-1} ,$$

while, at ξ_c, t reaches a finite value t_c. We then have

$$\Gamma,\Gamma_2\sim|\xi-\xi_c|^{-4},\quad z\sim|\xi-\xi_c|^{-3} . \quad (5.4)$$

This instability is present also in $2+\epsilon$ dimensions, except for a region given approximately by

$$\frac{\epsilon}{2t_0}\left[2-\frac{\Gamma_2^{(0)}}{\Gamma^{(0)}}\right]>1$$

in the initial values of $\Gamma_2^{(0)}$, $\Gamma^{(0)}$, and t_0. Moreover this behavior comes out to be independent of the nature (long-ranged or short-ranged) of the interactions and therefore it is also expected to apply to Hubbard-type models. The existence of a finite value λ_c of the scaling parameter defines in a natural way a length scale of the theory by the relation

$$q_c^2=\lambda_c\Lambda^2=\lambda_c(\tau_0D_0)^{-1} .$$

In order to obtain properties of the physical system at temperature T, one should integrate out all the fluctuating modes which are not cut off by the temperature. However, the different propagators in the theory have different cutoffs given by zT, z_2T, etc. Thus in principle we should integrate the RG equations to a cutoff λ equal to the smallest of these cutoffs, and from (5.4) this means setting $\tau_0^{-1}\lambda=zT^{13}$. However, this procedure cannot be carried through consistently because we are already outside the region of validity of the scaling equations when the larger cutoffs are encountered (see further discussion below). Nevertheless, carrying out this scaling, we find that (5.4) now implies

$$|\lambda-\lambda_c|\sim T^{1/3} , \quad (5.5)$$

and so $T=0$ when $\lambda=\lambda_c$. Hence the instability at a finite λ_c does not translate into a finite-temperature transition.

It is useful to spell out the difference between the previous solution and the one obtained using the old (incorrect) RG equations. In both cases, as the cutoff λ is reduced, the resistance t decreases while z, Γ, and Γ_2 diverge. With the previous equations,[2,10] z, Γ, and Γ_2 diverge logarithmically as $\lambda \to 0$. This leads to the conclusion that $t \to 0$ as $T \to 0$, even in the presence of disorder. With the new equations, z, Γ, and Γ_2 diverge at a finite λ_c with a power law, and their divergences preempt the tendency of t to decrease all the way to zero. The catastrophe of a dirty perfect conductor is thus avoided. However, it is not possible with either set of equations to obtain formally the expected metal-insulator transition (but see below).

If we now consider the spin susceptibility, according to Eq. (4.18), its group equation is completely determined by the previous flow equations:

$$\frac{d\frac{\chi^{\rm st}}{\chi^0}}{d\xi} = \frac{d}{d\xi}(z+\Gamma_2) = 2\frac{d\Gamma}{d\xi} = \frac{2t\Gamma_2}{z}(z+\Gamma_2) \; . \qquad (5.6)$$

This checks with what can be calculated directly from perturbation theory (see the Appendix) leading to a verification of Eq. (4.18).

It is clear that χ has the same divergence as Γ_2

$$\chi^{\rm st} \sim |\xi - \xi_c|^{-4} \sim T^{-4/3} \; , \qquad (5.7)$$

and

$$D_s = \frac{D}{z+\Gamma_2} \sim T^{4/3} \; , \qquad (5.8)$$

since D remains finite.

As discussed in Ref. 13, Eqs. (5.5)–(5.8) should be taken only as indicative since the RG equations have been derived under the assumption of weak coupling, a condition no longer satisfied near λ_c. In addition, because z_2 diverges with a faster power law than z, the equations cease to be valid once $\lambda < \lambda_s = z_2(\lambda_s)T$. Nevertheless they do allow the following qualitative physical picture. The enhancement of the spin susceptibility as λ approaches a finite value λ_c together with D_s going to zero can be taken as an indication of the formation of localized spin clusters.[13,14] No symmetry breaking is present since we cannot complete the renormalization-group iteration $\lambda \to 0$. It is natural to identify the length $L_c = q_c^{-1}$ previously introduced as the mean dimension of these clusters. The behavior in $T^{-4/3}$ of $\chi^{\rm st}$ even if approximate should suggest that those clusters should at least initially behave as localized magnetic moments with a residual interaction.

The system should then be stabilized by the inclusion of these self-generated magnetic moments and possibly be driven toward a metal-insulator transition via suppression of the effects of the triplet interaction amplitude. Consistent with the starting point, the same phenomenon should suppress the contributions due to Cooper channel if included.

ACKNOWLEDGMENTS

P.A.L. and M.M. acknowledge the support by National Science Foundation (NSF) Grant No. DMR-82-17956.

APPENDIX

In this appendix we calculate to first order in t_0 the susceptibility $\chi(\mathbf{q},\Omega)$. The purpose is to explicitly show that Eqs. (4.8) and (4.18) are indeed satisfied, at least perturbatively. At the same time it will be seen that they are the result of nontrivial cancellations, thus illustrating the power of having these equations as constraints imposed by conservation laws on the scaling theory. In fact, it is the apparent violation of Eq. (4.8) that led us to the modification of the RG equations of Ref. 2 into those of Refs. 13 and 14.

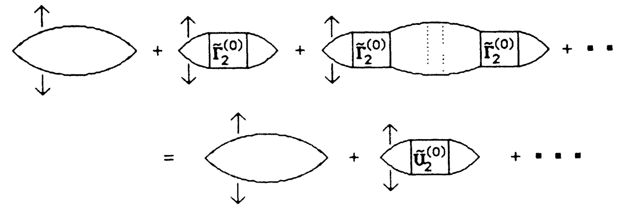

FIG. 7. Diagrams contributing to $\chi^{(0)}(\mathbf{q},\Omega)$, the susceptibility to zeroth order in t_0. Insertions of $\tilde{\Gamma}_2^{(0)}$'s are displayed explicitly. Insertions of $L_d^{(0)}$'s, where possible, are implicit and understood.

In perturbation theory it is more convenient to calculate the transverse susceptibility, which involves a spin flip at each vertex. We consider first the susceptibility to zeroth order in $t_0\xi$, $\chi^{(0)}(\mathbf{q},\Omega)$. In addition to the noninteracting graph, there are contributions from inserting a ladder summation of $\tilde{\Gamma}_2$'s (see Fig. 7), resulting in

$$\chi^{(0)}(\mathbf{q},\Omega) = N_0(1+N_0\tilde{\Gamma}_2^{(0)}) + \frac{i\Omega(1+N_0\tilde{\Gamma}_2^{(0)})^2 N_0}{-i\Omega(1+N_0\tilde{\Gamma}_2^{(0)}) + D_0 q^2} \; . \qquad (A1)$$

The reason why only $\tilde{\Gamma}_2$ appears is explained in Sec. IV. (A1) is written in a form to explicitly display the separation of $\chi^{(0)}(\mathbf{q},\Omega)$ into its static part $(\chi^{\rm st})^0$ (first two graphs of Fig. 7) and its dynamical part. Note that $\chi^0(\mathbf{q},\Omega)$ satisfies $\chi^0(0,\Omega)=0$, as required by spin conservation. Also note that (4.8) and (4.18) are satisfied with $D_s^{(0)} = D_0/(1+N_0\tilde{\Gamma}_2^{(0)})$.

To first order in $t_0\xi$, the same separation is retained by writing $\chi(\mathbf{q},\Omega)$ as

$$\chi(\mathbf{q},\Omega) = \chi^{\rm st} + \frac{i\Omega N_0 \Lambda_s^2 \zeta^2}{-i\Omega(z+N_0\zeta^2\tilde{\Gamma}_2) + Dq^2} \; . \qquad (A2)$$

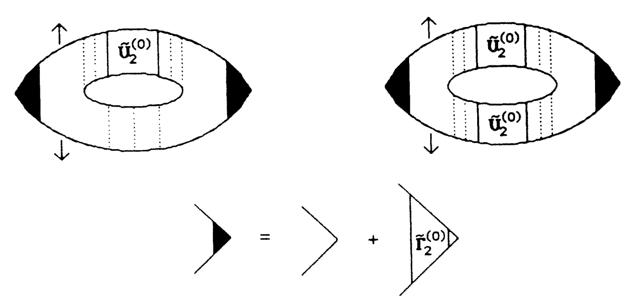

FIG. 8. First order in t contributions to $\chi^{\rm st}$.

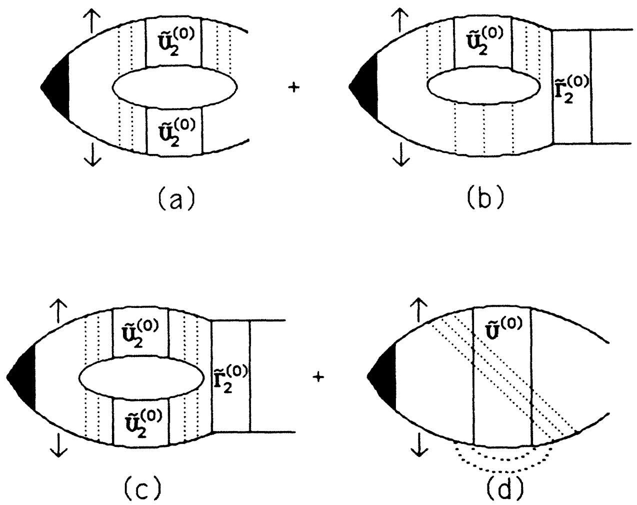

FIG. 9. First order to t contributions to the vertex Λ_s. Note that because of the spin's structure, it is not possible to have $\tilde{u}_2^{(0)}$ in place of $\tilde{u}^{(0)}$ in Fig. 9(d).

The factor $\zeta^2/[-i\Omega(z+N_0\zeta^2\tilde{\Gamma}_2)+Dq^2]$ is obtained from $1/[-i\Omega(1+N_0\tilde{\Gamma}_2^{(0)})+D_0q^2]$ by renormalizing the ladder summation of $\tilde{\Gamma}_2$ to first order in $t\xi$ and needs no further discussion. The $t\xi$ contribution to χ^{st} is given by the two graphs in Fig. 8 and equals

$$(\chi^{st})^{(1)}=2N_0(1+N_0\tilde{\Gamma}_2^{(0)})^2\frac{N_0\tilde{\Gamma}_2^{(0)}}{1+N_0\tilde{\Gamma}_2^{(0)}}t\xi\ . \qquad \text{(A3)}$$

Equation (A3) coincides with the perturbative expression for $2\Gamma=z+\Gamma_2$, provided we identify in the perturbative analysis, $z=z_0=1$ and $\Gamma_2=N_0\tilde{\Gamma}_2^{(0)}$. This confirms the general condition given by Eq. (4.18).

Finally, the vertex correction Λ_s as defined in Fig. 6 is modified to order $t\xi$ by the graphs shown in Fig. 9 which give

$$\Lambda_s^{(1)}=2(1+N_0\tilde{\Gamma}_2^{(0)})N_0\tilde{\Gamma}_2^{(0)}t\xi-(1+N_0\tilde{\Gamma}_2^{(0)})\zeta^{(1)}\ , \qquad \text{(A4)}$$

where $\zeta^{(1)}$ is the $t\xi$ contribution to ζ. This quantity, which contains $t\xi^2$, comes from Fig. 9(d). (A4) implies to first order, in t_0,

$$\Lambda_s\zeta=(1+N_0\tilde{\Gamma}_2^{(0)})+2(1+N_0\tilde{\Gamma}_2^{(0)})N_0\Gamma_2^{(0)}t\xi\ . \qquad \text{(A5)}$$

(A5) is seen to be identical to χ^{st} except for the overall factor of N_0, thus confirming the condition (4.7) also. Thus at least to first order in t_0, perturbation shows that the general picture of Sec. IV are indeed valid. On the other hand, this conclusion is arrived at after some nontrivial cancellations, and this fully reveals the value of proving those equations by global and local conservation laws. This is especially important if one is to attempt a scaling theory beyond first order in t.

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