

## Full splitting of the first zero-field steps in the $I$ - $V$ curve of Josephson junctions of intermediate length

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We report on the observation of full splitting of the first zero-field steps in the  $I$ - $V$  curves of Josephson transmission lines of intermediate length  $L \approx (3-5)\lambda_J$ , where  $\lambda_J$  is the Josephson penetration length. We study in detail how this splitting of the step into two branches depends on the temperature of the junction and on a weak applied magnetic field. We relate the splitting to excitations in the junctions whose behavior is described by the perturbed sine-Gordon equation.

Nonlinear effects in distributed Josephson junctions have recently attracted considerable interest.<sup>1</sup> Long and narrow Josephson junctions ( $L/\lambda_J \gg 1$ ,  $W/\lambda_J \ll 1$ , where  $\lambda_J$  is the Josephson penetration length) have been studied both theoretically and experimentally, and the characteristics of these junctions have been explained in terms of fluxon (soliton) dynamics.<sup>2-4</sup> For intermediate-length Josephson junctions ( $L \geq \lambda_J$ ) some numerical calculations and experimental studies have been carried out revealing a rather complicated dynamic behavior.<sup>5</sup>

The present paper is based on a detailed experimental investigation of Josephson transmission lines of intermediate length. It is, in particular, devoted to the dynamics of such a junction when it is biased on the first zero-field steps (ZFS  $\pm 1$ ) of the junction  $I$ - $V$  curve where a single fluxon/antifluxon moves back and forth along the junction. On this first current step we have observed two closely lying branches of the dc  $I$ - $V$  curve [ $\Delta V \approx 0.5 \mu\text{V}$  for  $V(\text{ZFS} \pm 1) \approx 40 \mu\text{V}$ ].<sup>6</sup> We report here on the temperature and magnetic field dependence of this phenomenon.

Sample preparation and experimental technique have been described in previous publications.<sup>7</sup> Several junctions have been investigated, all Nb-Nb<sub>x</sub>O<sub>y</sub>-Pb overlap Josephson junctions of intermediate normalized length. For this Brief Report we shall concentrate on a representative sample, a junction of length  $L = 397 \mu\text{m}$  and width  $W = 17.5 \mu\text{m}$ . The critical current density  $J$  and the Josephson penetration length  $\lambda_J$  were determined from the magnetic field dependence of the critical current  $I_c(B)$  using a procedure for intermediate length junctions described previously.<sup>8</sup> For the junction in question the critical current density (at 4.2 K) was  $26 \text{ A/cm}^2$ ,  $\lambda_J$  was  $91 \mu\text{m}$ , and  $l = L/\lambda_J$  was therefore 4.4. Geometrically, the overlap of the junction was perfect to within  $0.5 \mu\text{m}$ , the resolution of the optical microscope we used.

The maximum value of the critical current for the two directions of the bias current through the junction,  $I_c^+$  and  $I_c^-$ , were obtained for slightly different values of the strength of the applied magnetic field,  $B^+$  and  $B^-$ , respectively. At 5.4 K the difference  $B^+ - B^-$  was  $6.5 \mu\text{T}$ . This difference stems from a slight asymmetry in the junction geometry or in the bias current distribution. Zero magnetic

field was defined as  $B_0 = (B^+ + B^-)/2$ . The finite value of  $B_0 = 1.1 \mu\text{T}$  was required in order to compensate a residual field in the sample holder.

The branching of the first zero-field steps depends strongly on small changes in the magnetic field. This is illustrated for ZFS + 1 in Fig. 1, where a series of  $I$ - $V$  characteristics is shown for a number of constant settings of the applied magnetic field in the range from  $-20$  to  $26 \mu\text{T}$ . In the interval between the zero and  $+2 \mu\text{T}$  applied field no branching is observed, but for increasing  $|B|$  two branches are clearly seen. The height of the lower voltage branch decreases faster with increasing  $|B|$  than the higher voltage branch. The lower voltage branch was stable only for small fields and metastable for larger values of  $|B|$  (for the curves  $-3$  to  $-7$  and  $+4$  to  $+8$  in Fig. 1 the lower voltage branch could only be accessed by choosing a bias current level in the appropriate part of the step in zero applied field followed by an increase of the field to the desired value). It should be noted that the range of magnetic field used to study these changes in ZFS  $\pm 1$  is considerably smaller than the magnetic field  $B_{c1}$  required to reduce the critical current to zero. At  $T = 5.4 \text{ K}$  this value  $B_{c1}$  for the first zero of the  $I_c(B)$  diffraction pattern was  $88 \mu\text{T}$  as compared to the highest field of  $26 \mu\text{T}$  used to obtain the  $I$ - $V$  curves in Fig. 1. It should also be pointed out that the difference in magnetic fields required to observe the fully developed lower- and higher-voltage branches of the ZFS + 1 is rather small; e.g.,  $B = 6 \mu\text{T}$  for the curves  $+4$  and  $+1$  in Fig. 1. The small voltage splitting of the branches ( $\Delta V \approx 0.5 \mu\text{V}$ ) and the need for fine magnetic tuning may explain why this splitting of ZFS  $\pm 1$  has neither been observed experimentally until now nor clearly found in the numerical simulations. The fine structure in the bottom of the step reported previously<sup>4,9</sup> was not observed in this particular junction, although the experimental conditions for observing it should be more than adequate.

We have studied how the branching depends on the losses in the junctions. The losses were varied by changing the temperature. For each temperature the  $I$ - $V$  characteristic was recorded for values of the magnetic field that maximized the height of each branch. The results of such measurements on the junction are presented in Fig. 2. In the

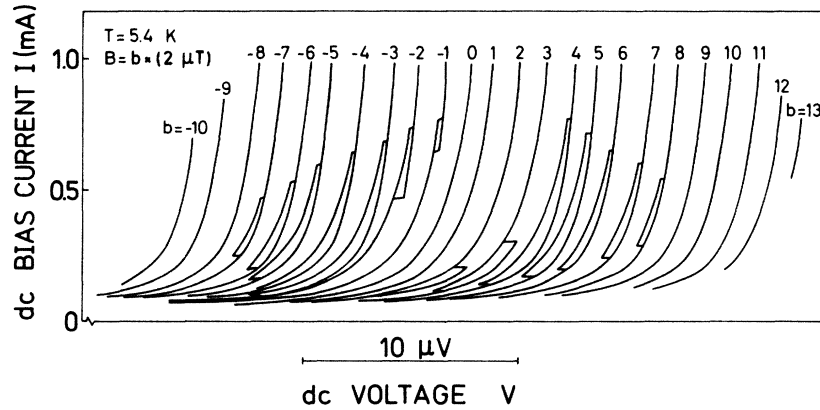


FIG. 1. The magnetic field dependence of the splitting of the first zero-field step (ZFS+1). The figure shows a series of measured  $I$ - $V$  curves of ZFS+1 taken for a number of different settings of the applied magnetic field (in the plane of the junction),  $B_{\text{appl}} = b \times (2.0 \mu\text{T})$ ,  $b = 0, \pm 1, \pm 2, \pm 3, \dots$ . The voltage at the step is about  $40 \mu\text{V}$ . The curves have been displaced along the voltage axis for clarity of presentation. The temperature is 5.4 K. Overlap junction with dimensions  $W \times L = 17.5 \times 397 \mu\text{m}^2$ , critical current density (5.4 K)  $J = 21 \text{ A/cm}^2$ ,  $\lambda_J = 100 \mu\text{m}$ , and normalized length  $l = 4.0$ .

intermediate temperature region two branches are fully developed (cf. curve No. 3). At lower temperatures the higher voltage branch had the smaller height (curve No. 4). In that case the higher voltage branch could only be traced out by accessing the branch at a higher temperature, e.g., at 5.3 K, followed by a lowering of the temperature to the appropriate lower temperature (e.g.,  $T = 4.2 \text{ K}$ , curve No. 4 in Fig. 2). At even lower temperatures the higher voltage branch was stable only in the very bottom of ZFS  $\pm 1$ , e.g., at 1.8 K the maximum observed height of that branch was about  $25 \mu\text{A}$  (the voltage difference between the two branches was here up to  $2.2 \mu\text{V}$ ). At higher temperatures the splitting of the branches was smaller (curve No. 2). Above  $T = 6 \text{ K}$  (curve No. 1) no branching could be ob-

served with the resolution of the voltage measurement ( $\Delta V \approx 0.1 \mu\text{V}$ ).

Using the results of perturbation theory based on the sine-Gordon equation<sup>3,10</sup> we have fitted theoretical  $I$ - $V$  curves to the branches of ZFS  $\pm 1$ , although we realize that neither of the branches may be purely solitonlike. The fitting procedure has been described in an earlier publication.<sup>11</sup> It is, strictly speaking, only valid for long junctions. The fits yield the loss parameters  $\alpha$  and  $\beta$ , where  $\alpha$  is the quasiparticle resistance of the junction and  $\beta$  is the surface resistance of the superconducting films. At a given temperature the two branches differ in the value of the  $\beta$  parameter, which is always smaller for the higher voltage branch, e.g., at  $T = 4.2 \text{ K}$ :  $\beta$  (low) =  $3.7 \times 10^{-3}$  and  $\beta$  (high) =  $1.0 \times 10^{-3}$  while  $\alpha$  (low)  $\approx \alpha$  (high)  $\approx 1.2 \times 10^{-2}$ .

The 8–12 GHz microwave radiation from the Josephson junction biased on the two branches was also measured by using a microstrip antenna placed  $12 \mu\text{m}$  from one end of the junction. The main result here was that on the higher voltage branch of the ZFS  $\pm 1$  a substantial microwave signal could be detected (up to 25 dB above the noise level of our receiver), whereas almost no signal could be seen when the junction was biased on the lower voltage branch. At magnetic fields where the lower voltage branch was fully developed (e.g., curve No. 1 in Fig. 1) no radiation was observed (total noise figure of the receiver was 4.5 dB, spectral resolution 1 kHz). Only at higher magnetic field strengths, in the region where the lower voltage branch was metastable (curve No. 3 in Fig. 1) was weak radiation detected with a maximum power up to about 10 dB above the noise. As far as the linewidth of the radiation is concerned our measurements show only small differences between the two branches. The measured linewidth typically varied from about 50 kHz to a few kilohertz when the bias current was swept up along the two branches of the step.<sup>7,12</sup>

We conclude by offering the following somewhat speculative interpretations of the observed splitting of ZFS  $\pm 1$ . The splitting may be accounted for by the two distinct modes seen in recent numerical simulations based on the perturbed sine-Gordon equation for the case of intermediate

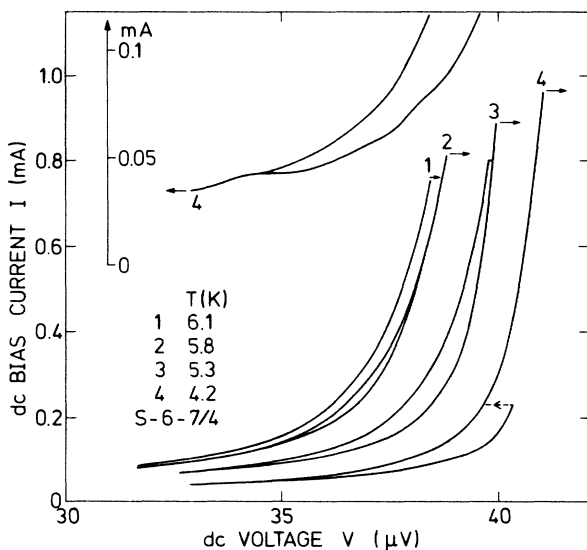


FIG. 2. Same junction as in Fig. 1: Temperature dependence of the splitting of the first zero-field step. The applied magnetic field was optimized for each branch. The 4.2-K curve is also shown on a blown-up current scale (same voltage axis as the other curves) to show the rather smooth bottom part of the branches.

length junctions.<sup>9,5</sup> Alternatively, the splitting phenomenon could be a result of the interaction between the soliton and a new type of (nondispersive) waves which propagate "on top of" the soliton and which may act as a synchronizing excitation in the junction.<sup>13</sup> Considering the magnetic field lines around the junction (which may be thought of as being equivalent to two weak boundary solitons along the two longer edges of the junction), we imagine that such weakly dispersive modes could form standing waves along the edges which could give rise to a phase shift in the direction perpendicular to the longer dimension of the junction,<sup>14</sup> thereby causing either constructive or destructive interference, which again could explain the observed difference in the

detected microwave power levels on the two branches. Clearly, both explanations require additional work before the mechanism responsible for the splitting reported here can be confirmed.

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