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Transverse magnetoconductivity of quasi-two-dimensional semiconductor layers in the presence of phonon scattering

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The transverse magnetoconductivity of quasi-two-dimensional layers, such as those occurring in heterojunctions of superlattices and in metal-oxide-semiconductor field-effect transistor inversion layers, is computed using the formalism of linear response theory, developed previously. The electron gas is considered to be quasi-two-dimensional, while the phonons are considered to be three dimensional. Explicit results for magnetophonon resonances are obtained for various kinds of phonon scattering (acoustic, nonpolar optical, polar optical, and piezoelectric). Collision broadening of the Landau levels is included to avoid divergences.

I. INTRODUCTION

Recent investigations[1] into the transverse magnetoconductivity and quantum Hall effect of two-dimensional and quasi-two-dimensional semiconductor layers have yielded valuable information regarding the quantum oscillation and the localization properties of the electrons in such systems. The energy spectrum of the electrons confined to semiconductor layers in the presence of a magnetic field applied perpendicularly to the layers is completely quantized. Therefore the density of states and the transverse magnetoconductivty of such systems diverge in the absence of scattering. At low temperatures several authors[2] calculated the transport and optical properties of the electrons in the inversion layers of metal-oxide-semiconductor field-effect transistors (MOSFET's) and superlattices by assuming the scattering of the electrons to occur by the surface roughness or the impurity ions. Theoretical calculations based on the assumption that the time spent by an electron near the scattering center is finite (and possibly even comparable to the mean free time of the electrons) have yielded reasonable quantitative agreement with the experimental data. However, at higher temperatures the scattering of the electrons by the lattice modes becomes important and a treatment of transport properties in superlattices and heterojunctions is called for.

The calculation of the relaxation times and mobility of electrons in the semiconductor layers in the presence of phonon scattering has been performed earlier by several authors.[3] To our knowledge a calculation of the transverse magnetoconductivity in the semiconductor layers in the presence of phonon scattering has not yet been given. However, the calculation of the relaxation times in high magnetic field in the quantum-well structures in the presence of phonon scattering has been reported earlier.[4]

The purpose of the present paper is to report on the computation of the transverse magnetoconductivity of quasi-two-dimensional semiconductor layers in the presence of phonon scattering. The basis of our treatment is the formalism of linear response theory as derived recently by one of us (C.M.V.V.) and co-workers.[5] The expression for the transverse magnetoconductivity is explicitly calculated for a quasi-two-dimensional electron gas confined to a square quantum-well-type structure interacting with three-dimensional phonons. The scattering of electrons with the acoustic modes, polar-optical phonons, nonpolar-optical phonons, and the piezoelectric coupling is considered in detail.

In order to obtain finite results for σ_{xx}, collision broadening of Landau levels is introduced so that the delta functions are replaced by Lorentzian shapes. For polar-optical and nonpolar-optical phonon scattering the magnetophonon resonance peaks occurring at $\omega_0=\mu\omega_c$ are calculated. The amplitudes of the magnetophonon resonances are expressed in terms of the width of the Landau levels in the limit of high magnetic fields where the coupling between Landau levels may be neglected.[6]

The magnetic field and temperature dependence of σ_{xx} for quasi-two-dimensional semiconductor layers for acoustic and piezoelectric phonons is essentially smooth and is given by the same dependence as that of τ_N on B and T; here $\gamma_N=1/\tau_N$ stands for the width of the Landau levels.

In Sec. II the theory of transverse magnetoconductivity for a quasi-two-dimensional semiconductor layer is presented; in Sec.III the calculation of σ_{xx} for all four types of phonon scattering is given and the results in the limiting cases are derived, including collision broadening of the Landau levels.

Results and discussions are given in Sec. IV. In the appendix we provide a discussion of Lorentzian broadening and the detailed derivation of γ_N for all four types of electron-phonon interaction.

II. QUASI-TWO-DIMENSIONAL SEMICONDUCTOR LAYERS: THEORY

The Hamiltonian of an electron-phonon system in the presence of a magnetic field applied perpendicularly (z direction) to the surface layers (x-y plane) is given by

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$$H=\sum_{N,\bar{x},p} E_{N\bar{x}p}a^{\dagger}_{N,\bar{x},p}a_{N,\bar{x},p}+\sum_{Q}\hbar\omega_{Q}(b^{\dagger}_{Q}b_{Q}+\tfrac{1}{2})+H_{e\text{-ph}} , \tag{1}$$

$$H_{e\text{-ph}}=\sum_{N,\bar{x},p}\sum_{N',\bar{x}',p'}\sum_{Q}V(\mathbf{Q})\langle N',\bar{x}',p'\,|\,e^{-i\mathbf{Q}\cdot\mathbf{r}}\,|\,N,\bar{x},p\rangle \times a^{\dagger}_{N',\bar{x}',p'}a_{N,\bar{x},p}(b^{\dagger}_{Q}+b_{-Q}) , \tag{2}$$

where $E_{N\bar{x}p}=(N+\frac{1}{2})\hbar\omega_c+p^2E_0$, N, $\bar{x}$, and p are the Landau quantum number, center of the cyclotron orbit, and the subband quantum number, respectively; $E_0=\hbar^2\pi^2/2m^*L_z^2$, where L_z is the separation between the layers, $\omega_c=eB/m^*c$, m^* being the spherical effective mass of the electrons assumed to be the same in all directions, $\mathbf{Q}(q_x,q_y,q_z)$ is the phonon's propagation vector, $a_{N,\bar{x},p}$ and $a^{\dagger}_{N,\bar{x},p}$ are the annihilation and creation operators for the electrons in the state $|\,N,\bar{x},p\rangle$; similarly, $b^{\dagger}_{Q}$ and b_{Q} are the creation and annihilation operators for the phonons in the state $|\,N_Q\rangle$; finally, $V(\mathbf{Q})$ is the electron-phonon perturbation potential. The vector potential of the constant magnetic field $\mathbf{B}$ in the Landau gauge is given by $\mathbf{A}=(0,Bx,0)$. The wave functions of the free electrons are given by

$$\psi_{N,\bar{x},p}(r)=\langle \mathbf{r}\,|\,N,\bar{x},p\rangle=\frac{1}{(2^N N!\sqrt{\pi}\lambda)^{1/2}}\exp\left[-\left[\frac{x-\bar{x}}{\lambda}\right]^2\right]H_N\left[\frac{x-\bar{x}}{\lambda}\right]e^{-i\bar{x}y/\lambda^2}\frac{1}{\sqrt{L_z}}\sin\left[\frac{p\pi}{L_z}\right]z , \tag{3}$$

where $\lambda=(\hbar/m^*\omega_c)^{1/2}$, $H_N(x)$ is the Hermite polynomial of the Nth degree, and $\bar{x}=-\lambda^2k_y$ is the center of the cyclotron orbit. $N_{\mathbf{Q}}$ stands for the distribution function of the phonons in the mode $\mathbf{Q}$,

$$N_{\mathbf{Q}}=\left[\exp\left[\frac{\hbar\omega_{\mathbf{Q}}}{k_BT}\right]+1\right]^{-1} . \tag{4}$$

The expression for the transverse magnetoconductivity derived earlier by one of us (C.M.V.V.) and co-workers[5] may easily be written for the present system and is given by

$$\sigma_{xx}=\frac{\pi e^2\beta\lambda^2}{\hbar(AL_z)}\sum_{N,N'}\sum_{k_y}\sum_{p,p',Q}X\,|\,V(Q)\,|^2\,|\,I_{NN'}(X)\,|^2\,|\,F_{pp'}(q_z)\,|^2 \times f_{Np}(1-f_{N'p'})\{N_0(\mathbf{Q})\delta(E_{Np}-E_{N'p'}+E_{\mathbf{Q}})+[N_0(\mathbf{Q})+1]\delta(E_{Np}-E_{N'p'}-E_{\mathbf{Q}})\} , \tag{5}$$

where $X=\frac{1}{2}\lambda^2q_{\perp}^2$, $E_{\mathbf{Q}}=\hbar\omega_{\mathbf{Q}}$ is the energy of a phonon of frequency $\omega_{\mathbf{Q}}$, $E_{Np}=E_{N\bar{x}p}$,

$$\begin{aligned}|\,I_{NN'}(X)\,|^2&=\frac{N'!}{N!}e^{-X}X^{N-N'}[L_{N'}^{N-N'}(X)]^2,\quad N'\le N\\&=\frac{N!}{N'!}e^{-X}X^{N'-N}[L_{N}^{N'-N}(X)]^2,\quad N\le N' ,\end{aligned} \tag{6}$$

$$F_{pp'}(q_z)=\frac{2}{L_z}\int_0^{L_z}e^{iq_zz}\sin\left[\frac{p\pi}{L_z}z\right]\sin\left[\frac{p'\pi}{L_z}z\right]dz , \tag{7}$$

and where $V(\mathbf{Q})$ depends on the nature of the electron-phonon interactions. The functions $L_N^M(X)$ are the associated Laguerre polynomials given by

$$L_N^M(X)=\frac{e^XX^{-M}}{N!}\frac{d^N}{dX^N}(e^{-X}X^{N+M}) ; \tag{8}$$

f_{Np} denotes the Fermi-Dirac distribution function

$$f_{Np}=f(E_{Np})=[\exp(E_{Np}/k_BT)+1]^{-1} .$$

Equation (7) was evaluated earlier for an infinite barrier by Ridley,[3] and the result is given for $p=p'$, by

$$|\,F_{p,p'}(q_z)\,|^2=\frac{1}{4}\left[4\left[\frac{\sin(q_zL_z/2)}{(q_zL_z/2)}\right]^2+\left[\frac{\sin[(q_z-2p\pi/L_z)(L_z/2)]}{(q_z-2p\pi/L_z)(L_z/2)}\right]^2+\left[\frac{\sin[(q_z+2p\pi/L_z)(L_z/2)]}{(q_z+2p\pi/L_z)(L_z/2)}\right]^2\right] , \tag{9}$$

and for $p \neq p'$ by

$$|F_{p,p'}(q_z)|^2 = \frac{1}{4}\left[\frac{\sin\{[q_z \pm (\pi/L_z)(p' \pm p)](L_z/2)\}}{[q_z \pm (\pi/L_z)(p' \pm p)](L_z/2)}\right]^2. \tag{10}$$

In the following we shall evaluate Eq. (5) for the transverse magnetoconductivity for different interactions. The sum over **Q** will be replaced by

$$\int (AL_z/8\pi^3)d^3Q = \int (AL_z/8\pi^3)2\pi q_\perp \, dq_\perp \, dq_z ,$$

where A is the area of the layer. Also, we note $\sum_{k_y}$ gives $1/2\pi\lambda^2$.

III. CALCULATION

A. Acoustic-phonon scattering

In this section we consider the elastic scattering of the electrons by acoustic phonons. In this limit we have $E_q \approx 0$; therefore the arguments of the delta functions in Eq. (5) contain terms like

$$E_{Np} - E_{N'p'} = (N - N')\hbar\omega_c + [p^2 - (p')^2]E_0$$

and thus the transverse magnetoconductivity diverges for intra-Landau-level and intrasubband transitions ($E_{Np} = E_{N'p'}$). Following the collision-broadening model, which has been successfully applied for electron-impurity scattering,[2] we shall replace the delta function by Lorentzians (for a detailed discussion of Lorentzian broadening and the results for γ_N, see the Appendix),

$$\delta(E_{Np} - E_{N'p'}) \Longrightarrow \frac{1}{\pi}\left[\frac{\hbar\gamma_{NN'}}{(E_{Np} - E_{N'p'})^2 + \hbar^2\gamma_{NN'}^2}\right]. \tag{11}$$

$\gamma_{NN'} = 1/\tau_{NN'}$ is the inverse relaxation time of the electrons, which, in general, depends on the temperature, magnetic field, and the Landau-level index N and N'.[4] We also have $\omega_Q = u_0 Q$, u_0 being the velocity of sound and $|V(Q)|^2 = (C^2/2\rho u_0)Q$, where ρ is the density of the material and C is the deformation-potential constant. We may also simplify the calculations further by making the high-temperature approximation for $N_0(Q)$,

$$N_0(Q) \approx N_0(Q) + 1 \approx \frac{k_B T}{\hbar\omega_Q} = \frac{k_B T}{\hbar u_0 Q} . \tag{12}$$

In the above approximation, the integral over q_z as given by Ridley[3] yields

$$\frac{L_z}{2}\int_{-\infty}^{\infty} |F_{pp'}(q_z)|^2 dq_z = \frac{\pi}{L_z}(2 + \delta_{pp'}) , \tag{13}$$

and the integral over $q_\perp$ is given by[5]

$$\int_0^{\infty} X\,|I_{NN'}(X)|^2 dX = \frac{N'!}{(N!)^2}(3N - N' + 1)[(2N - N')!] . \tag{14}$$

Substituting the values of the above integrals, the equation for the transverse magnetoconductivity takes the form

$$\sigma_{xx} = \frac{\pi e^2}{\hbar^2 L_z}\sum_{\substack{N,N'\\p,p'}}\frac{1}{2\pi^2}\left[\frac{C^2}{2\rho u_0^2}\right]\frac{N'!}{(N!)^2}(3N - N' + 1)[(2N - N')!](2 + \delta_{pp'})f_{Np}(1 - f_{N'p'})\frac{\hbar\gamma_{NN'}}{(E_{Np} - E_{N'p'})^2 + \hbar^2\gamma_{NN'}^2} . \tag{15}$$

For the case of $p = p' = 1$ and $N = N'$, we have

$$\sigma_{xx} = \frac{e^2}{\hbar}\left[\frac{C^2}{2\rho u_0^2}\right]\left[\frac{1}{2\pi}\right]\left[\frac{3}{L_z}\right]\sum_N (2N + 1)\frac{f_N(1 - f_N)}{\hbar\gamma_N} , \tag{16}$$

or

$$\sigma_{xx} = \frac{e^2}{\hbar}\left[-\frac{C^2}{2\rho u_0^2}\right]\left[\frac{3}{2\pi L_z}\right]\sum_N (2N + 1)f_N(1 - f_N)\tau_N, \quad \gamma_N = 1/\tau_N . \tag{17}$$

Thus the behavior of σ_{xx} is determined essentially by the behavior of τ_N, which varies smoothly with temperature and magnetic field.

B. Nonpolar-optical phonon scattering

In this case

$$E_Q = \hbar\omega_0 \text{ and } |V(Q)|^2 = \frac{\hbar D^2}{2\Omega\rho\omega} = D' , \tag{18}$$

Ω being the volume of the quasi-two-dimensional layer ($\Omega = AL_z$). Thus E_Q and $V(Q)$ are independent of Q and the transverse magnetoconductivity diverges at the energy given by $E_{N'p'} = E_{Np} \pm \hbar\omega_0$, where ω_0 is the constant frequency of the optical phonon. In order to overcome this divergence the collision-broadening treatment is adopted and the q_z and $q_\perp$ integrals are performed in a way similar to that in the previous case. Equation (5) then takes the following form:

$$\sigma_{xx}=\frac{\pi e^2\beta}{\hbar^2 L_z}\sum_{\substack{N,N'\\p,p'}}\frac{D'}{2\pi^2}f_{Np}(1-f_{N'p'})\frac{N'!}{(N!)^2}(3N-N'+1)(2N-N')(2+\delta_{pp'})$$
$$\times\left[N(\omega_0)\frac{\hbar\gamma_{NN'}}{(E_{N'p'}-E_{Np}-\hbar\omega_0)^2+\hbar^2\gamma_{NN'}^2}+[N(\omega_0)+1]\frac{\hbar\gamma_{NN'}}{(E_{N'p'}-E_{Np}+\hbar\omega_0)^2+\hbar^2\gamma_{NN'}^2}\right], \tag{19}$$

where the collision-broadening parameter $\gamma_{NN'}$ is assumed to be the same for the emission and absorption processes.

In the single-level approximation $N=N'$, $p=p'=1$, Eq. (19) simplifies to

$$\sigma_{xx}=\frac{\pi e^2}{\hbar^2}\frac{3\beta}{L_z}\frac{D'}{2\pi^2}\sum_N f_N(1-f_N)(2N+1)[2N(\omega_0)+1]\frac{\gamma_N}{\hbar(\omega_0^2+\gamma_N^2)} . \tag{20}$$

From Eq. (19) we note that σ_{xx} has peaks at the magnetophonon resonances $\omega_0=\mu\omega_c$, where μ is an integer; we note that Eq. (19) gives a quantitative measure of the amplitudes of the magnetophonon oscillations.

C. Polar-optical phonon scattering

The perturbing interaction is given by

$$|V(Q)|^2=\frac{A'}{Q^2},\quad Q^2=q_\perp^2+q_z^2 \tag{21}$$

where A' is a constant independent of Q. Substituting the above potential in Eq. (5) and performing the q_z integral, we have

$$\sigma_{xx}=\frac{\pi^2 e^2\beta}{\hbar L_z}A'\sum_{N,N'}\sum_{p,p'}\sum_{\pm}f_{Np}(1-f_{N'p'})[N(\omega_0)+\tfrac{1}{2}\pm\tfrac{1}{2}]\mathcal{F}_{NN'}^{pp'}\frac{\hbar\gamma_{NN'}}{(E_{N'p'}-E_{Np}\pm\hbar\omega_0)^2+\hbar^2\gamma_{NN'}^2}, \tag{22}$$

where

$$\mathcal{F}_{NN'}^{pp'}(\lambda)=\int_0^\infty dX\,X\,|I_{NN'}(X)|^2\left[\frac{1+\delta_{pp'}}{[X+\frac{1}{2}\lambda^2(p-p')^2E_0^2]}+\frac{1}{[X+\frac{1}{2}\lambda^2(p+p')^2E_0^2]}\right]. \tag{23}$$

Equation (22) is the two-dimensional analog of magnetophonon resonance in three dimensions, containing resonance peaks at the phonon frequencies given by $\omega_0=\mu\omega_c$, where μ is an integer. Such magnetophonon resonances in GaAs/Ga_xAl_{1-x}As superlattice systems seem to have been observed earlier. It is interesting to note that σ_{xx} is finite for $p=p'=1$, whereas the relaxation times diverge as noted earlier.[4] According to Eqs. (22) and (23) the main features of the magnetophonon resonance are determined by the function $\mathcal{F}_{NN'}^{pp'}$ and the Lorentzian factors of width $\gamma_{NN'}$, which, in turn, are determined from a number of scattering mechanisms. For the extreme quantum limit $N=N'=0$ and $p=p'=1$, we have

$$\mathcal{F}_{0,0}^{11}=3-\alpha e^{\alpha}\mathrm{E}_1(\alpha),\quad \alpha=2\lambda^2E_0^2$$

and (24)

$$\mathrm{E}_1(\alpha)=\int_\alpha^\infty\frac{e^{-x}}{x}dx .$$

Introducing Eq. (24) into Eq. (22), we have

$$\sigma_{xx}=\frac{\pi e^2\beta A'}{\hbar L_z}\sum_{\pm}f_{0,1}(1-f_{0,1})[N(\omega_0)+\tfrac{1}{2}\pm\tfrac{1}{2}]$$
$$\times[3-\alpha e^{\alpha}\mathrm{E}_1(\alpha)]\frac{\hbar\gamma_0}{\hbar^2\omega_0^2+\hbar^2\gamma_0^2} . \tag{25}$$

Equation (25) thus determines the amplitudes of the magnetophonon resonant peaks. These amplitudes vanish when $f_{0,1}=0$, $f_{0,1}=1$, or $3-\alpha e^{\alpha}\mathrm{E}_1(\alpha)=0$, which determines a critical value of the magnetic field at which the localization of the electron occurs.

D. Piezoelectric phonon scattering

For the sake of simplicity we consider the following spherically symmetric form of the electron piezoelectric phonon interaction rather than the commonly used anisotropy form:[7]

$$|V(Q)|^2=\frac{K}{Q}=\frac{K}{(q_\perp^2+q_z^2)^{1/2}},\quad K=\text{const} . \tag{26}$$

Following the same approximation as for acoustic-phonon scattering [Eq. (12)], the q_z and $q_\perp$ integrals occurring in Eq. (5) are similarly performed and the result for σ_{xx} takes the form

$$\sigma_{xx}=\frac{\pi e^2}{\hbar^2 L_z}\sum_{\substack{N,N'\\p,p'}}\frac{K}{4\pi^2}f_{Np}(1-f_{N'p'})(2+\delta_{pp'})$$
$$\times\frac{N'!}{(N!)^2}(2N-N')!$$
$$\times\frac{\hbar\gamma_{NN'}}{(E_{Np}-E_{N'p'})^2+\hbar^2\gamma_{NN'}^2} . \tag{27}$$

For $N=N'$ and $p=p'=1$ we have

$$\sigma_{xx}=\frac{3e^2K}{\hbar^2L_z4\pi}\sum_N f_{N,1}(1-f_{N,1})\left[\frac{\tau_N}{\hbar}\right]. \quad (28)$$

Thus the behavior of σ_{xx} is essentially determined by the behavior of $\tau_N=1/\gamma_N$ for piezoelectric phonon scattering. It may be noted that unlike polar-optical and nonpolar-optical phonon scattering, no resonances are expected for the present case.

IV. RESULTS AND DISCUSSION

In the limit of high magnetic fields, where the coupling between Landau levels is neglected, σ_{xx} is inversely proportional to the thickness of the layers for all types of phonon scattering. For a quantum-well-type structure, as realized in superlattices, σ_{xx} is directly proportional to the relaxation time of the electrons in the Nth Landau level in the elastic scattering approximation and in the high-temperature limit, for acoustic phonons, employing collision broadening.

Collision broadening of the Landau levels is also necessary to obtain finite results for σ_{xx} for nonpolar-optical and polar-optical phonon scattering. Including collision broadening in the manner of replacing a delta function by Lorentzians, two-dimensional magnetophonon resonances at $\omega_0=\mu\omega_c$ (μ is an integer) are predicted. The amplitude of the magnetophonon resonances for polar-optical phonons is obtained in closed form, by use of Eqs. (22) and (25). Equation (25) is valid only in the extreme quantum limit. For nonpolar-optical phonon scattering a closed expression for σ_{xx} is obtained without making any approximation for the phonon distribution function [see Eq. (19)].

For piezoelectric phonon scattering with inclusion of collision broadening, σ_{xx} follows the same type of dependence on the relaxation time as that for the acoustic-phonon scattering, but with a different dependence on N and N'.

In conclusion, we have presented a quantum-transport theory of transverse magnetoconductivity in quasi-two-dimensional semiconductor layers in the presence of phonon scattering. Two-dimensional magnetophonon resonances are calculated by a straightforward application of the collision broadening of Landau levels. The amplitude of such resonances are calculated in terms of the width of the Landau levels. Other applications of the present theory will be presented in a later publication.

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APPENDIX

Here we discuss the Lorentzian broadening and derive explicit expressions for the half-width γ_N occurring in the expressions for the transverse conductivity σ_{xx} for all four types of electron-phonon interaction in the quasi-two-dimensional semiconductor layers. A delta function is expressed in terms of the unperturbed Green function $G^0_\lambda(E)$, $\lambda=(N,x,p)$, as

$$\delta(E-E_\lambda)=(1/\pi)\mathrm{Im}G^0_\lambda(E), \quad (A1)$$

$$G^0_\lambda(E)=\langle\lambda|(E-H_0)^{-1}|\lambda\rangle=(E-E_\lambda)^{-1}. \quad (A2)$$

However, if we use the full retarded Green function $G_\lambda(E)=\langle\lambda|(E-H)^{-1}|\lambda\rangle$ instead of $G^0_\lambda(E)$ in Eq. (A1), we have

$$G_\lambda(E)=(E-E_\lambda-\Sigma_\lambda)^{-1}, \quad (A3)$$

$$\Sigma_\lambda=\Delta_\lambda+i\gamma_\lambda, \quad (A4)$$

where Σ_λ is the electron self-energy and we have neglected the phonon self-energy. Equations (A3) and (A4) yield

$$\mathrm{Im}\,G_\lambda(E)=\gamma_\lambda/[(E-E_\lambda-\Delta_\lambda)^2+\gamma_\lambda^2], \quad (A5)$$

where γ_λ and Δ_λ represent the half-width and the energy shift of the Lorentzian centered at the energy $E=E_\lambda$. Therefore, under the approximation of replacing $G^0_\lambda(E)$ by $G_\lambda(E)$, the delta function is replaced by the Lorentzian given by Eq. (A5).

In the limit of strong electron-phonon interaction, as is the usual case for the system under consideration, with the assumption of a high magnetic field near resonance, $\omega_0=\mu\omega_c$, γ_N is found to be given by the following expression:

$$(\gamma_N^\pm)^2=\hbar^{-2}\int\frac{d^3Q}{(2\pi)^3}[N_0(Q)+\tfrac{1}{2}\pm\tfrac{1}{2}]\times|V(Q)|^2\,|I_{NN}(X)|^2\,|F_{11}(q_z)|^2, \quad (A6)$$

where all the symbols are defined in Sec. II.

1. Electron—acoustic-phonon interaction

Making use of the electron-phonon interaction $V(Q)$ and the dispersion relation given in Sec. III along with the approximation implied by Eq. (12), Eq. (A6) is easily evaluated to yield the following result:

$$\gamma_N^+=\gamma_N^-=\gamma_N=\{(c^2/2\rho u_0^2)[k_BT/(2\pi\hbar)^2]\times(3\pi/L_z)(1/l^2)\}^{1/2} \quad (A7)$$

where we used the integral

$$\int_0^\infty|I_{NN}(X)|^2dX=1. \quad (A8)$$

From Eq. (A7) we thus find that γ_N is proportional to the square root of temperature and the square root of the magnetic field in the high-magnetic-field approximation.

2. Nonpolar phonon scattering

For the electron-nonpolar-phonon interaction, $V(\mathbf{Q})$ and $\omega_{\mathbf{Q}}$ are independent of $\mathbf{Q}$. Assuming $\omega_{\mathbf{Q}}=\omega_0$, Eq. (A6) is reduced to

$$\gamma_N^{\pm}=\{[D'/(2\pi\hbar)^2](3\pi/L_z)\times[N_0(\omega_0)+\tfrac{1}{2}\pm\tfrac{1}{2}](1/l^2)\}^{1/2} . \quad \text{(A9)}$$

3. Electron–polar-phonon interaction

Making use of $V(Q)$ given by Eq. (21) and replacing ω_Q by ω_0, we have, from Eq. (A6),

$$(\gamma_N^{\pm})^2=(\hbar^{-2}/2)[A'/(2\pi)^2][N_0(\omega_0)+\tfrac{1}{2}+\tfrac{1}{2}]\int_0^{\infty}dX\int_{-\infty}^{\infty}dq_z\left[|I_{NN}(X)|^2\Big/\left[X+\frac{l^2q_z^2}{2}\right]\right]|F_{11}(q_z)|^2 . \quad \text{(A10)}$$

If we use Ridley's momentum-conservation approximation[3] (MCA) to perform the q_z integral in the above equation, the X integral diverges and, thus, $\gamma_N^{\pm}$ diverges in the MCA. If the MCA is not made, it implies a broadening mechanism. Therefore we do not use the MCA, implying $\Delta q_z=2\pi/L_z n_z$, $n_z=0,\pm1,\pm2,\ldots$, and the above integral is reduced to

$$(\gamma_N^{\pm})^2=[A'/(2\pi\hbar)^2](\pi/2L_z)[N_0(\omega_0)+\tfrac{1}{2}\pm\tfrac{1}{2}]\times\int_0^{\infty}dX\,|I_{NN}(X)|^2/(X+\alpha_0) , \quad \text{(A11)}$$

where $\alpha_0\cong l^2/L^2$. The above integral cannot be evaluated for arbitrary N. Therefore we evaluate expression (A11) for the case of the ultraquantum limit ($N=0$), yielding the following formula:

$$\gamma_0^{\pm}=\{[A'/(2\pi\hbar)^2][N_0(\omega_0)+\tfrac{1}{2}\pm\tfrac{1}{2}]\times(1/2L_z)[e^{\alpha_0}E_1(\alpha_0)]\}^{1/2} , \quad \text{(A12)}$$

where $E_1(\alpha_0)$ is an exponential integral and is defined by Eq. (24).

4. Piezoelectric phonon coupling

Making use of the electron–piezoelectric-phonon interaction given by Eq. (26), and the approximation implied by the Eq. (12), we have

$$(\gamma_N^{+})^2=(\gamma_N^{-})^2=(\gamma_N)^2=[k_BT/(2\pi\hbar)^2](K/2\hbar u_0)\int_0^{\infty}dX\int_{-\infty}^{\infty}dq_z\,|I_{NN}(X)|^2|F_{11}(q_z)|^2/(X+l^2q_z^2/2) . \quad \text{(A13)}$$

The above integral is identical to that occurring in subsection 3 above. Following the same discussion as in subsection 3 for the ultraquantum limit, the integral in Eq. (A13) is reduced to the following form:

$$\gamma_0^{+}=\gamma_0^{-}=\gamma_0=[k_BT/(2\pi\hbar)^2]^{1/2}(\pi K/\hbar u_0L_z)^{1/2}[e^{\alpha_0}E_1(\alpha_0)]^{1/2} . \quad \text{(A14)}$$

It may be noted that the magnetic field dependence of γ_0 in the above equation is contained in the factor $e^{\alpha_0}E_1(\alpha_0)$.

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