Real-space inversion of the dielectric-response function of a superlattice

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We present a technique for inverting the dielectric-response function of a superlattice in the random-phase approximation allowing for realistic superlattice eigenstates and multiple subbands. In the case of an infinite superlattice obeying cyclic boundary conditions we show how to construct an analytic expression for the inverse response function in closed form whose roots give the plasmon modes.

INTRODUCTION

Recently there has been much interest in the theoretical calculation of the plasmon dispersion relationship of a superlattice (Bloss,[1] Sarma and Quinn,[2] Giuliani, Qin, and Quinn,[3] and Tselis and Quinn[4]). Much of the work has approached this problem using expressions for the dielectric-response function in reciprocal space. The plasmons are given by the frequencies for which the determinant of the infinite dielectric matrix goes to zero. In this paper we take the alternative approach of calculating the inverse response function using a real-space formalism which gives the plasmon modes as its poles. The advantage of this technique is that the resulting expressions are in the form required to calculate interaction effects so that the contribution of the plasmon poles can be seen directly in, for example, the self-energy. It is important to realize that for many applications the knowledge of the plasmon poles themselves is insufficient. One must also have the screened interaction. This is given by the expression

$$\int \epsilon(r,r'',\omega)v(r''-r')dr''$$

and so involves an accurate knowledge of the inverse response function and its spatial variation. This in turn requires that the model used for ϵ and hence ϵ^{-1} includes realistic eigenstates.

THE MODEL

Our model regards the superlattice as being a structure composed of a large number of regularly spaced, two-dimensional quantum wells separated by layers of dielectric materials. We assume that the behavior of the electrons parallel to the layers is free-electron-like. Perpendicular to the layers in the superlattice direction we assume extreme tight-binding types of wave functions with flat bands. Physically this means that we have taken there to be no overlap integral between electrons in adjacent wells, which is quite a good approximation to the intended experimental situation for the energies involved. We take the case when the number of subbands of importance (formed by the quantization within the wells) is two, although the formalism presented can be generalized in an obvious way to deal with an arbitrary number of bands. Thus we write our model wave functions as

$$\psi_{\mathbf{k}}^{(\alpha)}(\mathbf{r})=\left[\frac{1}{N\Omega}\right]^{1/2}\sum_{n}\phi^{(\alpha)}(x-na)e^{ik_{\perp}na}e^{i\mathbf{k}_{\parallel}\cdot\boldsymbol{\rho}}\,, \qquad (1)$$

where a is the superlattice spacing, (α) is an index denoting conduction or valence subband, and all the other symbols have their usual meaning. In this calculation we choose the Fermi level to be such that only the lowest (valence) subband is occupied.

CALCULATION OF THE DIELECTRIC RESPONSE FUNCTION

Our calculation follows along similar lines to Ortuno and Inkson,[4] Sterne and Inkson,[5] and Inkson and Shama.[6] We start with the expression dielectric-response function in the random-phase approximation[7]

$$\epsilon(\mathbf{r},\mathbf{r}',\omega)=\delta(\mathbf{r}-\mathbf{r}')-\int v(\mathbf{r}-\mathbf{r}'')P(\mathbf{r}'',\mathbf{r}',\omega)d\mathbf{r}''\,, \qquad (2)$$

where v is the Coulomb interaction and P the polarizability is given by

$$P(\mathbf{r},\mathbf{r}',\omega)=\sum_{K,K'}\frac{2(n_K-n_{K'})}{E_K-E_{K'}-\hbar\omega-i\delta}\times\psi_K(\mathbf{r})\psi_{K'}^*(\mathbf{r})\psi_K^*(\mathbf{r}')\psi_{K'}(\mathbf{r}')\,, \qquad (3)$$

K being an index denoting both wave vector and band index. This calculation can be made more accurate by dividing the Coulomb interaction by a scalar dielectric constant to take into account the effect of the valence polarizability and of the low-lying core energy levels. Substituting our wave functions into this expression, assuming that the functions $\phi(x)$ are real and using the condition on the Fermi level, we obtain

$$P(\mathbf{r},\mathbf{r}',\omega)=\frac{2}{N^2\Omega^2}\sum_{\mathbf{k}_{||},\mathbf{k}'_{||}}\frac{n^{(v)}_{\mathbf{k}_{||}}-n^{(v)}_{\mathbf{k}'_{||}}}{\varepsilon^v_{\mathbf{k}_{||}}-\varepsilon^v_{\mathbf{k}'_{||}}-\hbar\omega-i\delta}e^{i(\mathbf{k}_{||}-\mathbf{k}'_{||})\cdot(\rho-\rho')}$$
$$\times\sum_{p,q,r,s}\sum_{k_\perp,k'_\perp}\phi^v(x-pa)\phi^v(x-qa)\phi^v(x'-ra)\phi^v(x'-sa)e^{i(k_\perp pa-k'_\perp qa-k_\perp ra+k'_\perp sa)}$$
$$+\frac{2}{N^2\Omega^2}\sum_{\mathbf{k}_{||},\mathbf{k}'_{||}}\frac{n^{v}_{\mathbf{k}_{||}}}{\varepsilon^v_{\mathbf{k}_{||}}-\varepsilon^c_{\mathbf{k}'_{||}}-\hbar\omega-i\delta}e^{i(\mathbf{k}_{||}-\mathbf{k}'_{||})\cdot(\rho-\rho')}$$
$$\times\sum_{p,q,r,s}\sum_{k_\perp,k'_\perp}\phi^v(x-pa)\phi^c(x-qa)\phi^v(x'-ra)\phi^c(x'-sa)e^{i(k_\perp pa-k'_\perp qa-k_\perp ra+k'_\perp sa)}$$
$$+\frac{2}{N^2\Omega^2}\sum_{\mathbf{k}_{||},\mathbf{k}'_{||}}\frac{-n^{v}_{\mathbf{k}'_{||}}}{\varepsilon^c_{\mathbf{k}_{||}}-\varepsilon^v_{\mathbf{k}'_{||}}-\hbar\omega-i\delta}e^{i(\mathbf{k}_{||}-\mathbf{k}'_{||})\cdot(\rho-\rho')}$$
$$\times\sum_{p,q,r,s}\sum_{k_\perp,k'_\perp}\phi^c(x-pa)\phi^v(x-qa)\phi^c(x'-ra)\phi^v(x'-sa)e^{i(k_\perp pa-k'_\perp qa-k_\perp ra+k'_\perp sa)} \quad (4)$$

where

$$\varepsilon^v_{\mathbf{k}_{||}}=\frac{\hbar^2\mathbf{k}^2_{||}}{2m^*},\quad \varepsilon^c_{\mathbf{k}_{||}}=\frac{\hbar^2\mathbf{k}^2_{||}}{2m^*}+E_g\ .$$

The four terms in this expression arise from the two intersubband and two intrasubband contributions, respectively.

We Fourier transform this expression with respect to $\rho-\rho'$ and simplify using the fact that there is negligible overlap of the wave functions between wells to obtain

$$P(q_{||},x,x',\omega)=P^{vv}(\mathbf{q}_{||},\omega)\sum_s A^{vv}(x-sa)A^{vv}(x'-sa)+P^{cv}(q_{||},\omega)\sum_s A^{cv}(x-sa)A^{cv}(x'-sa)\ , \quad (5)$$

where

$$A^{vv}(x-sa)=\phi^v(x-sa)\phi^v(x-sa)\ , \quad (5a)$$

$$A^{cv}(x-sa)=\phi^c(x-sa)\phi^v(x-sa)\ , \quad (5b)$$

$$P^{vv}(q_{||},\omega)=\frac{2}{\Omega}\sum_{\mathbf{k}_{||}}\frac{n_{\mathbf{k}_{||}}-n_{\mathbf{k}_{||}+\mathbf{q}_{||}}}{\varepsilon_{\mathbf{k}_{||}}-\varepsilon_{\mathbf{k}_{||}+\mathbf{q}_{||}}-\hbar\omega-i\delta}\ , \quad (5c)$$

and

$$P^{cv}(\mathbf{q}_{||},\omega)=\frac{2}{\Omega}\sum_{\mathbf{k}_{||}}\frac{n_{\mathbf{k}_{||}}}{\varepsilon_{\mathbf{k}_{||}}-\varepsilon_{\mathbf{k}_{||}+\mathbf{q}_{||}}-E_g-\hbar\omega-i\delta}-\frac{2}{\Omega}\sum_{\mathbf{k}_{||}}\frac{n_{\mathbf{k}_{||}+\mathbf{q}_{||}}}{\varepsilon_{\mathbf{k}_{||}}+E_g-\varepsilon_{\mathbf{k}_{||}+\mathbf{q}_{||}}-\hbar\omega-i\delta}\ . \quad (5d)$$

$P^{vv}(q_{||},\omega)$ is the usual intraband polarization propagator for a two-dimensional electron gas while $P^{cv}(q_{||},\omega)$ is an interband propagator. The summation involved in (5c) is evaluated in the usual manner by transformation into an integral. The problem reduces to evaluating

$$P^{vv}(q_{||},\omega)=\frac{2}{(2\pi)^2}\frac{k_f^2}{\hbar q v_f}\left\{\int_0^1 dx\int_{-\pi}^{+\pi}d\theta\frac{x}{-x\cos\theta-z-u-i\alpha}-\int_0^1 dx\int_{-\pi}^{+\pi}d\theta\frac{x}{-x\cos\theta+z-u-i\alpha}\right\}, \quad (6)$$

where

$$x=\frac{k}{k_f},\quad z=\frac{q_{||}}{2k_f},\quad u=\frac{\omega}{q_{||}v_f}\ ,$$

which have been performed in the literature.[8] In a similar fashion one can write

$$P^{cv}(q_{||},\omega)=\frac{2}{(2\pi)^2}\frac{k_f^2}{\hbar q v_f}\left\{\int_0^1 dx\int_{-\pi}^{+\pi}d\theta\frac{x}{-x\cos\theta-z-u-E_g/\hbar q_{||}v_f-i\alpha}-\int_0^1 dx\int_{-\pi}^{+\pi}d\theta\frac{x}{-x\cos\theta+z-u+E_g/\hbar q_{||}v_f-i\alpha}\right\}, \quad (7)$$

which is identical to the expression for $P^{vv}(q_{||},\omega)$ but with z replaced by $z+E_g/\hbar q_{||}v_f$.

Using the convolution theorem we now express ϵ as

$$\epsilon(x,x',\mathbf{q}_{\|},\omega)=\delta(x-x')-\int v(q_{\|},x,x'')P(x'',x',q_{\|},\omega)dx''$$
$$=\delta(x-x')-\sum_{s,\nu}\int \frac{2\pi e^2}{q_{\|}}e^{-q_{\|}|x-x''|}[P(q_{\|},\omega)A(x'',sa)]^{\nu}A^{\nu}(x'-sa)dx'' , \tag{8}$$

where ν is an index denoting either (vv) or (cc). This is now in a suitable form to use the real-space inversion technique.

INVERSION OF THE DIELECTRIC-RESPONSE FUNCTION AND THE PLASMON MODES

We proceed to invert using the expression for the inverse of a separable matrix (Ortuno and Inkson)

$$\underline{A}=\underline{1}-\underline{B}\,\underline{C}\Longrightarrow A^{-1}=\underline{1}+\underline{B}(\underline{1}-\underline{C}\,\underline{B})^{-1}\underline{C} , \tag{9}$$

which gives for ϵ^{-1}

$$\epsilon^{-1}(x,x',q_{\|},\omega)=\delta(x-x')+\sum_{\substack{s,t\\ \nu,\nu'}}\int v(q_{\|},x,x'')[P(q_{\|},\omega)A(x''-sa)]^{\nu}Q_{st}^{\nu\nu'}A^{\nu}(x'-ta)dx'' , \tag{10}$$

where

$$Q^{-1}{}_{st}^{\nu\nu'}=\delta_{st}^{\nu\nu'}-\int\int[P(q_{\|},\omega)A(x-sa)]^{\nu}v(q_{\|},x,x')A^{\nu'}(x'-ta)dx\,dx' . \tag{10a}$$

This step reduces the problem of calculating ϵ^{-1} from inversion of ϵ which has continuous labels to inversion of $\underline{Q}^{-1}{}_{st}^{\nu\nu'}$ which has discrete labels and is usually more localized than ϵ^{-1}. We further simplify the inversion problem by imposing Born—von Kármán boundary conditions in the superlattice direction.

An illuminating way to write out $Q^{-1}{}_{st}^{\nu\nu'}$ is to treat it as a matrix Q_{st}^{-1} but with each element replaced by a 2×2 matrix. Using the fact that $A^{\nu}(x)=0$ for $x>a/2$ and $x<-a/2$, we find the following tight-binding-like structure:

$$\begin{bmatrix} x^{\nu\nu'} & e^{-qa}y^{\nu\nu'} & e^{-2qa}y^{\nu\nu'} & \cdots & e^{-2qa}z^{\nu\nu'} & e^{-qa}z^{\nu\nu'} \\ e^{-qa}z^{\nu\nu'} & x^{\nu\nu'} & e^{-qa}y^{\nu\nu'} & \cdots & & e^{-2qa}z^{\nu\nu'} \\ e^{-2qa}z^{\nu\nu'} & e^{-qa}z^{\nu\nu'} & x^{\nu\nu'} & \cdots & & \\ \vdots & \vdots & \vdots & & & e^{-qa}y^{\nu\nu'} \\ e^{-qa}y^{\nu\nu'} & e^{-2qa}y^{\nu\nu'} & & & e^{-qa}z^{\nu\nu'} & x^{\nu\nu'} \end{bmatrix} , \tag{11}$$

where

$$x^{\nu\nu'}=\delta^{\nu\nu'}-P^{\nu}(\mathbf{q}_{\|},\omega)\frac{2\pi e^2}{q_{\|}}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}A^{\nu}(x)e^{-q|x-x'|}A^{\nu'}(x')dx\,dx' , \tag{11a}$$

$$y^{\nu\nu'}=-P^{\nu}(q_{\|},\omega)\frac{2\pi e^2}{q_{\|}}\int_{-a/2}^{a/2}A^{\nu}(x)e^{qx}dx\int_{-a/2}^{a/2}A^{\nu'}(x')e^{-qx'}dx' \tag{11b}$$

and

$$z^{\nu\nu'}=-P^{\nu}(q_{\|},\omega)\frac{2\pi e^2}{q_{\|}}\int_{-a/2}^{a/2}A^{\nu}(x)e^{-qx}dx\int_{-a/2}^{a/2}A^{\nu'}(x')e^{qx'}dx' . \tag{11c}$$

The matrix $Q_{st}^{\nu\nu'-1}$ can be case into block diagonal form using a unitary transformation. We define

$$\delta^{\nu\nu'}B_{t,k}=\frac{1}{\sqrt{N}}e^{ikta}\delta^{\nu\nu'} , \tag{12}$$

where $k=2\pi n/Na$ and is restricted in the usual way to lie in the first Brillouin zone of the superlattice. Transforming Q^{-1} this gives

$$\underline{B}^{\dagger}\underline{Q}^{-1}\underline{B}=\delta_{k,k'}\Gamma^{-1\nu\nu'}(k) , \tag{13}$$

where

$$\underline{\Gamma}^{-1\nu\nu'}=\underline{X}^{\nu\nu'}+\underline{Y}^{\nu\nu'}\sum_{r=1}^{\infty}e^{-(qa-ikra)r}+\underline{Z}^{\nu\nu'}\sum_{r=1}^{\infty}e^{-(qa+ika)r} . \tag{13a}$$

On performing the summation we obtain

$$\underline{\Gamma}^{-1\nu\nu'}=\underline{X}^{\nu\nu'}+\underline{Y}^{\nu\nu'}\frac{e^{-qa+ika}}{1-e^{-qa+ika}}+\underline{Z}^{\nu\nu'}\frac{e^{-qa-ika}}{1-e^{-qa-ika}} . \tag{13b}$$

Inversion of $\underline{B}^{\dagger}\underline{Q}^{-1}\underline{B}$ is now simply a matter of inverting each of the 2×2 submatrices down the block diagonal, and we obtain for $\underline{Q}_{st}^{\nu,\nu'}$

$$\underline{Q}_{st}^{\nu\nu'}=\sum_k\underline{B}_{s,k}\underline{\Gamma}^{\nu\nu'}(k,\omega)\underline{B}_{k,t}^{\dagger} . \tag{14}$$

Substituting into our expression for ϵ^{-1} we find

$$\epsilon^{-1}=\delta(x-x')+\sum_{\substack{v,v'\\ s,k,t}}\int v(q_{\parallel},x,x'')[P(q_{\parallel},\omega)A(x''-sa)]^{v}B_{sk}\Gamma^{vv'}(k,q_{\parallel},\omega)B^{*}_{k,t}A^{v'}(x'-ta)dx'' \ . \tag{15}$$

The energy structure of ϵ^{-1} is now contained in the matrix $\Gamma^{vv'}$. The plasmon modes are given by the values of $k,q_{\parallel},\omega$ for which ϵ^{-1} becomes infinite or when $\det(\Gamma^{-1}(q_{\parallel},k,\omega))=0$

If we restrict ourselves to the case when only the lower band is of importance, $\underline{X}$, $\underline{Y}$, and $\underline{Z}$ reduce to simple scalars. In addition $Y=Z$ and so we can write the condition for plasmons as

$$0=1-\frac{2\pi e^2}{q_{\parallel}}P^{vv}(q_{\parallel},\omega)\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}A^{vv}(x)e^{-q_{\parallel}|x-x'|}A^{vv}(x')dx\,dx'$$
$$-\frac{2\pi e^2}{q_{\parallel}}P^{vv}(q_{\parallel},\omega)\left\{\frac{\sinh(q_{\parallel}a)}{\cosh(q_{\parallel}a)-\cos(ka)}-1\right\}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}A^{vv}(x)e^{-q_{\parallel}(x-x')}A^{vv}(x')dx\,dx \ . \tag{16}$$

Specializing further to the case where $A^{vv}(x)=\delta(x)$, all the integrals are unity and we derive the well-known result[1]

$$0=1-\frac{2\pi e^2}{q_{\parallel}}P^{vv}(q_{\parallel},\omega)\frac{\sinh(q_{\parallel}a)}{\cosh(q_{\parallel}a)-\cos(ka)} \ . \tag{17}$$

In the small $q_{\parallel}$ limit $P^{vv}(q_{\parallel},\omega)$ is given as

$$P^{vv}(q_{\parallel},\omega)=\frac{N_s q_{\parallel}^2}{m^*\omega^2} \ , \tag{18}$$

where N_s is electron density per unit area, which yields for ω^2

$$\omega^2=\frac{2\pi N_s e^2 q_{\parallel}}{m^*}\frac{\sinh(q_{\parallel}a)}{\cosh(q_{\parallel}a)-\cos(ka)} \ . \tag{19}$$

Similar arguments can be applied when the response is dominated by interband transitions. The equation analogous to (16) for this case is

$$0=1-\frac{2\pi e^2}{q_{\parallel}}P^{cv}(q_{\parallel},\omega)\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}A^{cv}(x')e^{-q_{\parallel}|x-x'|}A^{cv}(x)dx\,dx'$$
$$-\frac{2\pi e^2}{q_{\parallel}}P^{cv}(q_{\parallel},\omega)\left[\frac{\sinh(q_{\parallel}a)}{\cosh(q_{\parallel}a)-\cos(ka)}-1\right]\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}A^{cv}(x)e^{-q_{\parallel}(x-x')}A^{cv}(x')dx\,dx' \ . \tag{20}$$

Following the notation of Bloss[1] we find, working to first order in $q_{\parallel}$,

$$\frac{2\pi e^2}{q_{\parallel}}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}A^{cv}(x)e^{-q_{\parallel}|x-x'|}A^{cv}(x')dx\,dx'\approx 2\pi e^2 l_{10}-2\pi e^2 q_{\parallel}z_{10}^2 \tag{21a}$$

and

$$\frac{2\pi e^2}{q_{\parallel}}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}A^{cv}(x)e^{-q_{\parallel}(x-x')}A^{cv}(x')dx\,dx'\simeq -2\pi e^2 q_{\parallel}z_{10}^2 \ , \tag{21b}$$

where

$$l_{10}=\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}A^{cv}(x)\,|x-x'|\,A^{cv}(x')dx\,dx' \tag{21c}$$

and

$$z_{10}=\int_{-a/2}^{a/2}A^{cv}(z)z\,dz \ . \tag{21d}$$

The small $q_{\parallel}$ form of P^{cv} is

$$P^{cv}(q_{\parallel},\omega)=\frac{2N_s E_g}{\omega^2-E_g^2} \ . \tag{21e}$$

Combining all these terms together we obtain for the plasma frequency

$$\omega^2\approx E_g^2+4\pi N_s e^2 E_g l_{10}\left[1-\frac{q_{\parallel}z_{10}}{l_{10}}\times\left[\frac{\sinh(q_{\parallel}a)}{\cosh(q_{\parallel}a)-\cos(ka)}\right]\right] \ , \tag{22}$$

which is identical to Bloss's equation 15.

In Fig. 1 we show the kind of results for the plasma frequencies which Eqs. (16) and (20) yield. We have assumed in the calculation that

$$\phi^v(x)=0, \quad x>a/4 \text{ and } x<-a/4 \ ,$$

$$\phi^v(x)=\frac{2}{\sqrt{a}}\cos\left[\frac{2\pi x}{a}\right], \quad -a/4<x<a/4 \ ,$$

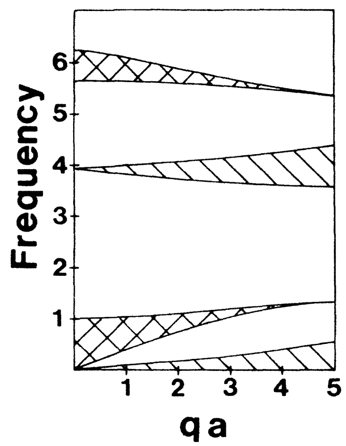

FIG. 1. Theoretical calculation of the plasmon dispersion relation of a superlattice. The frequency is expressed in units of classical plasma frequency. See the text for values of energy gap, etc., used in calculation. The lowest band represents the region where it is possible for an electron to undergo a real excitation process within the conduction band. The next band shows the plasmon dispersion relation resulting from the lower subband. The third curve is for real excitations across the gap. The final band shows the intersubband plasmon dispersion relation.

and

$$\phi^c(x)=0,\quad x>a/4 \text{ and } x<-a/4\;,$$

$$\phi^c(x)=\frac{2}{\sqrt{a}}\sin\left[\frac{4\pi x}{a}\right],\quad -a/4<x<a/4\;.$$

We have taken the value of the superlattice spacing to be 400 Å, the electron density to be 10^{12} electrons/cm^2, the energy of the gap to be 20 meV, the static dielectric response to be 13, and the effective mass of the electrons to be 1. The computation was done using the full expressions for P^{vv} and P^{cv} and analytic forms for the integrals involved in (6) and (7). Our results are very similar to those given by Bloss (1983) and Tselis and Quinn[9] for the upper band and qualitatively the same as those given by Giuliani, Qin, and Quinn for the lower subband.

However this calculation does not give the exact zeros of the determinant of $(\Gamma^{-1})^{\nu\nu'}$

$$0=(\Gamma^{-1})^{11}(\Gamma^{-1})^{22}-(\Gamma^{-1})^{12}(\Gamma^{-1})^{21}\;. \tag{23}$$

The approximations involved essentially take the determinant of (Γ^{-1}) to be

$$\det(\Gamma^{-1})=(\Gamma^{-1})^{11}(\Gamma^{-1})^{22}\;, \tag{23a}$$

and then find the zeros of $\det(\Gamma^{-1})$ by setting each of $(\Gamma^{-1})^{11}$ and $(\Gamma^{-1})^{22}$ equal to zero. We would hope to obtain a slightly better approximation to the plasma frequencies by including the effects of $(\Gamma^{-1})^{12}$ and $(\Gamma^{-1})^{21}$ in the computation. This calculation has been done, and we found that the effect of the extra two terms for the above values of electron density, energy gap, etc; was to slightly depress the frequencies of the lower band and raise those of the upper, but everywhere the effect was very small.

CONCLUSIONS

We have here a method of inverting the dielectric response of a superlattice which gives as its poles the plasmon frequencies. The formalism can easily deal with effects due to the finite widths of the quantum wells and can also treat the case where there is more than one subband of importance. More important, however, is that we now have an expression for ϵ^{-1} in closed form. The inverse response function is at the center of most interaction effects.[7] The electron-electron and electron-phonon both depend explicitly upon ϵ^{-1}. The electron self-energy can also be related to the inverse response function through the screened electron-electron interaction, and a knowledge of this quantity leads to the quasiparticle properties and nonlocal potential corrections to the band structure.

It should be pointed out that recently Jain and Allen[10] have calculated an expression for the density correlation function which is closely related to the inverse response function (7), assuming δ function electron density profiles. Our inverse response function reduces to this expression in the appropriate limit.

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