PHYSICAL REVIEW B VOLUME 33, NUMBER 1 1 JANUARY 1986

Amorphization of the Ising ferromagnet with a transverse field

T. Kaneyoshi
Department of Physics, Nagoya University, 464 Nagoya, Japan
(Received 11 February 1985)

The amorphization of a crystalline Ising ferromagnet with a transverse field is investigated by the use of the effective-field theory with correlations. It is proposed that a number of interesting phenomena may be observable in the effect of a transverse field on the thermodynamic quantities for amorphization, namely the longitudinal and transverse magnetizations and the initial parallel susceptibility, although at the present time we do not have any experimental results.

I. INTRODUCTION

The Ising model with a transverse field was originally introduced by de Gennes[1] as a variable model for hydrogen-bonded ferroelectrics such as the KH_2PO_4 type. Since then, it has been applied to several other systems; for example, cooperative Jahn-Teller systems like $DyVO_2$ (Ref. 2) and ferromagnets with strong uniaxial anisotropy in a transverse magnetic field.[3] The model Hamiltonian is given by

$$\mathscr{H}=-\sum_i \Omega_i S_i^x - \tfrac{1}{2}\sum_{i,j} J_{ij} S_i^z S_j^z ,$$

where S_i^x and S_i^z are spin-$\frac{1}{2}$ operators at site i, Ω_i represents a transverse field, J_{ij} is an exchange interaction, and the sums extend over the points of a lattice.

In two or more dimensions the transverse Ising model has a finite transition temperature, which can be depressed to zero temperature by increasing the transverse field to a critical value Ω_c. An exact solution for the one-dimensional case has been obtained,[4] where no phase transition is verified at finite temperature, but at $T=0$ the system is ordered for Ω less than some critical value Ω_c. Thus, the critical frontier starts at some Ω_c for $T=0$, and ends at the Ising critical point for $\Omega=0$, and it separates the paramagnetic region ($\langle S^z\rangle=0$) from the ferromagnetic one ($\langle S_i^z\rangle\neq 0$) by a second-order phase transition. At all temperatures, however, there is an order with $\langle S_i^x\rangle\neq 0$.

In the last decade, there has been interest in the problem of disorder in the transverse Ising model, which may apply to KD_2PO_4-KH_2PO_4 mixed systems[5] and diluted vanadates.[6] The bond- and site-diluted transverse Ising models have been studied by a variety of sophisticated techniques.[7–9] For the diluted systems, attention has been, in particular, directed to the Harris conjecture[10] that the critical transverse field as a function of concentration at zero temperature should display discontinuity at the percolation concentration.

On the other hand, the magnetism of structurally disordered alloys has become a subject of both experimental and theoretical interest in solid-state physics. A number of experimental and theoretical investigations have led to the conclusion that long-range magnetic order may exist in amorphous systems. At the same time, because of the disordered structure, many interesting physical properties not observed in the corresponding crystalline magnets are now becoming apparent.[11]

Theoretically, there exist a great number of sophisticated techniques for discussing amorphous magnets. Because of the difficulties inherent in theoretical description of such complicated magnetic systems, it is sometimes necessary to make some simplifications. For studying such systems, therefore, the lattice model has often been applied, in which the structural disorder is replaced by a random distribution of the exchange integral.[12]

In a series of works[13] we have investigated the amorphization of pure and diluted crystalline Ising ferromagnets by using both the effective-field theory with correlations (EFT) (Ref. 14) and the lattice model of amorphous magnets. Due to amorphization, some interesting effects on the relevant thermodynamic quantities appear in the thermal behavior; the susceptibility shows the effect of the eventual coexistence, in the system, of an infinite cluster with finite clusters. The magnetization exhibits reentrant phenomena as a function of temperature for selected values of the structural fluctuation. Except for this special case, showing the reentrant phenomena, the reduced magnetization curve falls below that of the corresponding crystalline ferromagnets, a phenomenon which is generally observed in amorphous and dilute ferromagnets.[11]

In this paper the effect of a transverse field on the amorphization of a pure crystalline Ising ferromagnet is investigated, within the same framework as used in the previous cases,[13] in order to clarify how the relevant thermodynamic quantities, namely the phase diagram, magnetization, and initial susceptibility, are influenced by a transverse field. From these investigations, a number of interesting effects of amorphization appear in the thermal behavior; the reduced longitudinal magnetization curve for a small value of Ω falls below that of the corresponding crystalline ferromagnet upon increasing the structural fluctuation, a phenomenon generally observed in amorphous ferromagnets with $\Omega=0$. For a large value of Ω, on the other hand, the behavior of the reduced magnetization curve is different from that for a small value of Ω. Only for small values of Ω may the longitudinal magnetization exhibit the effect of frustration, namely the reentrant phenomena. Thus, for a small value of Ω, we find some characteristic behavior in the thermodynamic quantities. For a mixture of weak bonds, in particular, the longitudinal magnetization indicates the eventual coexistence of weakly coupled spins in the infinite strongly coupled cluster. The initial parallel susceptibility exhibits a peculiar behavior below the transition temperature, in connection with the existence of weakly coupled spins.

The outline of this paper is as follows. In Sec. II we present the formulation of this model in the EFT. In Sec. III the theory is applied to the amorphization of the crystalline ferromagnet in a square lattice, in order to discuss the effect of a transverse field on the amorphization. In Sec. IV the numerical results are shown and discussed. In Sec. IV A the results of the results of the pure system are compared with those obtained from the standard mean-field approximation and other techniques, in order to clarify the present framework.

II. FORMULATION

The system consists of N identical spins, which is now described by the following Hamiltonian:

$$\mathscr{H} = -\sum_i \Omega_i S_i^x - \tfrac{1}{2}\sum_{i,j} J_{ij} S_i^z S_j^z - \sum_i H S_i^z , \quad (1)$$

where S_i^x and S_i^z are defined as before and H is the applied magnetic field. J_{ij} is the exchange interaction between spins at sites i and j. In order to describe the structural disorder in a simple way, the lattice model of amorphous magnets is used; the nearest-neighbor exchange interactions are given by independent random variables as

$$P(J_{ij}) = \tfrac{1}{2}[\delta(J_{ij} - J - \Delta J) + \delta(J_{ij} - J + \Delta J)] . \quad (2)$$

In a series of works on the amorphization of pure and diluted crystalline Ising ferromagnets with $\Omega_i = 0$,[13] the starting point for the statistics of the systems was the exact relations due to Callen.[15] However, when we include the transverse field in the problems, or when we take the Hamiltonian (1), we cannot apply the Callen identities to the present problem. For the Ising model in a transverse field, as discussed in the Appendix, the expectation values including the longitudinal and transverse spin operators S_i^x and S_i^z at a site i are approximately given by[16]

$$\langle \{f_i\} \mu_i^z \rangle = \left\langle \{f_i\} \frac{\theta_i}{H_i} \tanh(\tfrac{1}{4}\beta H_i) \right\rangle , \quad (3)$$

$$\langle \{f_i\} \mu_i^x \rangle = \left\langle \{f_i\} \frac{2\Omega_i}{H_i} \tanh(\tfrac{1}{4}\beta H_i) \right\rangle , \quad (4)$$

where $\langle \cdots \rangle$ indicates the canonical thermal average, $\mu_i^z = 2S_i^z$, $\mu_i^x = 2S_i^x$, and $\beta = 1/k_B T$. θ_i and H_i are defined by

$$\theta_i = 2H + \sum_j J_{ij} \mu_j^z \quad (5)$$

and

$$H_i = (4\Omega_i^2 + \theta_i^2)^{1/2} . \quad (6)$$

The parameter $\{f_i\}$ represents any function of the Ising variables so long as it is not a function of the site i. In the limit of $\Omega_i = 0$, $\langle \{f_i\} \mu_i^x \rangle = 0$, $H_i = \theta_i$, and Eq. (3) reproduces the Callen identity for the Ising model.

In particular, setting $\{f_i\} = 1$, the equations reduce to the longitudinal and transverse magnetizations for the Ising model in a transverse field. Expanding the functions appearing on the right-hand sides of Eqs. (3) and (4) as a formal series in the spin variables and neglecting correlations of H_i, the mean-field-approximation (MFA) result is also obtained;[17]

$$\sigma_i^z = \langle \mu_i^z \rangle = \left[\frac{\sum_j J_{ij} \sigma_j^z}{H_i^{\mathrm{MFA}}} \right] \tanh(\tfrac{1}{4}\beta H_i^{\mathrm{MFA}}) , \quad (7)$$

$$\sigma_i^x = \langle \mu_i^x \rangle = \left[\frac{2\Omega_i}{H_i^{\mathrm{MFA}}} \right] \tanh(\tfrac{1}{4}\beta H_i^{\mathrm{MFA}}) , \quad (8)$$

with

$$H_i^{\mathrm{MFA}} = \left[4\Omega_i + \left[\sum_j J_{ij} \sigma_j^z \right]^2 \right]^{1/2} \quad \text{for } H = 0 .$$

Thus, the relations (3) and (4) approximately derived are expected to give fairly nice results for small values of Ω_i (for more detailed discussions, see following sections).

In order to write Eqs. (3) and (4) in a form which is particularly amenable to approximation, let us introduce the differential-operator technique[18] as follows:

$$\langle \{f_i\} \mu_i^z \rangle = \left\langle \{f_i\} \exp\left[D \sum_j J_{ij} \mu_j^z \right] \right\rangle F_i(x) \bigg|_{x=0} , \quad (9)$$

$$\langle \{f_i\} \mu_i^x \rangle = \left\langle \{f_i\} \exp\left[D \sum_j J_{ij} \mu_j^z \right] \right\rangle G_i(x) \bigg|_{x=0} , \quad (10)$$

with

$$\left\langle \{f_i\} \exp\left[D \sum_j J_{ij} \mu_j^z \right] \right\rangle = \left\langle \{f_i\} \prod_j [\cosh(DJ_{ij}) + \mu_j^z \sinh(DJ_{ij})] \right\rangle ,$$

where the functions $F_i(x)$ and $G_i(x)$ are given by

$$F_i(x) = \frac{2H + x}{[4\Omega_i^2 + (2H+x)^2]^{1/2}} \times \tanh\left[\frac{\beta}{4} [4\Omega_i^2 + (2H+x)^2]^{1/2} \right] , \quad (11)$$

$$G_i(x) = \frac{2\Omega_i}{[4\Omega_i^2 + (2H+x)^2]^{1/2}} \times \tanh\left[\frac{\beta}{4} [4\Omega_i^2 + (2H+x)^2]^{1/2} \right] , \quad (12)$$

and $D = \partial/\partial x$ is a differential operator.

When the formulation is applied to an isotropic lattice, the operator functions can be generally written as

$$\left\langle \{f_i\} \exp\left[D \sum_j J_{ij} \mu_j^z \right] \right\rangle = K(D) + L(D) , \quad (13)$$

where $K(D)$ and $L(D)$ are given by even and odd functions of the operator D; for a square lattice with nearest-neighbor interactions, the functions are

$$K(D)=\langle\{f_i\}\rangle\left[\prod_{\delta=1}^{4}\cosh(DJ_{i+\delta})\right]+\frac{1}{2!}\sum_{\delta=1}^{4}\sum_{\delta'=\delta}\langle\{f_i\}\mu_{i+\delta}^z\mu_{i+\delta'}^z\rangle\sinh(DJ_{i+\delta})\sinh(DJ_{i+\delta'})$$
$$\times\left[\prod_{l(\neq\delta,\delta')}\cosh(DJ_{i+l})\right]+\left\langle\{f_i\}\prod_{\delta=1}^{4}\mu_{i+\delta}\right\rangle\left[\prod_{\delta=1}^{4}\sinh(DJ_{i+\delta})\right] \tag{14}$$

and

$$L(D)=\sum_{\delta=1}^{4}\langle\{f_i\}\mu_{i+\delta}^z\rangle\sinh(DJ_{i+\delta})\left[\prod_{\delta(\neq l)}\cosh(DJ_{i+\delta})\right]+\frac{1}{3!}\sum_{\delta=1}^{4}\sum_{\delta'(=\delta)}\sum_{\delta''(\neq\delta,\delta')}\langle\{f_i\}\mu_{i+\delta}^z\mu_{i+\delta'}^z\mu_{i+\delta''}^z\rangle$$
$$\times\sinh(DJ_{i+\delta})\sinh(DJ_{i+\delta'})\sinh(DJ_{i+\delta''})$$
$$\times\left[\prod_{l(\neq\delta,\delta',\delta'')}\cosh(DJ_{i+l})\right], \tag{15}$$

where the subscripts δ refer to the nearest-neighbor sites of the ith site. Equations (9) and (10) can generate many kinds of equations for spin-correlation functions, upon substituting appropriate Ising variable functions for $\{f_i\}$. Among them, upon setting $\{f_i\}=1$, Eqs. (9) and (10) reduce to

$$\sigma_i^z=\langle\mu_i^z\rangle=\left\langle\exp\left[D\sum_j J_{ij}\mu_j^z\right]\right\rangle F_i(x)\Big|_{x=0}$$
$$=L(D)\bar{F}_0(x)\,|_{x=0}+HK(D)\bar{F}_1(x)\,|_{x=0} \tag{16}$$

and

$$\sigma_i^x=\langle\mu_i^x\rangle=\left\langle\exp\left[D\sum_j J_{ij}\mu_j^z\right]\right\rangle G_i(x)\Big|_{x=0}$$
$$=K(D)\bar{G}_0(x)\,|_{x=0}+HL(D)\bar{G}_1(x)\,|_{x=0}\,, \tag{17}$$

where the last lines of Eqs. (16) and (17) were derived by expanding $F_i(x)$ and $G_i(x)$ with H and retaining terms linear in H. The functions $\bar{F}_0(x)$ and $\bar{G}_0(x)$ are given by setting $H=0$ into $F_i(x)$ and $G_i(x)$. The functions $\bar{F}_1(x)$ and $\bar{G}_1(x)$ are defined by

$$\bar{F}_1=\frac{\partial F_i(x)}{\partial H}\Big|_{H=0}$$

and

$$\bar{G}_1=\frac{\partial G_i(x)}{\partial H}\Big|_{H=0}\,.$$

Now, in a quenched random-bond magnet, the disorder lies in the exchange bonds; the equations given above are then the expressions for a particular configuration of exchange bonds, and hence it is necessary to take the random average $\langle\cdots\rangle_r$ over all possible bond configurations. However, it is clear that, if we try to treat exactly all the spin-spin correlations present in Eqs. (16) and (17), and to perform properly the configurational averages which are still to be done, the problem becomes mathematically intractable. We shall therefore proceed as follows; as discussed in the previous works,[13,14,19] we take the configurational average of both sides of Eqs. (16) and (17), then completely decouple the multispin-correlation functions, namely

$$\langle\langle\mu_i^z\mu_j^z\cdots\mu_l^z\rangle\rangle_r\cong\langle\langle\mu_i^z\rangle\rangle_r\langle\langle\mu_j^z\rangle\rangle_r\cdots\langle\langle\mu_l^z\rangle\rangle_r$$

and use the fact that the distribution laws associated with different bonds are independent of each other. Within these approximations it is clear that the strict criticality of the system is lost (in particular, the critical exponents are going to be the classical ones, and the real dimensionality of the system is only partially taken into account through the coordination number z). Nevertheless, the present framework is, as already mentioned in a number of works,[13,14,19] quite superior to the standard MFA; this point has been verified in several models[13,19] and, for the present one, will be exhibited further on.

For the ferromagnetic square lattice with random nearest-neighbor interactions, the averaged magnetizations then satisfy, upon using Eqs. (14) and (15),

$$m^z=\langle\sigma_i^z\rangle_r$$
$$=4A_1m^z+4A_2(m^z)^3+H[B_1+6B_2(m^z)^2+B_3(m^z)^4] \tag{18}$$

and

$$m^x=\langle\sigma_i^x\rangle_r$$
$$=C_1+6C_2(m^z)^2+C_3(m^z)^4\quad\text{for } H=0\,, \tag{19}$$

where the coefficients A_i $(i=1,2)$, B_i $(i=1-3)$, and C_i $(i=1-3)$ are given by

$$A_1=[\langle\cosh(DJ_{i,i+\delta})\rangle_r]^3\langle\sinh(DJ_{i,i+\delta})\rangle_r\bar{F}_0(x)\,|_{x=0}\,,$$
$$A_2=[\langle\sinh(DJ_{i,i+\delta})\rangle_r]^3\langle\cosh(DJ_{i,i+\delta})\rangle_r\bar{F}_0(x)\,|_{x=0}\,,$$
$$B_1=[\langle\cosh(DJ_{i,i+\delta})\rangle_r]^4\bar{F}_1(x)\,|_{x=0}\,,$$
$$B_2=[\langle\cosh(DJ_{i,i+\delta})\rangle_r]^2[\langle\sinh(DJ_{i,i+\delta})\rangle_r]^2$$
$$\times\bar{F}_1(x)\,|_{x=0}\,,$$
$$B_3=[\langle\sinh(DJ_{i,i+\delta})\rangle_r]^4\bar{F}_1(x)\,|_{x=0}\,,$$

and (20)

$$C_1=[\langle\cosh(DJ_{i,i+\delta})\rangle_r]^4\overline{G}_0(x)\,|_{x=0}\,,$$

$$C_2=[\langle\cosh(DJ_{i,i+\delta})\rangle_r]^2[\langle\sinh(DJ_{i,i+\delta})\rangle_r]^2\times\overline{G}_0(x)\,|_{x=0}\,,$$

$$C_3=[\langle\sinh(DJ_{i,i+\delta})\rangle_r]^4\overline{G}_0(x)\,|_{x=0}\,.$$

In the Ising model with a transverse field, at high temperatures the S^z components are disordered, and at temperatures below a critical value T_c an ordered phase is set up with $m^z\neq 0$, although in all temperature regions there exists an order with $m^x\neq 0$. Therefore, it is interesting to study the behavior of the initial parallel susceptibility in connection with the phase change. The initial parallel susceptibility $\chi_{\parallel}$ is defined by

$$\chi_{\parallel}=\lim_{H\to 0}\left[\frac{\partial m^z}{\partial H}\right]. \tag{21}$$

In this section we have discussed the effective-field theory with correlations in a transverse Ising ferromagnet with random bonds. In the following sections, for simplicity, we shall study the thermodynamic quantities of the amorphization of the transverse Ising ferromagnet in a square lattice using this framework.

III. AMORPHIZATION OF THE FERROMAGNETIC SQUARE LATTICE

By means of Eq. (2), the random-bond averages are then given by

$$\langle\cosh(DJ_{i,i+\delta})\rangle_r=\cosh(2DJ\delta)\cosh(DJ)\,,$$
$$\langle\sinh(DJ_{i,i+\delta})\rangle_r=\cosh(2DJ\delta)\sinh(DJ)\,, \tag{22}$$

where the parameter δ is defined by $\delta=\Delta J/2J$. As mentioned in Ref. 13, the result (22) can also be obtained by using the so-called "lattice model" of amorphous magnets. In the model, the parameter δ is called the structural fluctuation and measures the amount of fluctuation of the exchange interactions coming from the structural disorder in amorphous magnets.

For $H=0$ the averaged magnetization m^z in the amorphization of the ferromagnetic square lattice is given by, from Eq. (18),

$$m^z=\left[\frac{1-4A_1}{4A_2}\right]^{1/2}. \tag{23}$$

On the other hand, the averaged transverse magnetization m^x can be evaluated from Eq. (19), upon substituting Eq. (23) into it. Therefore, the critical ferromagnetic frontiers can be derived from the condition

$$4A_1-1=0\,, \tag{24}$$

by which the phase diagrams and transition temperatures can be determined as functions of $\Omega_i=\Omega$ and δ. Then, by applying a mathematical relation $e^{\alpha D}f(x)=f(x+\alpha)$, the coefficients A_i, B_i, and C_i in Eqs. (23) and (19) can be expressed as a sum of the functions $\overline{F}_0(x)$, $\overline{F}_1(x)$, and $\overline{G}_0(x)$ with an appropriate argument x. For instance, the coefficient A_1 is given by

$$A_1=(\tfrac{1}{2})^7\{\overline{F}_0(4J+8J\delta)+4\overline{F}_0(4J+4J\delta)+6\overline{F}_0(4J)+4\overline{F}_0(4J-4J\delta)+\overline{F}_0(4J-8J\delta)+2[\overline{F}_0(2J+8J\delta)+4\overline{F}_0(2J+4J\delta)+6\overline{F}_0(2J)+4\overline{F}_0(2J-4J\delta)+\overline{F}_0(2J-8J\delta)]\}\,. \tag{25}$$

Differentiating both sides of Eq. (18) with H, we obtain the initial parallel susceptibility as

$$\chi_{\parallel}=\frac{B_1+6B_2(m^z)^2+B_3(m^z)^4}{1-4A_1-12A_2(m^z)^2}\,. \tag{26}$$

We are now in a position to examine numerically the physical properties of the amorphization of the Ising ferromagnetic square lattice with a transverse field. The numerical results will be given in the next section.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section the effects of a transverse field on the physical properties of the amorphization of the ferromagnetic square lattice are studied numerically. Before discussing the effects, however, it will be worthwhile to examine the pure Ising ferromagnetic square lattice with a transverse field within the present formulation, in order to clarify our results and compare them with those obtained from the MFA and other methods. Therefore, in Sec. IV A we show the numerical results for the pure Ising ferromagnetic square lattice with a transverse field, and then the effects of a transverse field on the amorphization are examined in Secs. IV B and IV C. In particular, some new phenomena for the amorphization are obtained in Sec. IV C.

A. Pure case

As has been discussed by many authors, when Ω increases from zero, the transition temperature T_c falls from its value in the Ising model and reaches zero at a critical value Ω_c. Therefore, in order to compare our result with those of other works, the critical temperature is plotted against transverse field Ω in Fig. 1 by solving Eq. (24) for the pure system with $\delta=0$. In Fig. 1 the results obtained from the MFA, the high-temperature-series expansion (SE),[20] and the renormalization-group[8] (RG) results are also depicted. For comparison, the critical

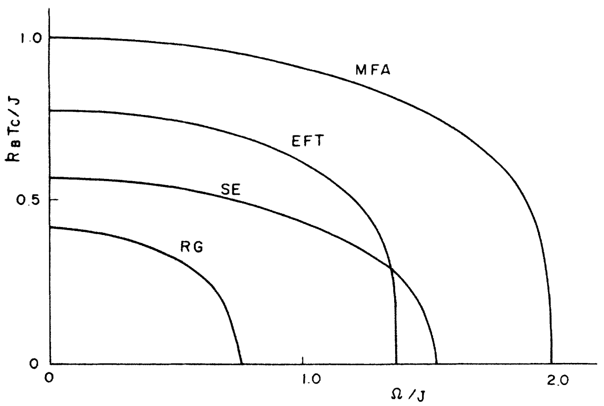

FIG. 1. Transition temperature for the pure ($\delta=0$) square lattice plotted as a function of transverse field Ω. EFT denotes the result for the present approximation. For comparison, the standard molecular-field approximation (MFA), series expansion (SE), and real-space renormalization-group (RG) results are also plotted.

values of Ω_c obtained from such methods are collected in Table I. As seen from the figure, within the present formulation, near the critical transverse field we obtain a considerably more pronounced rate of decrease of T_c with Ω than the corresponding decrease predicted by SE. The result comes from the starting formulas (3) and (4) we took; in deriving (3) and (4) we introduced a decoupling approximation (A6) (see Appendix), as if the contribution from the transverse field could be treated as a "mean-field" type of contribution. However, the overall behavior of our result (EFT) is considerably better than that of RG, in comparison with those of SE. Moreover, for the special case of the pure Ising model ($\Omega=0$), Eq. (24) reduces to

$$\tanh(\beta J)+2\tanh\left[\frac{\beta J}{2}\right]=2\ , \tag{27}$$

which is nothing but the equation obtained from the Zernike approximation,[21] as discussed in Refs. 13 and 19. The transition temperature of (27) is determined to be $4k_BT_c/J=3.0898$, which is to be compared with the exact result $4k_BT_c/J=2.2692$,[22] as well as $4k_BT_c/J=2.8854$ of the Bethe-Peierls approximation (the MFA leads to $4k_BT_c/J=4$). Thus, the present formulation based on the approximate relations (3) and (4) is expected to give fairly good results for small values of Ω. In the following, therefore, the numerical results are obtained for the values of Ω less than Ω_c.

In Figs. 2 and 3 the thermal behavior of the longitudinal and transverse magnetizations (σ^z and σ^x) and the initial parallel susceptibility are presented for some values of Ω, by solving Eqs. (23), (19), and (26) with $\delta=0$. The results clearly show that the larger the transverse field, the smaller the longitudinal magnetization σ^z; the role of the transverse field is essentially to inhibit the ordering of the S^z component. Thus, the values of the initial parallel susceptibility and transverse magnetization σ^x increase even at $T=0$, upon increasing the value of Ω. The initial parallel susceptibility diverges at each transition temperature, in connection with the phase transition.

TABLE I. Critical values of Ω.

	RG	EFT	SE	MFA
Ω_c/J	0.77	1.37	1.52	2.0

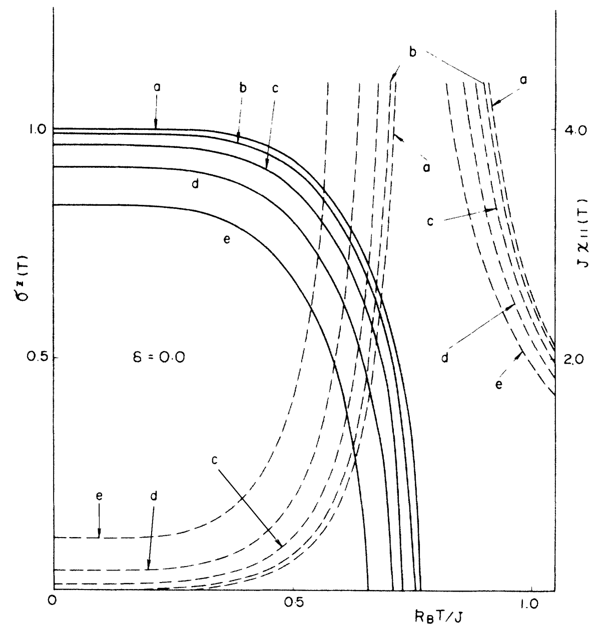

FIG. 2. Thermal behavior of longitudinal magnetization σ^z (solid line) and initial parallel susceptibility $\chi_{\parallel}$ (dashed line) for the pure ($\delta=0$) ferromagnetic square lattice with selected values of Ω: (a) $\Omega=0.1J$, (b) $\Omega=0.3J$, (c) $\Omega=0.5J$, (d) $\Omega=0.7J$, and (e) $\Omega=0.9J$

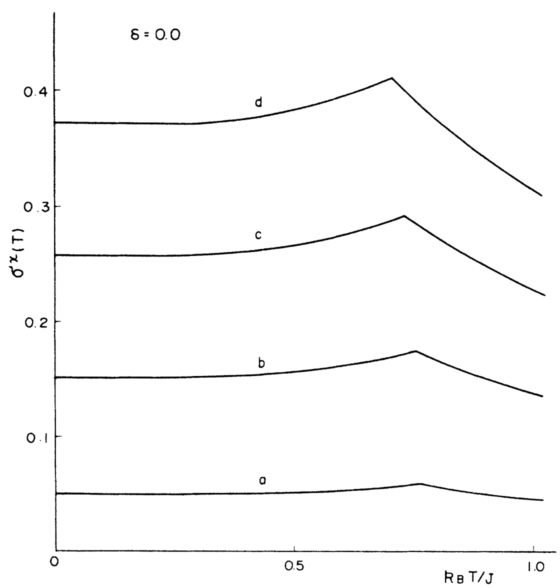

FIG. 3. Thermal behavior of transverse magnetization σ^x for the pure ($\delta=0$) ferromagnetic square lattice with the corresponding values of Ω in Fig. 2: (a) $\Omega=0.1J$, (b) $\Omega=0.3J$, (c) $\Omega=0.5J$, and (d) $\Omega=0.7J$.

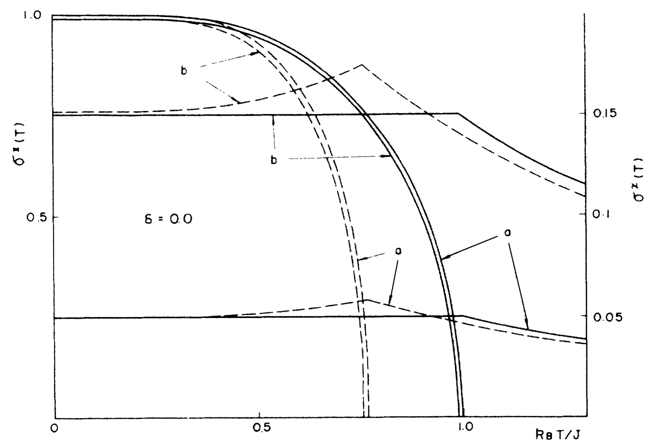

FIG. 4. Thermal dependences of longitudinal and transverse magnetizations (σ^z and σ^x) for the pure ($\delta=0$) ferromagnetic square lattice with (a) $\Omega=0.1J$ and (b) $\Omega=0.3J$. The solid lines are the results of the MFA and the dashed lines are the present results (EFT).

In Fig. 4 the MFA results (σ^z and σ^x) for two selected values of Ω ($=0.1J$ and $0.3J$) are shown for the sake of comparison. As seen from the figure, the present results for σ^x are qualitatively different from the MFA behavior; below the transition temperature of σ^z the transverse magnetization σ^x is very sensitive to temperature, in contrast with the constant behavior predicted by the MFA, namely Eqs. (7) and (8). Our effective-field results should be compared with the Monte Carlo calculations reported by Prelovsek and Sega[23] for the special case of spin ∞, which show similar behavior (see Fig. 3 of Ref. 23).

B. Amorphization

In a series of works[13] we have studied the amorphization of the Ising ferromagnetic square lattice with $\Omega=0$. In this subsection we will investigate the effects of a transverse field on the amorphization.

By solving Eq. (24), the critical frontiers in the (T,δ) space are plotted in Fig. 5 for selected values of Ω. In a series of works with no transverse field,[13] we discussed that two possible different values of T_c exist for a given value of δ in the range $0.5<\delta<0.565$ (the dotted curve in Fig. 5); for $\delta>0.5$ one bond in (2) becomes negative, so that the effect of frustration may appear in the system. If we admit the existence of a spin-glass phase slightly below the two T_c's, the result may support the reentrant phenomenon, that is, the transition from the spin-glass phase to the paramagnetic phase passing through the ferromagnetic one is possible. On the other hand, as seen from Fig. 5, the possibility of the reentrant phenomenon is at first preserved for small values of Ω, but it disappears at a value near $\Omega=0.6J$; if the reentrant phenomenon exists in a system, the result implies that when we apply an appropriate transverse field to the system, there exists a critical transverse field at which the reentrant phenomenon disappears.

By solving Eq. (23), the thermal behavior of m^z for two systems with $\Omega=0.1J$ and $1.0J$ is depicted in Figs. 6 and 7, upon changing the value of δ. As seen from Fig. 6, for the system with $\delta=0.55$ in Fig. 5, the magnetization, which does not exist until a certain temperature, starts to increase, passes through a maximum value, and decreases to zero with increasing temperature; as mentioned above, the reentrant phenomena may be obtained for $\Omega=0.1J$, although it is impossible for $\Omega=1.0J$, as shown in Fig. 7.

In Fig. 8 the reduced magnetization curves of m^z are depicted for the two systems ($\Omega=0.1J$ and $1.0J$). For the small value of Ω ($\Omega=0.1J$), as observed in the amorphization of Ising ferromagnets with $\Omega=0$,[13] the reduced

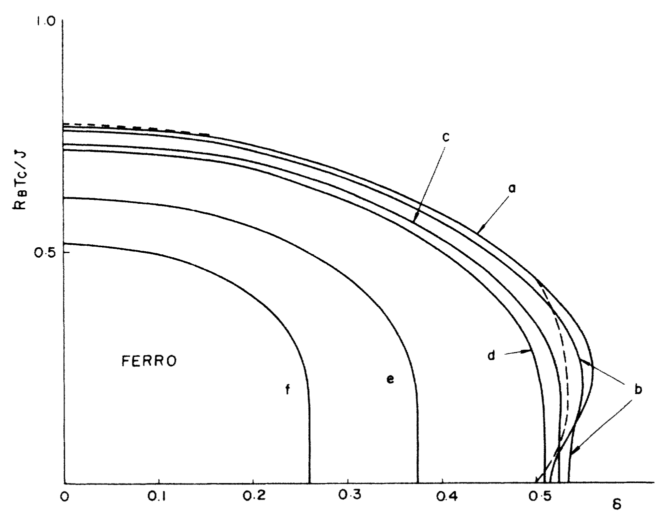

FIG. 5. Phase diagrams in the (T,δ) space for the amorphization of the transverse Ising ferromagnetic square lattice with selected values of Ω: (a) $\Omega=0.1J$, (b) $\Omega=0.3J$, (c) $\Omega=0.5J$, (d) $\Omega=0.6J$, (e) $\Omega=1.0J$, and (f) $\Omega=1.2J$. The dashed line is the result for $\Omega=0$ (Ref. 13).

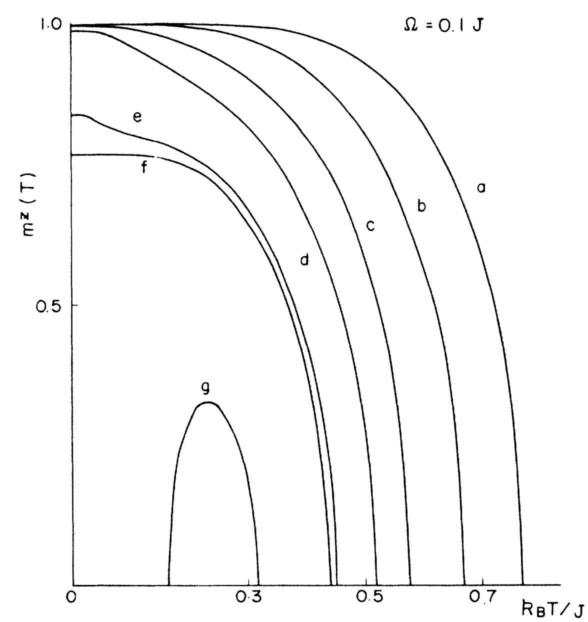

FIG. 6. Thermal dependences of the average magnetization m^z for the amorphization of the ferromagnetic square lattice with $\Omega=0.1J$: (a) $\delta=0$, (b) $\delta=0.3$, (c) $\delta=0.4$, (d) $\delta=0.45$, (e) $\delta=0.495$, (f) $\delta=0.5$, and (g) $\delta=0.55$.

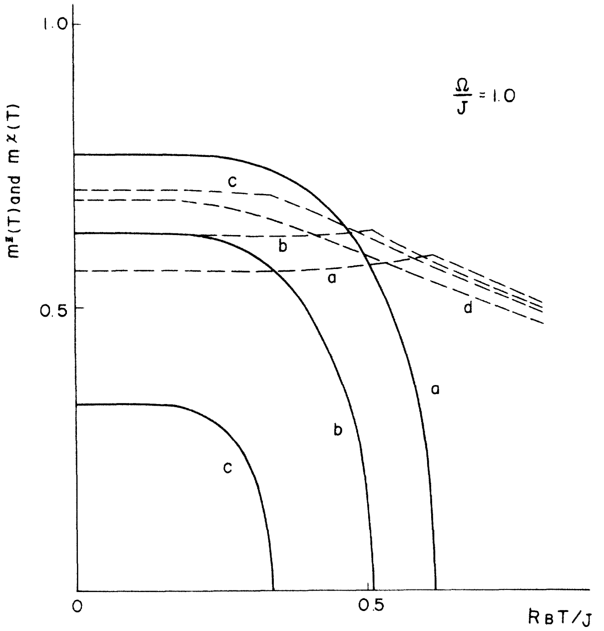

FIG. 7. Thermal dependences of the averaged magnetizations m^z (solid line) and m^x (dashed line) for the amorphization of the ferromagnetic square lattice with $\Omega=1.0J$: (a) $\delta=0.0$, (b) $\delta=0.25$, (c) $\delta=0.35$, and (d) $\delta=0.55$.

magnetization curve falls below that of the corresponding crystalline ferromagnet with $\delta=0$ upon increasing the structural fluctuation δ (except $\delta=0.5$, which corresponds to bond dilution), a phenomenon which is, in general, observed in amorphous ferromagnets.[11] For the large value of Ω ($\Omega=1.0J$), however, the behavior is completely different from that for the small value of Ω; the reduced magnetization curve ($\delta=0.35$) is above that of $\delta=0$.

By solving Eqs. (23) and (19), in Figs. 9 and 7 the thermal dependences of m^x are depicted for the two systems ($\Omega=0.1J$ and $1.0J$), upon changing the value of δ. The curve labeled $\delta=0.5$ in Fig. 9 corresponds to the bond-diluted system with the bond concentration $p=\frac{1}{2}$. Curve (d) in Fig. 7 decreases monotonically with increasing temperature, since the system is in the paramagnetic state for whole temperature range.

In Fig. 10 the thermal dependences of the initial parallel susceptibility (or inverse parallel susceptibility) for $\Omega=0.1J$ are shown by solving Eq. (26) for the corresponding values of δ in Fig. 6. Above the transition temperature the inverse parallel susceptibility changes almost linearly with temperature, except for the very narrow region near T_c, independent of the value of δ we take. On the other hand, in Fig. 11 the thermal dependences of the inverse parallel susceptibility for $\Omega=1.0J$ are depicted for the corresponding values of δ in Fig. 7. For a large value of Ω, however, the deviation from linearity in the vicinity of T_c is more clearly observed than that for a small value of Ω, when the structural fluctuation δ increases. Such a deviation from linearity in the vicinity of T_c was also observed in the bond- (or site-) diluted Ising model with a transverse field.[9] Curve (d) in Fig. 11 corresponds to the paramagnetic phase, so that the inverse parallel susceptibility does not go to zero.

As shown in Figs. 6 and 10, the thermal behavior of m^z and $\chi_{\parallel}$ expresses some characteristic behavior when the structural fluctuation δ increases from $\delta=0.4$ to $\delta=0.5$. For instance, the curve labeled $\delta=0.495$ in Fig. 6 at first decreases rather rapidly from the saturation value at $T=0$, and then follows the curve of $\delta=0.5$. This type of behavior was also seen more clearly in the amorphization

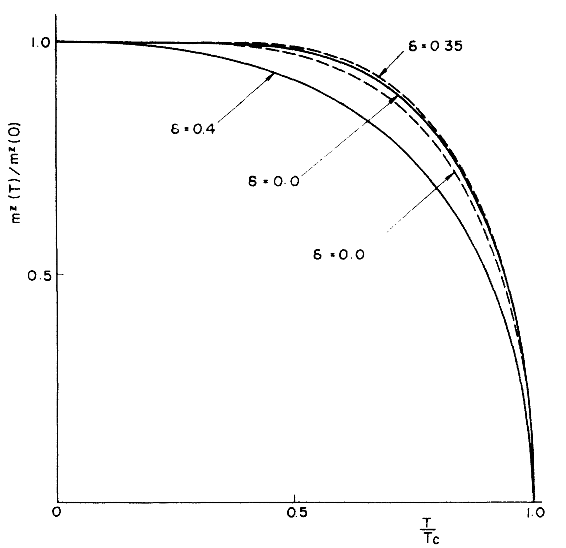

FIG. 8. Reduced magnetization curves of m^z for the two systems with $\Omega=0.1J$ (solid line) and $\Omega=1.0J$ (dashed line).

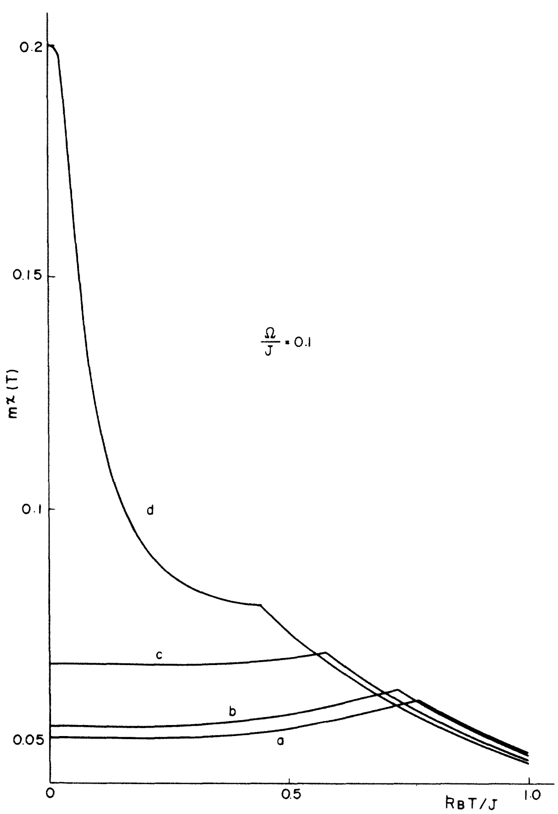

FIG. 9. Thermal dependences of m^x for the system $\Omega=0.1J$ with (a) $\delta=0.1$, (b) $\delta=0.3$, (c) $\delta=0.4$, and (d) $\delta=0.5$.

of the Ising ferromagnet with $\Omega=0$ (Ref. 13) or in the bond-mixing Ising ferromagnet with $\Omega=0$ (Ref. 24); in the bond-mixed Ising ferromagnet, the probability distribution function $P(J_{ij})$ is, instead of (2), given by

$$P(J_{ij})=P\delta(J_{ij}-J)+(1-P)\delta(J_{ij}-J') ,$$

where P is the concentration of the bond mixture. When we define a parameter α as $\alpha=J'/J$, the amorphization discussed in this work corresponds to $P=\frac{1}{2}$ and

$$\alpha=\frac{1-2\delta}{1+2\delta} .$$

As mentioned above, $\delta=0.5$ just corresponds to the bond-dilution problem at $P=\frac{1}{2}$. That the structural fluctuation δ is near the value of $\delta=0.5$ indicates a mixture of very weak bonds in the system. Therefore, the characteristic behavior of m^z seen in the curve labeled $\delta=0.495$ implies that two types of spins exist, the weakly coupled spins and the spins in the infinite cluster. As discussed in the previous work with $\Omega=0$,[24] the existence of weakly coupled spins in the infinite strongly coupled cluster has shown some particularly interesting behavior of the thermodynamic quantities. In the following we investigate the effect of a transverse field on the characteristic behavior of some thermodynamic quantities, namely longitudinal and transverse magnetizations (m^z and m^x) and the initial parallel susceptibility.

C. New phenomena

Now, let us investigate in detail the system with $\Omega=0.1J$ for values of δ in the ranges $0.4<\delta<0.5$ and $\delta>0.5$ since, as shown in Fig. 5, the critical frontier in the (T,δ) space becomes narrow upon increasing the value of Ω, and for a value larger than $\Omega=0.6J$ it is impossible to find the ferromagnetic phase in the vicinity of $\delta=0.5$.

As shown in Fig. 12, when the value of δ (α) changes from $\delta=0.4$ ($\alpha=\frac{1}{9}$) to $\delta=0.5$ ($\alpha=0$), the longitudinal magnetization m^z rapidly decreases from the saturation value at $T=0$ and then follows the magnetization curve of $\delta=0.5$ ($\alpha=0$); as mentioned above, the result implies that two types of spins exist, the weakly coupled spins and the spins in the infinite cluster. The initial susceptibility diverges at each critical temperature. However, when the value of δ (α) changes from $\delta=0.4$ ($\alpha=\frac{1}{9}$) to $\delta=0.5$ ($\alpha=0$), the initial susceptibility exhibits a peculiar behavior below its critical temperature, in connection with the characteristic behavior of m^z; for instance, the curve labeled (c) expresses a maximum and a minimum below its critical temperature. Moreover, the behavior of m^x for $\Omega=0.1J$ is depicted in Fig. 13. As shown in the figure, the transverse magnetization m^x also expresses a characteristic behavior below each transition temperature of m^z; in particular, the results labeled $\delta=0.42$ and 0.45 show weak minima below the transition temperatures. Thus, as discussed in the previous works with $\Omega=0$,[13,24] the peculiar behavior of magnetization and susceptibility is closely related to the appearance of the weakly coupled spins in the system, upon decreasing the value of α toward zero, although the effects of a transverse field on the quantities are very different from those of systems with $\Omega=0$.

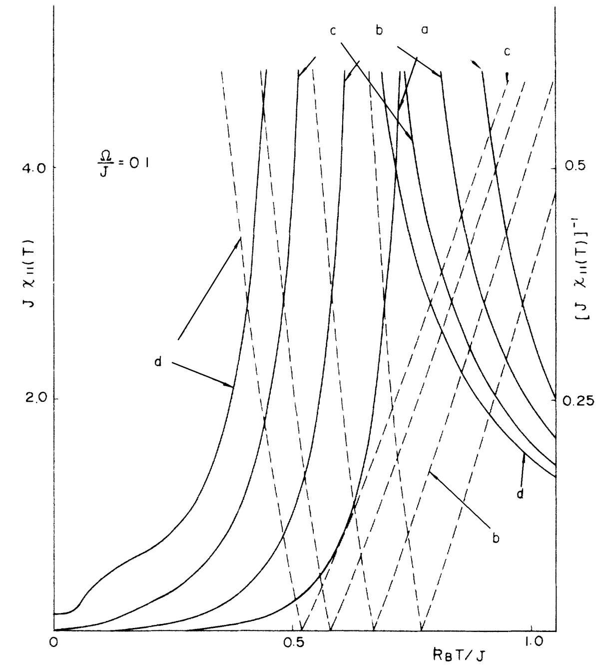

FIG. 10. Thermal dependences of $\chi_{\parallel}$ (solid line) and $\chi_{\parallel}^{-1}$ (dashed line) for the system $\Omega=0.1J$ with (a) $\delta=0.0$, (b) $\delta=0.3$, (c) $\delta=0.4$, and (d) $\delta=0.45$.

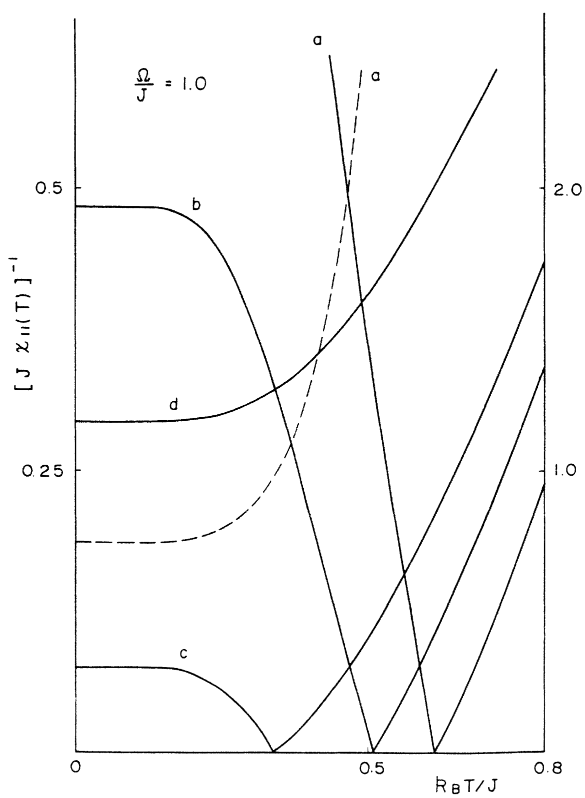

FIG. 11. Thermal dependences of $\chi_{\parallel}^{-1}$ (solid line) and $\chi_{\parallel}$ (dashed line) for the system $\Omega=1.0J$ with (a) $\delta=0.0$, (b) $\delta=0.25$, (c) $\delta=0.35$, and (d) $\delta=0.55$.

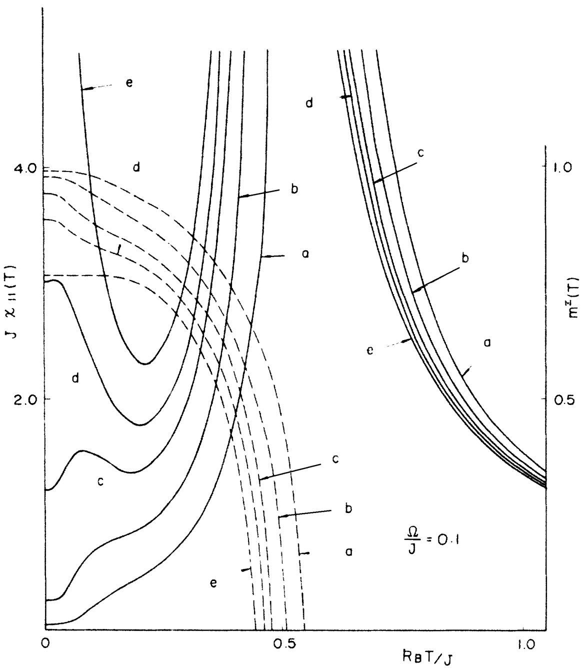

FIG. 12. Averaged magnetization m^z (dashed line) and parallel susceptibility (solid line) versus temperature for the system $\Omega=0.1J$ with (a) $\delta=0.43$, (b) $\delta=0.46$, (c) $\delta=0.48$, (d) $\delta=0.49$, and (e) $\delta=0.5$.

As discussed above, when the value of δ becomes larger than 0.5, one bond in (2) becomes negative. The effect of frustration starts to appear in the system with small values of Ω; as shown in Fig. 5, the reentrant phenomenon is obtained, for instance, for the system $\Omega=0.1J$ with $\delta=0.55$. Therefore, in Fig. 14, the thermal behavior of m^z, m^x, and the inverse parallel susceptibility is depicted

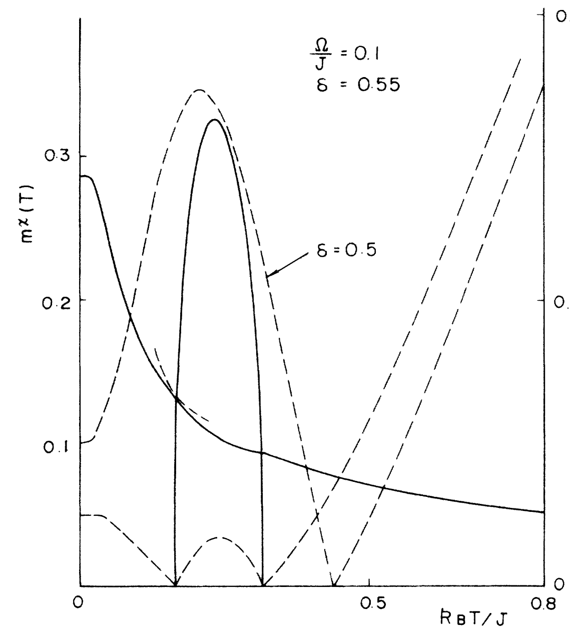

FIG. 14. Thermal dependences of m^x (solid line) and $\chi_{\parallel}^{-1}$ (dashed line) for the system $\Omega=0.1J$ with $\delta=0.55$. The behavior of $\chi_{\parallel}^{-1}$ for the system $\Omega=0.1J$ with $\delta=0.5$ is also depicted.

for the system $\Omega=0.1J$ with $\delta=0.55$. The initial susceptibility diverges at two transition temperatures at which the magnetization m^z starts to appear. The transverse magnetization m^x exhibits two kinks at the transition temperatures of m^z.

Finally, the thermal behavior of m^x and $\chi_{\parallel}$ for $\Omega=0.1J$ are depicted in Fig. 15 for selected values of δ, namely 0.6, 0.8, and 1.0, for which the magnetization m^z

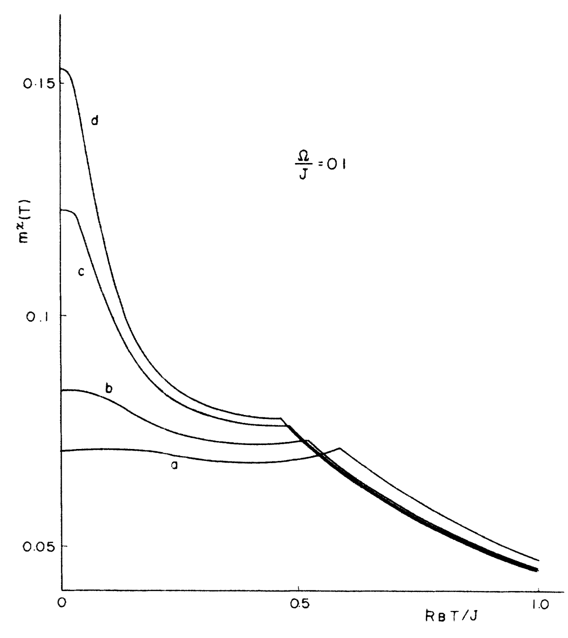

FIG. 13. Averaged magnetization m^x versus temperature for the system $\Omega=0.1J$ with (a) $\delta=0.42$, (b) $\delta=0.45$, (c) $\delta=0.48$, and (d) $\delta=0.49$.

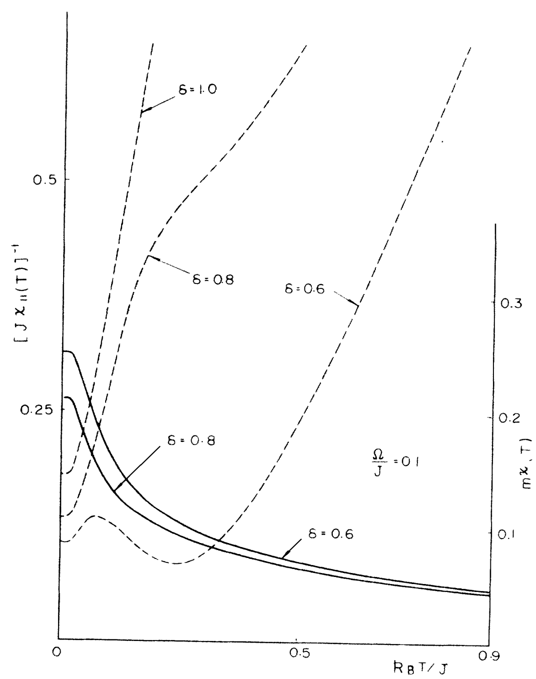

FIG. 15. Thermal dependences of $\chi_{\parallel}^{-1}$ (dashed line) and m^x (solid line) for the system $\Omega=0.1J$ with $\delta=0.6$, 0.8, and 1.0.

reduces to $m^z=0$. As seen from the figure, even if $m^z=0$, the initial susceptibility expresses a peculiar behavior for $\delta=0.6$ and 0.8, although the transverse magnetization m^x decreases monotonically with temperature; the results for $\delta=0.6$ and 0.8 may be related to the reoccurrence of reentrant phenomena. In fact, the result of $\chi_{\parallel}$ for $\delta=0.1$ does not express such a behavior and is similar to curve (d) in Fig. 7.

V. CONCLUSIONS

In this work we have studied the amorphization of the Ising ferromagnetic square lattice with a transverse field by starting from relations (3) and (4) and using the effective-field theory with corrections introduced by the present author. In Secs. II and III we have proved that relations (3) and (4) derived approximately give fairly nice results, especially for small values of Ω, and thermodynamic quantities beyond the MFA can be obtained.

In this work some interesting effects on the amorphization for the transverse Ising ferromagnetic square lattice arise in the thermal behavior. As far as we know, the results obtained in Sec. IV C are the first. The longitudinal magnetization may exhibit reentrant phenomena only for small values of Ω. Upon increasing the structural fluctuation δ, the reduced magnetization curves for a small value of Ω fall below those of the corresponding crystalline ferromagnet with $\delta=0$, a phenomenon which is generally observed in amorphous ferromagnets with $\Omega=0$. However, for a large value of Ω, the behavior of the reduced magnetization curve is different from that for a small value of Ω. For a value of δ in the range $0.4<\delta<0.5$, the longitudinal magnetization for a small value of Ω indicates the eventual coexistence of weakly coupled spins in the infinite strongly coupled cluster. In the range of δ, the initial parallel susceptibility for a small value of Ω exhibits a peculiar behavior below the transition temperature of m^z, in connection with the existence of weakly coupled spins. Thus, we would like to propose that a number of interesting phenomena may be included in the effect of a transverse field on the amorphization of a crystalline Ising ferromagnet, although at the present time we do not have any experimental observation.

APPENDIX

In this appendix, we derive Eqs. (3) and (4) following Ref. 16. The expectation value including the longitudinal or transverse spin operator at a site i is given by

$$\langle \{f_i\} S_i^\alpha \rangle = \mathrm{Tr}[\{f_i\} S_i^\alpha \exp(-\beta\mathcal{H})]/\mathrm{Tr}[\exp(-\beta\mathcal{H})] , \tag{A1}$$

where $\alpha=z$ or x. The total Hamiltonian given by (1) can be separated into two parts,

$$\mathcal{H}=\mathcal{H}_i+\mathcal{H}' , \tag{A2}$$

where $\mathcal{H}_i$ includes all parts of $\mathcal{H}$ associated with the lattice site i,

$$\mathcal{H}_i=-\left[H+\sum_j J_{ij}S_j^z\right]S_i^z-\Omega_i S_i^x , \tag{A3}$$

and $\mathcal{H}'$ represents the remaining part of the Hamiltonian and does not contain spin variables on the site i. By noticing that $\mathcal{H}_i$ and $\mathcal{H}'$ do not commute, we can obtain the following result for the expectation value,

$$\langle \{f_i\} S_i^\alpha \rangle = \left\langle \{f_i\} \frac{\mathrm{Tr}_{\{i\}} S_i^\alpha \exp(-\beta\mathcal{H}_i)}{\mathrm{Tr}_{\{i\}} \exp(-\beta\mathcal{H}_i)} \right\rangle - \left\langle \{f_i\} \left[\frac{\mathrm{Tr}_{\{i\}} S_i^\alpha \exp(-\beta\mathcal{H}_i)}{\mathrm{Tr}_{\{i\}} \exp(-\beta\mathcal{H}_i)} - S_i^\alpha \right] \bar{\Delta} \right\rangle , \tag{A4}$$

with

$$\bar{\Delta}=1-\exp(-\beta\mathcal{H}_i)\exp(-\beta\mathcal{H}')\exp[\beta(\mathcal{H}_i+\mathcal{H}')] , \tag{A5}$$

where $\mathrm{Tr}_{\{i\}}$ means the partial trace with respect to lattice site i.

Equation (A4) is an exact relation, although it is difficult to deal with owing to the presence of the second thermal average. Accordingly, let us introduce an approximation as follows:

$$\left\langle \{f_i\} \left[\frac{\mathrm{Tr}_{\{i\}} S_i^\alpha \exp(-\beta\mathcal{H}_i)}{\mathrm{Tr}_{\{i\}} \exp(-\beta\mathcal{H}_i)} - S_i^\alpha \right] \bar{\Delta} \right\rangle \cong \left\langle \{f_i\} \left[\frac{\mathrm{Tr}_{\{i\}} S_i^\alpha \exp(-\beta\mathcal{H}_i)}{\mathrm{Tr}_{\{i\}} \exp(-\beta\mathcal{H}_i)} - S_i^\alpha \right] \right\rangle \langle \bar{\Delta} \rangle , \tag{A6}$$

from which we can obtain a relation

$$\langle \{f_i\} S_i^\alpha \rangle = \left\langle \{f_i\} \frac{\mathrm{Tr}_{\{i\}} S_i^\alpha \exp(-\beta\mathcal{H}_i)}{\mathrm{Tr}_{\{i\}} \exp(-\beta\mathcal{H}_i)} \right\rangle . \tag{A7}$$

The decoupling (A6) can then be viewed as a zeroth-order approximation of the exact relation. In other words, (A7) can be assumed to be obtained from the approximation

$$\langle \{f_i\} (\bar{S}_i^\alpha - S_i^\alpha) \rangle = 0 , \tag{A8}$$

where

$$\bar{S}_i^\alpha = \frac{\mathrm{Tr}_{\{i\}} S_i^\alpha \exp(-\beta\mathcal{H}_i)}{\mathrm{Tr}_{\{i\}} \exp(-\beta\mathcal{H}_i)} .$$

[1]P. G. de Gennes, Solid State Commun. **1**, 132 (1963).
[2]R. J. Elliott, G. A. Gehring, A. P. Malozemoff, S. R. P. Smith, N. S. Staude, and R. N. Tyte, J. Phys. C **4**, L179 (1971).
[3]R. B. Stinchcombe, J. Phys. C **6**, 2459 (1973).
[4]P. Pfeuty, Ann. Phys. (N.Y.) **57**, 79 (1970).
[5]G. Samara, Ferroelectrics **5**, 25 (1973).
[6]U. Schmidt, Z. Phys. **267**, 277 (1974); E. J. S. Lage and R. B. Stinchocombe, J. Phys. C **9**, 3295 (1976); P. Prelovsek and R. Pirc, *ibid.* **10**, 861 (1977).
[7]R. J. Elliott and I. D. Saville, J. Phys. C **7**, 3145 (1974); **7**, 4293 (1974); R. B. Stinchcombe, *ibid.* **14**, L263 (1981); I. Tamura, E. F. Sarmento, and T. Kaneyoshi, *ibid.* **17**, 3207 (1984).
[8]R. R. dos Santos, J. Phys. C **15**, 3141 (1981); J. Phys. A **14**, L179 (1981).
[9]V. K. Saxena, Phys. Rev. B **27**, 6884 (1983).
[10]A. B. Harris, J. Phys. C **7**, 3028 (1974).
[11]T. Kaneyoshi, *Amorphous Magnetism* (Chemical Rubber Co., Boca Raton, FL, 1984).
[12]K. Handrich, Phys. Status Solidi (b) **32**, K55 (1969).
[13]T. Kaneyoshi, R. Honmura, I. Tamura, and E. F. Sarmento, Phys. Rev. B **29**, 5121 (1984); T. Kaneyoshi and I. Tamura, *ibid.* **30**, 359 (1984); **30**, 5246 (1984).
[14]T. Kaneyoshi and H. Beyer, J. Phys. Soc. Jpn. **49**, 1306 (1980).
[15]H. B. Callen, Phys. Lett. **4**, 161 (1963).
[16]E. F. Sarmento, I. Tamura, L. E. M. C. de Oliveira, and T. Kaneyoshi, J. Phys. C **71**, 2195 (1984).
[17]R. Blinc and B. Zeks, Adv. Phys. **21**, 693 (1972).
[18]R. Honmura and T. Kaneyoshi, J. Phys. C **12**, 3979 (1979).
[19]E. F. Sarmento and C. Tsallis, Phys. Rev. B **27**, 5784 (1983); R. Honmura, E. F. Sarmento, and C. Tsallis, Z. Phys. B **51**, 355 (1983); T. Kaneyoshi, I. Tamura, and E. F. Sarmento, Phys. Rev. B **28**, 6491 (1983).
[20]R. J. Elliott and C. Wood, J. Phys. C **4**, 2359 (1971).
[21]F. Zermike, Physica **7**, 565 (1940).
[22]L. Onsager, Phys. Rev. **65**, 117 (1944).
[23]P. Prelovsek and I. Sega, J. Phys. C **11**, 2103 (1978).
[24]T. Kaneyoshi, Z. Phys. B **60**, 35 (1985).