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Critical cutting force between flux vortices in a type-II superconductor

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Plots of the critical current against the square root of the magnetic field obtained from new experiments on thin type-II superconducting films under conditions of parallel magnetic field and current show regularly spaced steps. These novel results are interpreted as being due to the successive entry of planes of parallel vortices into an oscillating flux distribution associated with coherent flux cutting. The results allow the first estimate of the critical cutting force between vortices, a quantity of general significance for the dynamic response of vortices.

The flux-flow mechanism in type-II superconductors, in which the magnetic field and current are applied parallel to each other, has remained unclear in spite of considerable theoretical and experimental investigation in recent years.[1-6] In this "force-free" configuration, the quantized flux vortices which form the magnetic field can exist stably in an arrangement in which the current-induced Lorentz force $\mathbf{F}_J = \mathbf{J} \times \mathbf{B}$ is zero.[1] Any breakdown of this configuration leading to a dissipative voltage parallel to the current must, from the Josephson relation $\mathbf{E} = \mathbf{B} \times \mathbf{v}$, be due to the transport into the superconductor of magnetic flux perpendicular to the applied field $\mathbf{H}_0$. It has been widely appreciated that the continuous entry of this transverse flux without an accompanying buildup of longitudinal flux must be due to some process whereby flux vortices can cut through one another.[1,2,7] In a recent paper[8] we suggested that the complex structure of the critical current versus magnetic field (I_c vs B) plots obtained from experiments on thin superconducting films could be interpreted as evidence for coherent flux oscillations and flux cutting in the plane of the film, in general agreement with the model proposed for bulk behavior by Clem and Yeh.[5] This Rapid Communication reports more recent experimental results which not only confirm this interpretation, but also enable a direct estimate to be made of the cutting force between two inclined vortices. It is suggested that the low value of this force makes it likely that flux cutting is an important breakdown mechanism in certain hard superconductors.

Small sample areas were lithographically defined on $Pb_{64}Tl_{36}$ films as described previously.[8] In the experiment reported in this paper, the film thickness was 0.81 μm. The samples were mounted so that the plane of the film could be accurately aligned with the magnetic field. I_c vs B curves were plotted using a voltage feedback circuit to control the current at a constant voltage (0.6 μV). Plots were obtained at different temperatures by reducing the pressure in the cryostat. Results obtained on samples of the same thickness but different widths show that the critical current scales with the cross-sectional area of the sample, implying that breakdown of the critical state at the sample edges is not a significant effect. Samples mounted so that the plane of the film was normal to the magnetic field showed critical currents several orders of magnitude lower than those shown in the samples in parallel field, indicating that bulk pinning does not contribute significantly to the observed critical-current values.

The results of this experiment are shown in Fig. 1. These curves show a broad peak in the critical current which increases in magnitude and shifts to higher fields as the temperature is reduced. Superimposed on the ascending portion of each curve is a series of steps or subsidiary peaks whose size and position are independent of temperature. This temperature independence implies that the steps are caused not by pinning or by variations in the critical fields, but rather by some mechanism by which the stability of the vortex distribution is determined by the sample geometry. The model which relates these steps to coherent flux oscillations in the film has been described previously, so only a brief outline will be given here. It should also be noted that in the thicker films investigated in the experiments reported here, the behavior accords well with the model for bulk behavior[5] and experimental results on macroscopic samples.[9,10]

According to our model, the flux distribution within the superconductor consists of a series of interacting planes of vortices which are parallel to the planar surfaces of the film.

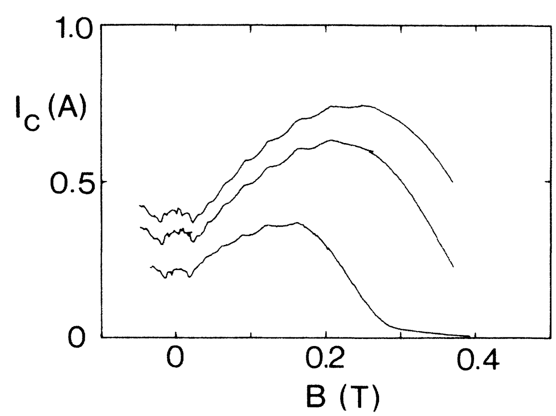

FIG. 1. Critical current against magnetic field curves for 0.81 ± 0.02 μm Pb-36 at.% Tl films. Magnetic field and current applied parallel to each other and to the plane of the film. Sample width = 122 μm, length = 64 μm. Curves were plotted during continuous magnetic field sweeps with current controlled to a 0.6 μV voltage criterion. Temperatures were stable to ±0.1 K during time of experiment. Lowest curve (4.2 K) is plotted as the average of 10 different runs (decreasing and increasing fields), other curves (3.1 K, 2.45 K) are single runs taken with increasing fields.

Those vortices within each plane are parallel to each other; those in the planes nearest the surfaces are inclined at the angle of the surface field (the sum of the applied field H_0 and the self-field of the current). The angle (θ) of the vortices to the applied field H_0 decreases towards the center of the film. The flow mechanism involves the motion of these planes of vortices in a direction normal to the plane of the film. This motion is oscillatory, with adjacent planes moving in opposite directions to each other. Throughout the motion, the interaction between vortices and the surfaces provides stability for these vortex planes; to this extent the ill-defined concept of "surface pinning" may be said to be important. However, geometrical considerations indicate that in the configuration of parallel field and current, the surface interaction cannot support the flux lattice against the oscillatory transport mechanism proposed, rather this support must arise directly from the interactions between the vortices.

At the start of one cycle, the vortices in planes which are approaching one another are inclined at different angles, those in the plane further from the center of the film being inclined at a greater angle to the applied field H_0. When two approaching planes are sufficiently close to one another, coherent cutting and cross joining will be initiated at each vortex intersection (as described in Ref. 8), momentarily resulting in a single double-density plane of parallel vortices inclined at an angle intermediate to the angles of the vortices in the incident planes. These planes are unstable with respect to separation into two new planes of parallel vortices which subsequently repel one another, move in opposite directions, and interact with their adjacent planes in a manner similar to that described above. The cutting and cross joining of these planes will result in a configuration identical to that at the start of the cycle, and hence the process is oscillatory. As a result of this oscillation or shuttling process, there is a net flow of transverse flux towards the center of the film, while there is no accompanying net motion of the longitudinal flux. The mean electric field generated by this flow is therefore dependent on the angle between vortices in adjacent rows and on the shuttling frequency.

Cutting between any two mutually inclined vortices introduces a high degree of local vortex curvature and so vortex motion during the cutting process itself, and also the propagation of cutting throughout the plane of intersection, may be expected to occur at velocities similar to those involved in the dissipation of an elastic instability following a depinning event. Such velocities are 2–3 orders of magnitude in excess of the shuttling velocity of the complete planes calculated on the basis of the experimental results (of the order of 1 ms^{-1}). This, coupled with a consideration of the interlocking of the vortex lattice which results from cutting between vortices in the two incident planes and which persists until cutting has taken place at all intersection points along a given vortex, suggests that the cutting between vortex planes occurs during a much shorter time interval than the time taken for the movement of a complete plane between cutting positions. This allows the modeling of the overall process as a coherent shuttling of the vortex planes between the cutting positions, with the cutting at those positions occurring simultaneously at all crossover points.

At fields above the bulk H_{c1} [the lower critical field (9 mT)[8]], the number of vortices within the film can be directly related to the applied magnetic field ($B=\mu_0 H$). This makes it possible to calculate the fields at which additional planes of vortices enter the shuttling system described above. We will show that these fields can be directly related to the peaks in Fig. 1.

In the shuttling flux system described, the maximum value of θ is the angle of the surface field. The largest experimental value of this angle is of the order of 12° and so we make the simplifying approximation that $\cos\theta=1$ throughout the calculation. Let d be the vortex spacing within the planes of vortices, and assume that D, the average plane spacing (normal to the plane of the film and hence normal to the vortex planes) equals $k^2 d$, where k is a structural constant of order unity (for a triangular Abrikosov lattice of parallel vortices $k^2=2/\sqrt{3}$). The total distance traveled by the vortex planes between cutting points is thus $2D$. We can then relate d to B as

$$B=\phi_0/k^2d^2 \ . \tag{1}$$

If the number of vortex planes within the film equals M, we can rewrite this relation to give

$$B^{1/2}=\phi_0^{1/2}kM/2a \ , \tag{2}$$

where $2a$ is the film thickness. This implies that successive vortex planes enter the shuttling system at a constant spacing $\phi_0^{1/2}k/2a$ of the square root of the field. Their entry affects the behavior of the system, and hence provides an explanation of the subsidiary peak structure. The addition of a new vortex plane would be expected to occur at the same point within each peak. This result shows that the addition of new vortex planes occurs singly, and not in pairs, as was inferred in Ref. 8.

The data of Fig. 1 are plotted against $B^{1/2}$ in Fig. 2; the peak spacing is indeed constant, and the spacing implies a value of k^2 of 0.90; this should be compared with the values of k^2 for triangular and square lattices (0.866 and 1.0, respectively). If this value is substituted into Eq. (2), then the field corresponding to the number of planes within the film can be calculated. For values of M greater than about 4, this field corresponds very closely with the field at the

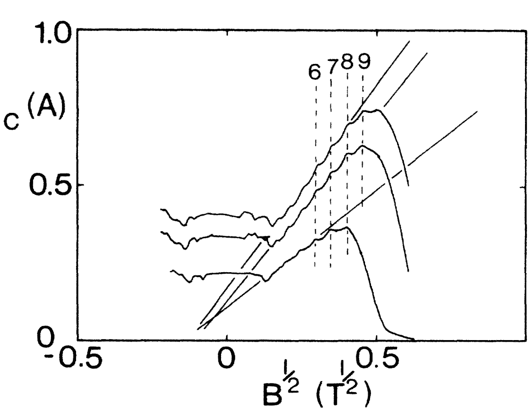

FIG. 2. The data of Fig. 1 replotted as critical current against square root of the magnetic field. The dotted lines are drawn to show the temperature independence of the high field steps; the numbers indicate the number of vortex planes associated with these points. The solid lines indicate the gradient of the linear increase in critical current at the top of each step.

top of each step, implying that these are the points at which additional planes enter the system, and allowing the identification of the number of planes corresponding to each peak as shown in Fig. 2.

It is also clear from the I_c vs $B^{1/2}$ plot in Fig. 2 that the step maxima lie on a straight line whose gradient depends inversely on the temperature. We can use the knowledge of the number of planes within the system to relate this linearity to a constant cutting force per vortex intersection. The small-angle approximation introduced earlier implies that longitudinal flux is approximately conserved in the cutting process, and also that there is a constant difference in the angle θ between vortices in adjacent mutually inclined planes.[11] Coupling this with the relationship between **B** and **H**, already introduced to make the approximation that the magnitude of the induction B is constant throughout the film, allows us to make the further approximation that the gradient of the transverse component of the induction (dB_y/dx) is also a constant. This implies that the transport current density J_z is independent of position within the film.

This current results in a Lorentz force, $\mathbf{f}_J = \mathbf{J} \times \hat{\mathbf{B}}\phi_0$ per unit length, acting on each vortex in a direction normal to the vortex planes. In order to calculate the force between approaching planes which must be overcome in order that cutting may be initiated, we assume a specific angle-independent critical cutting force f_c between two inclined vortices. The force between adjacent planes (inclined at θ_1 and θ_2) may then be calculated by simply counting the number of vortex crossover points. This results in a force on unit length of a single flux line of f_c/l, where l is the distance between intersections measured along that line:

$$l = d \cos\theta_2 / \sin(\theta_2 - \theta_1) = d/(\theta_2 - \theta_1) \quad . \tag{3}$$

Relating this to the difference in Lorentz force on the two lines (which, in the small-angle limit depends only on the Lorentz-force component $f_J = J\phi_0 \sin\theta$), we obtain the relationship between the critical current and magnetic field at the subsidiary maxima

$$J_c = 2 f_c k B^{1/2} / \phi_0^{3/2} \quad . \tag{4}$$

Since this criterion is independent of the angle and position of the vortex planes, it applies to all intersecting vortex planes and clearly predicts a linear variation of I_c with $B^{1/2}$, allowing the estimation of the critical cutting force at the three different temperatures as $f_c = 6.5 \times 10^{-13}$ N ($T = 2.45$ K), 5.9×10^{-13} N ($T = 3.1$ K), 4.5×10^{-13} N ($T = 4.2$ K).

This is the first direct measurement of the critical cutting force between inclined vortices, but an earlier indirect measurement of the same quantity produced a value which was within an order of magnitude of these results.[10] Both the form of the model described above and the weakness of the force itself provide evidence to support the flux-cutting model of Wagenleithner,[6] in which opposed inclined vortices curve strongly in the region of intersection, so as to reduce the repulsive energy between them. Wagenleithner specifically states that for small angles ($< 5°$), the maximum repulsive force f_c is independent of the intersection angle. This has been introduced as a simplifying assumption of the model proposed in this paper, and should be compared with the model for the cutting of straight lines of Ref. 3, which predicts a force which diverges as the intersection angle is reduced. The Ginzburg-Landau parameter κ was equal to 5.8 for the alloy used in the experiment;[11,12] using this and taking a 5° intersection angle, these models[3,6] predict values of f_c of the order of 3×10^{-10} N and 3×10^{-12} N, respectively. Though the second value is considerably closer to the experimental value, it still appears to be an overestimate.

This low experimental value for the cutting force suggests that vortex cutting may be an important effect not just in the force-free configuration, but in more general systems. As an example, the elementary pinning interaction in hard superconductors can exceed 10^{-11} N.[13] The relative weakness of the cutting interaction makes it probable that the breakdown of the critical state in such materials depends not on the unpinning of vortex lines which are pinned, but rather on the cutting of adjacent lines so that the flux lattice can flow round the pinning centers (a possible mechanism for the "pin avoidance" model of Kramer[14]).

The results presented in this paper can be summarized as follows. The equal spacing of the peaks in Fig. 2 provides direct evidence to support the model of coherent flux cutting described above. Increasing the applied magnetic field results in the successive entry of planes of vortices into the system. The linearity of the ascending portion of these curves implies the existence of an angle-independent critical cutting force per vortex intersection and allows a direct estimate of this force.

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