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Surfaces of superfluid ^{3}He-*B*

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The surface textures of superfluid ^{3}He-*B* are studied on the high-energy scale of superfluid condensation energy. With the use of the Ginzburg-Landau theory a new locally stable surface texture is found. It is suggested that this new texture is responsible for the crossover in dissipation regimes observed experimentally by Ling, Betts, and Brewer.

It is an old suspicion that the surfaces of superfluid ^{3}He-*B* may be coated by the *A* phase.[1] There are two recent experiments in which the surfaces may play a decisive role. In the first, Ling, Betts, and Brewer[2] observed different behaviors of dissipation above and below the tricritical pressure. The second is the persistent current experiment by Pekola *et al.*[3,4] Motivated by these experiments I have studied the order parameter of ^{3}He-*B* near surfaces. Using Ginzburg-Landau theory I found, in addition to the conventional surface texture, another local minimum of the free energy. This state has the *B* phase in the bulk but the *A* phase on the surface. This paper is a report of the properties of this new surface texture. In the end I will discuss how these results may apply to the experiments mentioned above.

Consider the following geometry: a surface in the y-z plane and a superfluid occupying the half-space $x > 0$. A phase gradient causes a flow parallel to the surface in the y direction. Otherwise, assume a translationally invariant order parameter both in the y and the z directions. In order to classify the solutions for the order-parameter matrix $A_{\delta j}(x,y,z) = \exp(iky) A_{\delta j}(x)$, it is useful to consider additional symmetries in the same way as Salomaa and Volovik have done in the case of vortices.[5] The possible symmetries are now $P_1 = R_z$, $P_2 = TR_y$, and $P_3 = P_1 P_2$, where R_z (R_y) denotes reflection with respct to the x-y plane (x-z plane) and T complex conjugation. In analogy to the case of vortices, there are five symmetry classes depending on the combinations of P's the order parameter satisfies. The number of real degrees of freedom in each class are the same as well (5, 9, 9, 10, and 18). The new feature is that one can change the phase gradient (current) continuously and even let it go to zero. In the case of zero current, new symmetries can appear. A careful analysis reveals five symmetry classes that are invariant under rotations around the x axis and 12 which are not.

Near the superfluid transition temperature the surface textures can be determined by the Ginzburg-Landau (GL) theory.[6] The GL equations were solved numerically using the most general form for the order parameter (18 real degrees of freedom). The specular reflection boundary condition[7] is assumed below unless explicitly stated otherwise. The strong-coupling β coefficients of Sauls and Serene[8] were used. The numerical values below are for a pressure of 24 bars, well below the tricritical pressure according to the Sauls-Serene β coefficients (~ 28.5 bars).

The standard GL equations can be written as $F_{\delta i}(x,A) = 0$, where the operator F is given by

$$F_{\delta j}(x,A) = -A_{\delta j}(x) + \text{third-order terms} + \text{gradient terms} \ .$$

The numerical method consists of discretizing the x coordinate and iterating the order parameter using the formula

$$A_{\delta j}^{(n+1)}(x_k) = A_{\delta j}^{(n)}(x_k) + CF_{\delta j}(x_k, A^{(n)}) \ ,$$

where C is an appropriate constant. This method is not the most efficient but it is simple to program. Computing times less than 10 sec were always enough.

Consider first the case of vanishing current. Figure 1 displays the two locally stable surface textures that were found. The first is the conventional texture.[9-12] It is the most symmetric state, and the only nonzero components of the order parameter are the real parts of A_{zz} and $A_{yy} = A_{zz}$ (I assume a real diagonal form of the bulk *B*-phase order parameter). Its energy per unit area is $0.76\xi(T) f_B^c$, where $\xi(T)$ is the temperature-dependent coherence length and f_B^c the superfluid condensation energy density of the bulk *B* phase. The energy is measured relative to the imaginary case of bulk *B* phase everywhere. This state is the absolute minimum of the free energy in the GL region.

Figure 1(b) displays the nonzero components of a new surface texture. It has reflection symmetry combined with complex conjugation in both the y and the z directions (TR_y and TR_z). Expecially, the rotational symmetry around the surface normal is broken. The free energy of this state is $1.20\xi(T) f_B^c$. It has the *B* phase in the bulk but the real A_{zz} and the imaginary A_{zy} form an *A* phase on the surface. The l vector points normal to the surface and the d vector is in the z direction. This state seems to be a strong-coupling effect because numerical iteration did not converge to this state at lower pressures (20 bars). This is understandable because the superfluid condensation energy of the *A* phase diminishes at lower pressures compared to the planar state, which covers the surface in the conventional state. On the high-pressure side, the bulk *A* phase nucleates from the new texture immediately at the tricritical point.[13] At 24 bars pressure the local stability was tested by making perturbations to the order parameter that break all the symmetries. For small perturbations the iteration always converged back to the new surface texture.

The diffusiveness of the surface scattering tends to suppress the components $A_{\delta y}$ and $A_{\delta z}$ of the order parameter, so that they extrapolate to zero at a finite distance b behind the wall. This reduces the range of pressures where the new surface texture is locally stable. In preliminary runs at 28 bars, $b = 2\xi(T)$ was the shortest extrapolation length where the new texture was found to be stable. Since for a given diffusivity the extrapolation length is independent of the temperature (near T_c), it follows that at some diffusivities and pressures the new state is locally stable at low temperature but not near T_c.

The flow properties of the two states are very different. In the conventional state the current is roughly what it would be if the bulk B phase prevailed everywhere. The new state is anisotropic: In the z direction there is excess current and in the y direction there is less current corresponding to the lengths $0.5\xi(T)$ and $2.8\xi(T)$, respectively.

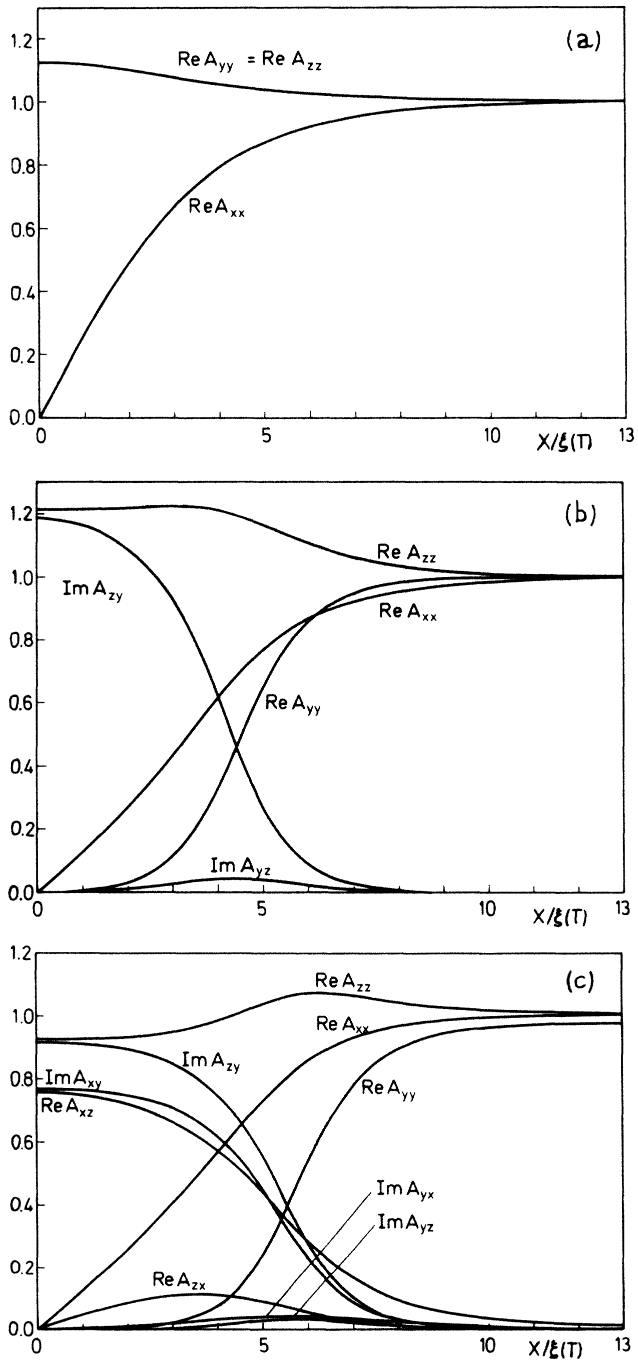

FIG. 1. The nonzero components of the order parameter (3×3 complex matrix $A_{\delta j}$). The surface is at $x=0$ and the bulk B phase is on the right. $A_{\delta j}$ is normalized to the unit matrix in the bulk. (a) The most symmetric texture ($v=0$), (b) the new surface texture ($v=0$), and (c) the new texture with superfluid velocity $v=0.1\hbar/2m_3\xi(T)$ in the y direction.

In other words, there is effectively a layer of thickness $2.8\xi(T)$ that has no current flow in the y direction. Because of this, the energy difference between the two states gets smaller with increasing flow. In fact, the free energies cross at the velocity 0.24, which is below the critical velocity 0.32 of the bulk B phase [the velocity is given in units of $\hbar/2m_3\xi(T)$], i.e., the new texture has lower energy at high velocities than the conventional texture. The flow tilts the d vector of the surface A phase from the z direction towards the surface normal, the angle being 35° at the velocity 0.1 [Fig. 1(c)].

The magnetic properties of the two states differ considerably. In bulk fluid the dipole-dipole interaction fixes the relative rotation of the spin space and the orbit space at 104°. Surfaces do not change the angle but impose boundary conditions on the direction of the rotation axis. The boundary conditions of the new texture differ from those of the conventional texture[14] at all magnetic fields: At low fields the dipole-dipole energy is minimized by a rotation that turns the z direction to the x direction, i.e., makes the d vector and the l vector of the surface A phase parallel. In high fields the field energy dominates the dipole-dipole energy. The new texture behaves magnetically as if there were normal layers of thicknesses 4.9, 5.4, and −7.5 on the surface when the field corresponds to the orbital x, y, and z directions, respectively [unit length $=\xi(T)$]. In other words, the lowest energy is achieved by a spin rotation that turns the y direction to the magnetic field direction. (In the conventional texture the rotation is from x to the field.) The new texture has a spontaneous magnetic moment which is directed normal to the surface (in orbital space).

In the experiment by Ling, Betts, and Brewer[2] the crossover in dissipation at the tricritical point may be caused by the new surface texture that lingers in the experimental cell after cooling through the bulk A phase. It is impossible for the GL theory to produce the observed power law for the critical current above the tricritical point,[15] but it is clear that the critical current is determined by nucleation and pinning of vortices on the surfaces, which depends on the surface texture. If the crossover is caused by the metastable state, it would imply that the extraordinary power law above the tricritical pressure disappears if one, for example, waits long enough within the B-phase region.

The initial motivation for this work was the experiments of Pekola *et al.*[3,4] They found a transition within the B phase both in open [nuclear magnetic resonance (NMR) experiment] and confined geometries (hydrodynamic experiment). There is a strong evidence that the former is caused by a transition in vortex cores. Out of the latter the experimentalists conclude two things: (1) The transition in the hydrodynamic experiment is due to the same vortex-core transition and (2) they measure a large latent heat associated with this transition. These claims are clearly contradictory; a vortex-core transition cannot have such a large latent heat. If one believes that the measured latent heat is real, one has to dismiss the vortex-core transition. The next candidate for the transition is then a surface transition (no transition is expected in the bulk). The surface-transition hypothesis would allow the differencies observed in the transition lines of NMR and the hydrodynamic experiment. It is not impossible that lower temperatures could stabilize the new surface texture; the energy difference between the two states is on the same order of magnitude as the energy difference between the o and the v vortex,[5] which at the present time seem the best candidates for the vortex-core transition.[16,17] The surface A phase could explain why the $B \rightarrow A$ transition superheats at low pressures but not at high pressures.[1]

The other possibility is not to believe in the latent heat. This is justified because the thermometer in the hydrodynamic experiment measures the effective superfluid density rather than the temperature. That would allow the vortex-core transition, which can nicely explain the independence of the transition line on the superfluid velocity. For the vortex-core transition one has to assume the presence of vortices, which is not necessarily the case in the hydrodynamic experiment because the measurements are made while the experimental cell is not rotating. The vortex-core-transition hypothesis gives no explanation for the difference of the transition lines in the two experiments. In conclusion, the transition in the hydrodynamic experiment is not yet understood.

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