Comment on "Excitations in the one-dimensional anisotropic classical Heisenberg chain"

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Several errors are indicated in the treatment of one-dimensional anisotropic classical Heisenberg chain by Zaspel. His essential conclusions are shown to be valid, but on the basis of completely different arguments.

Zaspel intends to prove in his paper[1] the possibility of the occurrence of both linear and nonlinear excitations in some anisotropic systems as shown experimentally.[2] He studies the system with the following classical Hamiltonian:

$$H=\int\left[\frac{\hat{J}}{2}\left[\frac{\partial \mathbf{S}}{\partial z}\right]^2+\hat{A}(S^z)^2\right]dz\ , \tag{1}$$

where we keep the same notations as Zaspel.[1] The problem arises in the formulation of the Hamilton's equations, since they can be strictly formulated from the Hamiltonian density for spin densities. Unfortunately, Zaspel[1] did not follow this approach so his equations are dimensionally incorrect. For the sake of comparison, we shall quote his initial equations. Introducing classical spin through components $\mathbf{S}=(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ u\equiv\cos\theta)$, he obtains the following set of equations of motion:

$$\phi_t=Ja\frac{u_{zz}}{1-u^2}-Ja\frac{uu_z^2}{(1-u^2)^2}-Jau\phi_z^2+2\frac{A}{a}u\ , \tag{2a}$$

$$u_t=Ja(1-u^2)\phi_{zz}-2Jauu_z\phi_z;\ \ \hat{J}\equiv Ja;\ \ \hat{A}\equiv\frac{A}{a}\ , \tag{2b}$$

which are not only dimensionally incorrect, but there also appears an incorrect sign in (2a).

We shall give here the rigorous treatment for general spin S: $\mathbf{S}=S(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ u\equiv\cos\theta)$. The equations obtained are

$$\phi_t=-JSa^2\frac{u_{zz}}{1-u^2}-JSa^2\frac{uu_z^2}{(1-u^2)^2}-JSa^2u\phi_z+2ASu\ , \tag{3a}$$

$$u_t=JSa^2(1-u^2)\phi_{zz}-2JSa^2uu_z\phi_z\ . \tag{3b}$$

This incorrect sign term is important for the more general treatment, but in this particular case, it does not influence the equation for ϕ_{tt}, because it is neglected as a small one in this calculation.

Another important mistake occurs during the derivation of Eq. (3b) of Ref. 1, which is crucial for the explanation of the heat capacity of tetramethyl ammonium manganese chloride (TMMC) on the basis of the linear spin-wave theory in the absence of the transverse field. To demonstrate this, we shall give a detailed account on the derivation of the equation corresponding to the above-mentioned one.

Differentiating Eq. (3a) with respect to t and z, we obtain

$$\phi_{tt}=-JSa^2\left[\frac{u_{zz}}{1-u^2}\right]_t-JSa^2\left[\frac{uu_z^2}{(1-u^2)^2}\right]_t-JSa^2u_t\phi_z^2-2JSa^2u\phi_z\phi_{zt}+2ASu_t\ , \tag{4}$$

$$\phi_{zt}=-JSa^2\left[\frac{u_{zz}}{1-u^2}\right]_z-JSa^2\left[\frac{uu_z^2}{(1-u^2)^2}\right]_z-JSa^2u_z\phi_z^2-2JSa^2u\phi_z\phi_{zz}+2ASu_z\ . \tag{5}$$

It can be easily seen that in the approximation which retains only the terms of order JAS^2a^2 ($\hat{J}\hat{A}$ in Zaspel's notation) in the final equation, the last term in (4) is important, as concluded by Zaspel.[1] On the other hand, the author missed the fact that the term $-2JSa^2u\phi_z\phi_{zt}$ in (4) also gives the contribution of the same order of magnitude, since, from (5) it follows that $\phi_{zt}=2ASu_z+O(JSa^2)$. Taking into account Eqs. (3b), (4), and (5), we obtain, finally,

$$\phi_{tt}-[c^2(u)\phi_z]_z-4JS^2Aa^2uu_z\phi_z=0 \tag{6}$$

with

$$c^2(u)=2AJS^2a^2(1-u^2)\ .$$

The equation for u_{tt},

$$u_{tt}-[c^2(u)u_z]_z=0\ , \tag{7}$$

is correct within the given approximation. Still, the statement that the general solution of this equation is of the form

$$u=f(z-c(u)t)+g(z+c(u)t)\ , \tag{8}$$

with f and g arbitrary is incorrect. In fact, as shown in Ref. 3, these two types of solutions belong only to two sets of particular solutions, and since the equation is nonlinear, the general solution cannot be obtained by the linear combination of these two.

For the particular case $u=0$, all the conclusions of Zaspel are valid, of course, because the equation now becomes linear.

Finally, let us note that it was not necessary to perform all these calculations in order to prove the existence of the linear excitations in this system in the absence of an external field, because, the sine-Gordon equation in this case has the nonlinear term which is proportional to the field, so it vanishes in the absence of an external field turning the equation into a linear one.

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[2]F. Borsa, M. G. Pini, A. Rettori, and V. Tognetti, Phys. Rev. B 28, 5173 (1983).
[3]S. E. Jones and W. F. Ames, Q. Appl. Math. 25, 302 (1967).