Remanent magnetization of the infinite-range Ising spin glass

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(Received 30 August 1985)

The remanent magnetization M_r and the excess energy ΔE are studied for the Sherrington-Kirkpatrick model of spin glasses using Monte Carlo simulations. At zero temperature T the system relaxes to a stationary nonequilibrium state with nonzero M_r and ΔE; both quantities are self-averaging. For $0 < T < T_c$ the simulations indicate that M_r and ΔE decay to zero as a power law in time.

The remanent magnetization is one of the most characteristic properties of spin glasses.[1] In thermal equilibrium the magnetization of spin glasses is zero since the magnetic moments point in random directions. However, after a magnetic field has been applied and then switched off the spin glass relaxes into a state with nonzero magnetization M_r which slowly decays to zero.[2,3] Hence M_r shows the existence of a large spectrum of metastable states and corresponding relaxation times which are characteristic for the dynamics of disordered materials in general.

A very successful model of spin glasses has been proposed by Edwards and Anderson (EA).[4] It is a simple Ising model with random and competing interactions. In fact, computer simulations of the two-dimensional (2D) model have shown that this model has a remanent magnetization.[5] It depends on time t, temperature T, the previously applied field h, and the history of the sample (field cooled versus zero-field cooled) in qualitative agreement with many details of the experiments.[6]

All of these properties exist without the presence of a spin-glass phase in thermal equilibrium since such a phase does not exist in two dimensions.[7] On the other hand, a spin-glass phase has infinite energy barriers[8] which may influence the nonequilibrium behavior of the system. For instance, in the infinite-range Ising ferromagnet in an external field below the spinodal line, a nonequilibrium state with the magnetization pointing opposite to the field is stable.[9] Similarly, in thermal equilibrium the infinite-range spin glass [the Sherrington-Kirkpatrick (SK) model[10]] is characterized by infinite energy barriers and nonergodic behavior below the critical temperature T_c.[11] Hence, one might expect that an initial state decays into a "valley" in phase space far from equilibrium where it is permanently trapped, leading to a stationary nonzero remanent magnetization. In fact, a nonzero stable remanent magnetization is obtained analytically for the SK model.[12]

In this article it is shown numerically that the infinite-range spin glass is not trapped in a state far from equilibrium. The remanent magnetization M_r decays to zero; in addition, the energy E decays to its equilibrium value E_0. Our data can be fitted by a power-law decay in time. Only at zero temperature do stable nonequilibrium states with nonzero M_r and $\Delta E = E - E_0$ exist. Both quantities are self-averaging, i.e., their sample-to-sample fluctuations disappear in the thermodynamic limit.

The Hamiltonian of the SK model is given by

$$H = -\sum_{\substack{i,j \\ (i<j)}} J_{ij} S_i S_j - h \sum_i S_i \ , \tag{1}$$

where $S_i = \pm 1$, $i = 1, \ldots, N$, the J_{ij} are independent random variables with a Gaussian distribution of zero mean and variance $(N-1)^{-1}$, and h is a uniform field. For $h = 0$, $N \to \infty$, a transition occurs at $T_c = 1$.[10] The dynamics is defined by the usual Glauber single-spin-flip relaxation.[5] A computer program for the SK model may be found in Ref. 11. The time t is given in units of Monte Carlo steps per spin (MCS/S).

Due to the infinite range of the couplings J_{ij} only rather small samples up to $N = 1024$ could be simulated (Ref. 11 uses $N \leq 192$). Hence, finite-size effects should be carefully studied. From the simulations of the equilibrium properties, two characteristic time scales τ and $\tau_{\rm erg}$ emerged, both of which diverge in the thermodynamic limit $N \to \infty$.[11] τ is interpreted as the time needed for diffusing to a neighboring state of the system and it diverges like (at $T = 0.4$)

$$\ln \tau = 2.58 N^{1/4} - 0.66 \ . \tag{2}$$

$\tau_{\rm erg}$ is the ergodic time scale needed to reverse an equilibrium state to its negative counterpart. It is much larger than τ and $\ln \tau_{\rm erg}$ diverges like $N^{1/2}$. Only data for time scales much shorter than τ may show the behavior of the infinite system.

First, let us consider the saturated remanent magnetization. The system starts from the ferromagnetic state $S_i = +1$ at $t = 0$; i.e., $M(t=0) = 1$ and $E(t=0) = 0$. At zero temperature the system decays in about 20 MCS/S to a stationary state. The stationary values of M_r and E depend on the system size N and fluctuate strongly for different bond configurations. However, from studying 10 000 samples for $N = 128$, 256, 512, and 1024 I find (i) the average values $\langle M_r \rangle$ and $\langle E \rangle$ extrapolate, linearly with $1/\sqrt{N}$, to the values

$$\langle M_r \rangle = 0.14 \pm 0.01; \ \langle E \rangle / E_0 = 0.92 \pm 0.01 \ , \tag{3}$$

where E_0 is the ground-state energy, $E_0 = -0.765 \pm 0.002$;[13] (ii) the fluctuations $\langle (M - \langle M \rangle)^2 \rangle$ and $\langle (E - \langle E \rangle)^2 \rangle$ extrapolate as $1/N$ to zero. Hence, a stable remanent magnetization exists at $T = 0$; it is about a factor of 2 smaller than M_r of the 2D model.[5,6]

Note that M_r is the overlap to the initial state $S_i = +1$; hence, it determines the distance $d = (1 - M_r)/2$ between the initial and the final state.[14] Therefore, from the result (4) the following picture of the phase space at $T = 0$ emerges: With probability one (for $N \to \infty$), *any* initial state relaxes a distance $d = 0.43$ in phase space to a state with an energy which is 8% above the ground-state energy. The ground state or other low-lying states have a vanishing small

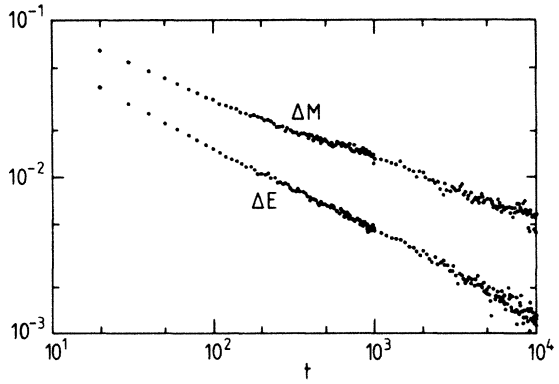

FIG. 1. Decay of the remanent magnetization M and energy E with time t plotted double logarithmically for $\Delta M = M - M_\infty$ and $\Delta E = E - E_\infty$. The data are shown for $N = 512$ and $T = 0.4$ and indicate a power-law decay with parameters given in Table I.

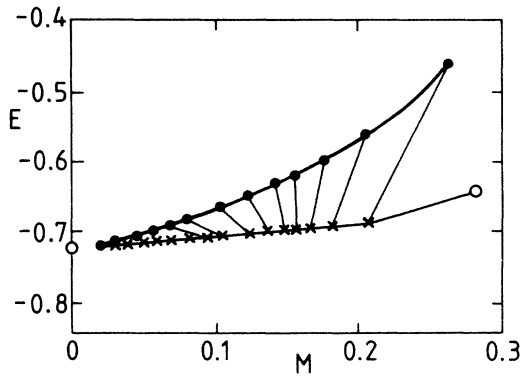

FIG. 2. Energy vs magnetization of the decay of the thermoremanent magnetization for cooling field $h = \infty$ (dots) and $h = 0.3$ (crosses) and for $N = 512$ and $T = 0.4$. The data of the same times are connected. The circles show thermal equilibrium and the field-cooled initial state ($h = 0.3$), respectively. The initial state for $h = \infty$ is at $E = 0$ and $M = 1$.

basin of attraction.

At higher temperatures M and E decay with time. Figure 1 shows the decay of M and E for $T = 0.4$ and $N = 512$. The data are averaged over 5000 bond configurations; for each configuration one run was performed. The statistical error is of the order of the fluctuations with time in Fig. 1. The data of Fig. 1 are consistent with a power-law decay,

$$M(t) - M_\infty = At^{-a}, \qquad E(t) - E_\infty = Bt^{-b}, \tag{4}$$

with

$$M(t) = \frac{1}{N}\sum_i S_i(t), \quad E(t) = \sum_{i<j} J_{ij} S_i(t) S_j(t) .$$

The parameters a, A, M_∞ and b, B, E_∞ are obtained from least-squares fits and are shown in Table I for different system sizes N. We see that M_∞ is rather small and decreases with increasing N. (Note that a finite random system has a magnetization of the order of $1/\sqrt{N}$; hence, a nonzero M_∞ of this order is expected.) $M(t)$ relaxes for time scales much shorter than the characteristic time τ of Eq. (2). $M(t)$ decreases with increasing size N. All this strongly indicates a vanishing stationary remanent magnetization $M_r = M_\infty(N \to \infty) = 0$, at least the value 10^{-2} is an upper bound.

The decay to thermal equilibrium is also indicated by the energy parameter E_∞. Namely, it agrees with the energy of a system which was cooled in $h = 0$ from $T = 2$ to $T = 0.4$ in steps of $\Delta T = 0.05$ over 20000 MCS/S. Cooling this system further to $T = 0$ gives an energy which extrapolates with $N \to \infty$ to the ground-state energy E_0. This shows that this cooling procedure gives the equilibrium energy (within the statistical accuracy) which is equal to the fit parameter E_∞ of Table I; hence, the power-law fit of Eq. (4) indeed extrapolates to a state of thermal equilibrium.

A similar behavior is obtained for the thermoremanent magnetization (TRM); i.e., when the initial state is obtained from cooling the system in an external field h. I have studied the decay of M and E after cooling the system in a field $h = 0.3$ over 20000 MCS/S from $T = 2$ to $T = 0.4$. Again the decay can be fitted by a power law, as in Fig. 1, but with smaller values of the exponents a and b. Similar to the two-dimensional case[6] the decay of the TRM is slower for smaller cooling fields. However, a reliable determination of the decay parameters of Eq. (4) was not possible due to the slow decay and larger statistical errors.

In a magnetization-energy diagram, for different initial states the system decays on different paths to equilibrium as shown in Fig. 2. Our results suggest a different power law for each path, even very close to equilibrium. Presumably such a singular behavior cannot be obtained from any kind of perturbation expansion around equilibrium.

Figure 3 shows the dependence of the short-time TRM, $M(t = 100$ MCS/S) as a function of temperature and cooling field. As in experiments[2] and the 2D model,[6] the TRM has a maximum as a function of field. It is interesting that even for such short times the critical temperature T_c can be estimated from the T dependence of the TRM.

Recent measurements of the TRM of Ag:Mn have been reported which show a stretched exponential decay rather

TABLE I. Parameters of the power-law decay, Eq. (3), obtained from least-squares fits for the saturated remanent magnetization at $T = 0.4$.

N	b	$\ln B$	E_∞	a	$\ln A$	M_∞
128	0.60	3.2	0.7056	0.36	2.51	0.017
256	0.55	3.0	0.7160	0.38	2.48	0.014
512	0.54	2.9	0.7222	0.40	2.47	0.012
1024	0.53	2.85	0.7257	0.38	2.49	0.011

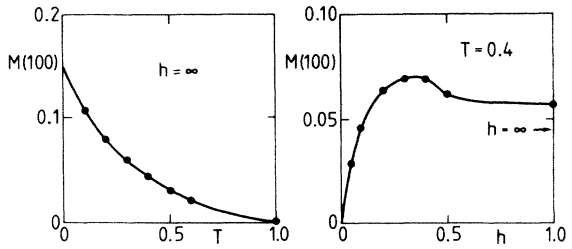

FIG. 3. Remanent magnetization (TRM) after $t = 100$ MCS/S as a function of temperature T and cooling field h. The statistical error is smaller than the data points.

than a power-law decay,[3] although earlier data on insulating spin glasses could be fitted by a power law.[2] Note, however, that these new experiments use an extremely small cooling field; hence, the initial state is very close to thermal equilibrium. In the present simulation, however, the cooling field is infinite or comparable to the coupling energy.

The present numerical simulations were performed on the Cray-XMP computer of the Kernforschungsanlage Jülich and took about 100 h of CPU time. Many useful discussions with H. Horner and A. Zippelius are gratefully acknowledged.

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