## Signs of magnetic surface anisotropy constants

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A method presented earlier fG. T. Rado, Phys. Rev. B 26, 295 (1982)) for determining magnetic surface anisotropy constants by means of ferromagnetic resonance is elucidated and discussed.

A previous paper' includes a method for determining magnetic surface anisotropy constants by means of ferromagnetic resonance (FMR). In the present paper that method is elucidated and discussed. The equation numbers used herein refer to equations presented in Ref. 1.

We show, first of all, that the quantity  $R$  introduced previously' contains an error which exists only in certain cases, but in those cases it changes some of the theoretical predictions of Ref. 1. One such case is that of the FMR experiments on iron<sup>2</sup> analyzed previously.<sup>1</sup> In that particular case the quantity R (defined by  $R = v_1/v_3$ ) is given not by the R of Eq. (4.22) but by  $(-1)$  times the R of Eq. (4.22). This correction is a previously overlooked consequence of the fact that in the experiments on iron analyzed in Ref. <sup>1</sup> both  $H + H_{\alpha} - 2Ak_i^2/M$  and  $B + H_{\beta} - 2Ak_i^2/M$  are positive when  $i = 1$ , but negative when  $i = 3$ .

Using the corrected value of  $R$  in Eq. (4.30) for analyzing the experimental data<sup>2</sup> used previously,<sup>1</sup> we now obtain  $K_s = +3.1$  ergs/cm<sup>2</sup> and  $K_{ss} = +1.6$  ergs/cm<sup>2</sup>, i.e., approximately  $(-1)$  times the values given in Ref. 1. We believe that an analysis of the data on the basis of Eq. (4.23), which is more accurate than Eq. (4.30), is not warranted at present. The main reason is that, even today, the measured magnetization of ultrathin iron crystals grown on GaAs (and covered by Al) is not weil established and apparently thickness dependent.<sup>3</sup> As noted previously,<sup>1</sup> moreover, the boundary conditions at the two surfaces of an iron crystal were different in the experiments<sup>2</sup> but were assumed to be the same in the theory. For these two reasons the theory,<sup>1</sup> including its present elucidation, is not strictly applicable to the experiments of Ref. 2.

Next we comment on the fact that our corrected  $K_s$  is positive (see above) and thus appears to contradict Neel's<sup>4</sup> theoretical prediction that  $K_s$  should be negative. Actually there is no contradiction, as shown below. While the surface anisotropy tensor is the same for each of the six different  $(110)$  type planes, it is represented by a different matrix for each of these six planes. In Ref. <sup>1</sup> it was assumed that the crystal surfaces are  $(110)$  planes, whereas Neel assumed them to be (011) planes. Starting with the matrix of the surface anisotropy tensor in a coordinate system in which the crystal surfaces are  $(110)$  planes, we rotate the coordinate system so that the fixed crystal surfaces previously labeled  $(110)$  must be labeled  $(011)$  after the coordinate rotation. After determining the transformation matrix  $(T)$  which describes this coordinate rotation, we use  $(T)$  to perform that similarity transformation which changes the surface anisotropy matrix appropriate for <sup>a</sup> (110) crystal surface to a surface anisotropy matrix appropriate for a (011) crystal surface. In this way we find that a positive  $K_s$ for (110) crystal surfaces becomes a negative  $K<sub>s</sub>$  for (011)

crystal surfaces. Thus, we now have a true agreement between the sign of our deduced  $K_s$  and the sign of Neel's predicted  $K_s$ . In contrast, the corresponding agreement found previously' was only an apparent agreement because it resulted from the omission of a minus sign in  $R$  (and hence in  $K_s$ ) and the omission of a transformation of the surface anisotropy matrix.

We note that the method described above leads to the expressions shown in Table I for the surface anisotropy energy density  $E_{\text{surf}}$  in bcc crystals having  $\{110\}$  type surfaces. Each of these expressions includes not only a term involving the constant  $K_s$  introduced by Néel,<sup>4</sup> but also a term involving the constant  $K_{ss}$  introduced in Ref. 1. As in the previou paper,<sup>1</sup> which contains the derivation of  $E_{\text{surf}}$  for a (110) surface, the coordinate axes  $x$ ,  $y$ ,  $z$  constitute a Cartesian system, and  $u_x$ ,  $u_y$ ,  $u_z$  denote the Cartesian components of a unit vector which is parallel to the magnetization. In each row of Table I, however, the  $x, y, z$  coordinate system is oriented differently, namely, in such a manner that a given [110}type crystal surface is described correctly by the Miller indices presented in that row.

We further note that Table I applies not only to  $\{110\}$ type surfaces of bcc crystals but also to (110} type surfaces of fcc and simple cubic (sc) crystals. For example,  $E_{\text{surf}}$  for (011) surfaces is given by  $-K_s u_y u_z + K_{ss} u_x^2$  for fcc crystal as well as for bcc crystals. This property of Table I is a consequence of the fact that the symmetry considerations used previously<sup>1</sup> for deriving  $E_{\text{surf}}$  are just as applicable to fcc and sc crystals as to bcc crystals. However, the meaning of  $K_s$  and  $K_{ss}$  is different for each of these three cubic crystal types. To see this, we recall that  $N\acute{e}el's<sup>4</sup>$  expression for  $E_{\text{surf}}$  of (011) surfaces of fcc crystals is proportional to  $(u_y - u_z)^2$  and thus contains a term in  $u_x^2$  (because  $(u_y - u_z)$  and thus comains a term in  $u_x$  (becaus  $u_y^2 + u_z^2 = 1 - u_x^2$ ) as well as a term in  $u_y u_z$ . Since Néel' derivation involves nearest-neighbor interaction only, it follows that our term  $K_{ss}u_x^2$  for (011) surfaces might arise solely from nearest-neighbor interactions in the case of fcc

TABLE I. Expressions for the surface anisotropy energy density  $E_{\text{surf}}$  in bcc crystals having {110} type surfaces. The quantities  $u_x$ ,  $u_y$ ,  $u_z$  are explained in the text.

Surface plane	$E_{\rm surf}$
(110)	$K_s u_x u_y + K_{ss} u_z^2$
(110)	$-K_s u_x u_y + K_{ss} u_z^2$
(011)	$K_s u_y u_z + K_{ss} u_x^2$
(011)	$-K_s u_y u_z + K_{ss} u_x^2$
(101)	$K_s u_z u_x + K_{ss} u_y^2$
(101)	$-K_s u_2 u_x + K_{ss} u_y^2$

crystals, even though this term was shown' to arise solely from next-nearest-neighbor interactions in the case of bcc crystals. While Neel's nearest-neighbor method<sup>4</sup> predicts that the value of the coefficient of  $u_x^2$  is one-half that of the coefficient of  $u_y u_z$ , the more general symmetry-based method<sup>1</sup> predicts that these two coefficients are independent of each other.

Although Ref. <sup>1</sup> is concerned with (110) type surfaces only, we now comment briefly on the form of  $E_{\text{surf}}$  for (100) and (111) surfaces. With the use of the symmetrybased method' we easily find for (100) surfaces that for bcc, fcc, and sc crystals the value of  $E_{\text{surf}}$  is proportional to

 $\cos^2\theta$ , where  $\theta$  is the angle between the magnetization and the normal to the crystal surface. For the case of (100) surfaces,  $\cos^2\theta$  equals  $u_x^2$ . The nearest-neighbor method<sup>4</sup> also yields  $E_{\text{surf}} \propto \cos^2 \theta$  for fcc and sc crystals but  $E_{\text{surf}} = 0$  for bcc crystals. For (111) surfaces the symmetry-based method' leads to  $E_{\text{surf}} \propto \cos^2 \theta$  for bcc, fcc, and sc crystals, whereas the nearest-neighbor method<sup>4</sup> also predicts  $E_{\text{surf}} \propto \cos^2 \theta$  for fcc crystals but  $E_{\text{surf}}=0$  for bcc and sc crystals. We find, therefore, that for (111) as well as for (100) surfaces the symmetry-based method leads to terms in  $E_{\text{surf}}$  which vanish on the basis of nearest-neighbor interactions but might arise from next-nearest-neighbor (or farther) interactions.

- it appears separately, but not where it appears as  $tanh(k_3L)$ . <sup>2</sup>G. A. Prinz, G. T. Rado, and J. J. Krebs, J. Appl. Phys. 53, 2087 (1982), see Fig. 7,
	- <sup>3</sup>T. R. McGuire, J. J. Krebs, and G. A. Prinz, J. Appl. Phys. 55, 2505 (1984).
	- <sup>4</sup>L. Néel, J. Phys. Radium 15, 225 (1954).
- <sup>1</sup>G. T. Rado, Phys. Rev. B 26, 295 (1982). Please note the following typographical errors: In Eq. (3.11), the term  $4A(\partial \theta_0/\partial n)$ should be replaced by  $-4A(\partial \theta_0/\partial n)$ . In Eq. (4.14), the term  $2A(\partial \theta_1/\partial n)$  should be replaced by  $-2A(\partial \theta_1/\partial n)$ . Equation (4.15) should read  $A(\partial \phi_1/\partial n)+\phi_1K_s = 0$ . In Eq. (4.21), the product  $k_3L$  should be replaced by  $Ak_3L$  in the two places where