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Landau levels above the optical-phonon continuum in two and three dimensions

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The position of the Landau levels perturbed by the electron-phonon interaction is calculated for energies above the LO-phonon continuum in two and three dimensions. In addition to the Landau levels at $n\hbar\omega_c$ (ω_c is the cyclotron frequency) a series of levels are found at $\hbar\omega_0 + l\hbar\omega_c$ (ω_0 is the LO-phonon frequency). The degeneracy between the levels $n\hbar\omega_c$ and $\hbar\omega_0 + l\hbar\omega_c$ is lifted by the LO-phonon interaction. Our results are valid for excitation energies smaller than $2\hbar\omega_0$, for zero temperature, and for weak polar materials.

In an earlier paper,[1] henceforth denoted I, we have calculated the correction due to the electron-phonon interaction to the different Landau levels for two- (2D) and three-dimensional (3D) electrons. The results of I are valid for arbitrary Landau level n, arbitrary magnetic field strength, zero temperature (T), weak electron-phonon coupling strength (α), and for energies below the LO-phonon continuum, i.e., $E_n - E_0 < \hbar\omega_0$ (E_n is the energy of the nth Landau level). Those results are a generalization of earlier work by Larsen[2,3] and Das Sarma.[4] In the present paper the results of I are generalized to $2\hbar\omega_0 > E_n - E_0 > \hbar\omega_0$.

For energies above the LO-phonon continuum the energy levels have a nonzero width and the electron can decay to the ground state by emission of an optical phonon. This is in contrast to the case $E_n - E_0 < \hbar\omega_0$, where the energy levels at $T=0$ are stable and have zero width. At present we are only concerned with the position of the energy levels and therefore only the real part of the energy is of interest here. Degenerate perturbation theory has to be used, and in order to guarantee the correct pinning behavior, improved Wigner-Brillouin perturbation theory (IWBPT) is used, as first proposed and applied to InSb by Larsen[5] and later on reinvestigated by Lindemann, Lassnig, Seidenbusch, and Gornik[6] for the case of GaAs.

The correction to the Landau levels $E_n = \hbar\omega_c(n+\frac{1}{2}) + \Delta E_n$ due to the electron–LO-phonon interaction is

$$\Delta E_n = -P\sum_{l=0}^{\infty}\sum_{\mathbf{q}}\frac{|M_{nl}(\mathbf{q})|^2}{\hbar\omega_0 + \hbar\omega_c(l-n) + \frac{\hbar^2 q_z^2}{2m_b} - \Delta_n} \quad , \tag{1}$$

where for IWBPT $\Delta_n = \Delta E_n - \Delta E_0$ and all other notations are as in I. For the ideal 2D case, no motion in the z direction is possible and the term $\hbar^2 q_z^2/2m_b$ in the denominator of Eq. (1) is absent. When $E_n - E_0 > \hbar\omega_0$ the denominator in Eq. (1) can become zero. In order to obtain the energy correction we then take the principal value indicated by $P(\ldots)$.

The q_x and q_y integration in Eq. (1) can be done analytically and we found for the 3D case the following result:

$$\Delta E_n = -\sqrt{2}\alpha\sum_{m=0}^{\infty}\int_0^{\infty} dt\,\frac{e^{-t^2/2\omega_c}}{2\mu_n + t^2}\left[1 - t\frac{\Theta(-\mu_n)}{\sqrt{-2\mu_n}}\right]S_{n,m}(t^2) \quad , \tag{2}$$

where

$$\mu_n = \omega_c(n-m) - 1 + \Delta_n \quad , \tag{3a}$$

$$S_{n,m}(x) = \left[\sum_{l=0}^{s}\binom{s}{l}\frac{(-x/2\omega_c)^l}{(|n-m|+l)!}\right]^2\left[\frac{x}{2\omega_c}\right]^{|n-m|}\delta(m,n) \quad , \tag{3b}$$

with $s = m\,(n \geq m)$, $n\,(n \leq m)$ and $\delta(m,n) = n!/m!\,(n \geq m)$, $m!/n!\,(n \leq m)$, and where units are used such that $\hbar = m_b = \omega_0 = 1$.

In the 2D case a less complicated result is found:

$$\Delta E_n = \frac{\alpha\sqrt{\omega_c}}{2}\sum_{m=0}^{\infty}\frac{\delta(m,n)}{\mu_n}\sum_{l,l'=0}^{s}\binom{s}{l}\binom{s}{l'}\frac{(-1)^{l+l'}\Gamma(|n-m|+l+l'+\frac{1}{2})}{(|n-m|+l)!(|n-m|+l')!} \quad , \tag{4}$$

with $\Gamma(x)$ the gamma function.

The solutions of Eq. (2) for the 3D and of Eq. (4) for the 2D Landau level E_1 are shown in Fig. 1 for $\alpha = 0.07$. The result is referred to the ground-state energy E_0. It is customary to relate $E_1 - E_0 = \hbar\omega_c^*$ to the position of the cyclotron resonance frequency ω_c^* from which the cyclotron mass $m^*/m_b = \omega_c/\omega_c^*$ is derived as plotted in Fig. 2. The result below the continuum (i.e., $E_1 - E_0 < \hbar\omega_0$) is obtained from I. From Figs. 1 and 2 we note the following: (1) Eq. (2) for the 3D case and for $\alpha = 0.07$ (0.02) has a solution only if $\omega_c/\omega_0 > 0.924$ (0.978). This confirms an earlier result by Larsen[7] within the WBPT approach and was shown[8] to be an artifact of the method. This was recently confirmed experimentally by Sigg, Bluyssen, and Wyder.[9] (2) For any magnetic field (in 3D only for $\omega_c/\omega_0 > 0.924$ when $\alpha = 0.07$) there exist two branches. When $\omega_c/\omega_0 < 1$ the lowest branch has the dominant oscillator strength and is interpreted as the cyclotron resonance peak which gives a polaron-cyclotron mass m^*, larger than the electron band mass m_b. For $\omega_c/\omega_0 > 1$ most of the oscillator strength is transferred to the upper branch (see also Ref. 10) which now is interpreted as the cyclotron resonance peak whose cyclotron mass is $m^* < m_b$ and approaches asymptotically m_b for $\omega_c \to \infty$. We also found that the upper branch does *not* ap-

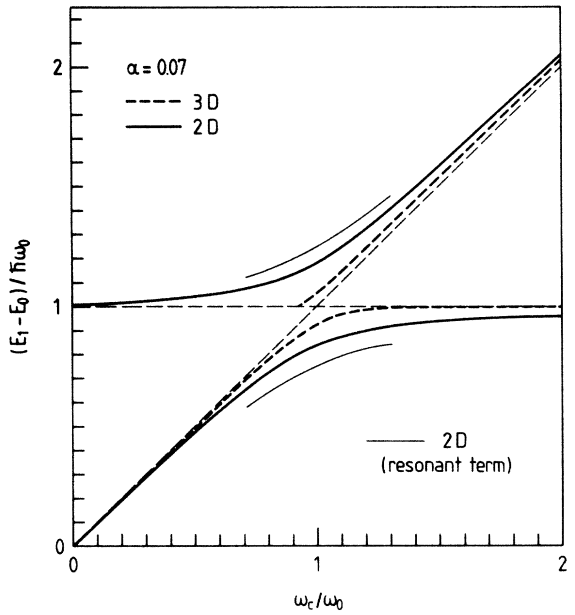

FIG. 1. Energy of the first Landau level (E_1) shifted with the ground-state energy (E_0) as a function of the magnetic field for the 2D (solid curve) and the 3D (dashed curve) case. If only the resonant term is included within WBPT the thin solid curve is obtained.

proach the unperturbed result when $\omega_c \to \infty$, i.e., the difference $(E_1 - E_0 - \hbar\omega_c)/\hbar\omega_0$ increases with increasing magnetic field strength, e.g., it is 0.0332 for $\omega_c/\omega_0 = 2$ and 0.0338 for $\omega_c/\omega_0 = 3$, but the relative difference tends to zero. (3) *The splitting* of the energy levels at $\omega_c = \omega_0$ *is not symmetric* around the unperturbed result, i.e., $E_1 - E_0 = \hbar\omega_c \pm \delta_\pm$ but where $\delta_+ \neq \delta_-$. For example, we found for $\alpha = 0.07$ $\delta_-/\hbar\omega_0 = 0.163$ (0.076) and $\delta_+/\hbar\omega_0 = 0.179$ (0.054) for the 2D (3D) case. A symmetrical splitting is found, i.e., $\delta_+ = \delta_-$,

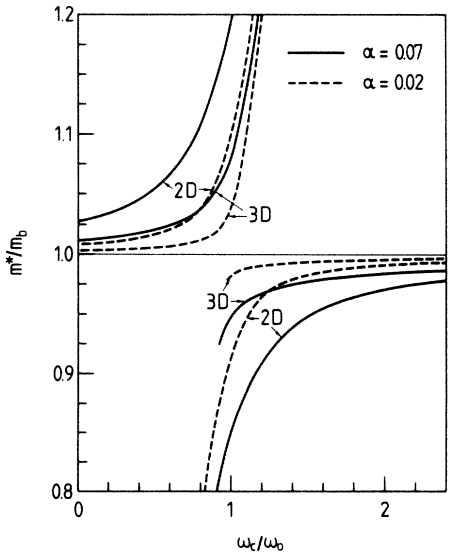

FIG. 2. Cyclotron polaron mass in 2D and 3D for two values of the electron-phonon coupling constant $\alpha = 0.02$ and $\alpha = 0.07$ which correspond respectively to InSb and GaAs.

if only the resonant term in Eq. (1) is included within WBPT (i.e., $\Delta_n = \Delta E_n$). The latter approach was followed, e.g., in Refs. 11–14 and it leads to the energy levels (for the 2D case) indicated in Fig. 1 by the thin full curves. Taking into account only the resonant term leads to a 48% overestimation of the energy level splitting for the 2D case with $\alpha = 0.07$ at $\omega_c = \omega_0$.

A careful analysis of Eqs. (2) and (4) reveals that more than one solution is possible. We found that for every n, Eq. (2) for magnetic fields larger than a critical value and Eq. (4) for any magnetic field strength exhibit an infinite number of solutions. These solutions correspond to different excited states. In Fig. 3 we have plotted the first three solutions for $n = 1, 2, 3$ in the 2D case when $\alpha = 0.07$. The series of energy levels closely follow the unperturbed energies $n\omega_c$, $n = 1, 2, 3, \ldots$ and $\omega_0 + l\omega_c$, $l = 0, 1, 2, 3, \ldots$ except when those energies cross each other. Thus the electron-phonon interaction lifts the degeneracy between these levels.

This has to be interpreted in terms of a noncrossing of energy levels and is different from a lifting of the degeneracy of the Landau levels (for an infinite system each Landau level is infinitely degenerate) itself, which is not possible by the electron–LO-phonon interaction as was demonstrated in Ref. 15.

Up to now the levels $\omega_0 + l\omega_c$ with $l = 1, 2, 3, \ldots$, called "LO-phonon-assisted harmonics," have only been investigated[16] in bulk InSb. Our present analysis shows that they should also be present in quasi-two-dimensional electron systems like the InSb inversion layer studied in Ref. 17.

In summary, the energy levels of 2D and 3D electrons subject to a magnetic field and interacting with LO phonons

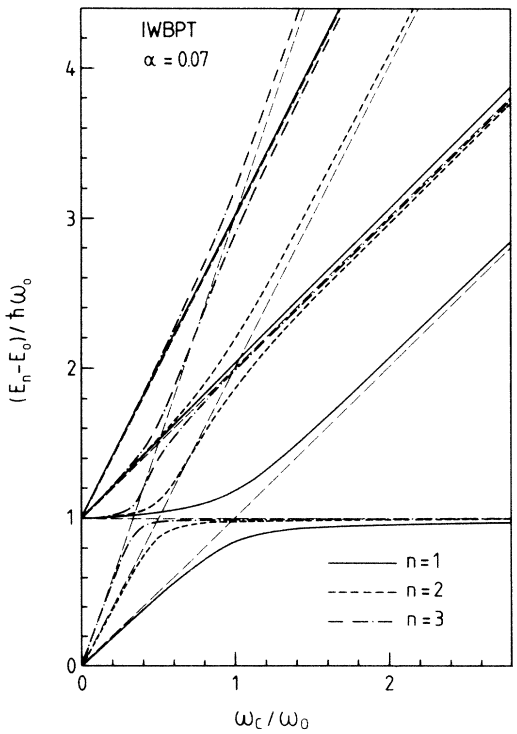

FIG. 3. The full energy spectrum for $\alpha = 0.07$ and the levels $n = 1, 2, 3$. The unperturbed energies are given by the thin dashed lines.

is investigated within degenerate second-order perturbation theory for electron energies above the LO-phonon continuum. We found two sets of energy levels with energies $n\hbar\omega_c$ and $\hbar\omega_0 + l\hbar\omega_c$ where $n = 1, 2, 3, \ldots$ and $l = 1, 2, 3, \ldots$. Because only single-phonon scattering processes are taken into account, our results are valid only below the two-LO-phonon branch. For clarity, the numerical results in Fig. 3 are also shown for $E_n - E_0 > 2\hbar\omega_0$.

In Ref. 12 Lassnig and Zawadzki found that the cyclotron resonance splitting observed[17] in the InSb inversion layer is smaller than the theoretical calculated splitting. Nevertheless, these authors could explain the experimental results quantitatively by assuming an electron density which is a factor of 2 smaller than the actual experimental electron density. Because in Ref. 12 only the resonant term was incorporated in the calculation, we suggest that this discrepancy may be due to the neglect of the nonresonant terms in Eq. (1) which diminishes the resonant splitting, as is demonstrated numerically in Fig. 1.

Experimentally[13,18] it was found that in the $Al_xGa_{1-x}As$-GaAs heterojunction, screening effects, finite width of the 2D layer, and band nonparabolicity may alter the polaron effect considerably. These effects were not taken into account in the present one-electron picture. In Ref. 13 Sigg, Wyder, and Perenboom found that screening of the electron-phonon interaction reduces the 2D polaron effects so strongly that they become smaller than the polaron effects in bulk material. They used Thomas-Fermi screening and incorporated only the resonant term in Eq. (1). In the present paper we found that the resonant term alone results in an overestimation of the resonant polaron effect. Consequently, we may conclude that the screening effects in Ref. 13 are weaker than anticipated by Sigg *et al.* This is in agreement with recent[19,20] findings that Thomas-Fermi screening overestimates the screening effects considerably, especially in the range of low and intermediate electron densities.

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