Disorder and the fractional quantum Hall effect: Activation energies and the collapse of the gap

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We examine the broadening of the collective excitations of a fractional quantum Hall state due to disorder. Because of the absence of screening at long wavelength in this regime, we believe that the broadening depends mostly on the ionized impurity contribution to the disorder potential. The broadening of the collective excitation spectrum reduces the minimum excitation energy and eventually the gap required for the occurrence of the fractional quantum Hall effect collapses. We present some results on the necessary conditions for the gap to remain finite. These depend on some exact sum rules for three-point correlation functions of isotropic states constructed entirely within the lowest Landau level. Finally the relationship between our results and the activation energies seen in the magnetotransport coefficients is discussed.

I. INTRODUCTION

Recently Girvin *et al.*[1] presented a theory of the collective excitation spectrum in the fractional quantum Hall regime. The theory is based on the expectation that a single mode will exhaust most of the oscillator strength available within the lowest Landau level and is supported by small system numerical calculations.[2] The wave function for the collective mode at wave vector k is

$$\psi_k[z]=N_k \sum_i \exp\left(-ik\frac{\partial}{\partial z_i}\right)\exp\left(-i\frac{k^*}{2}z_i\right)\psi_0[z]$$

$$=N_k\bar{\rho}_k\psi_o[z]\ , \tag{1}$$

where $\psi_0[z]$ is an incompressible liquid[3] ground-state wave function and the derivatives operate only on the polynomial part of the wave function.[4] In Eq. (1) $\bar{\rho}_k$ is the projection of the density operator

$$\rho_k=\sum_k \exp[-i(k^*z_i+kz_i^*)/2]\ ,$$

onto the lowest Landau level, the two-dimensional wave vectors and positions are given in complex notation and N_k is a normalization constant (see below). [Lengths throughout are in units of $a_L=(\hbar c/eB)^{1/2}$ and we let $\hbar=1$]. Physically $\bar{\rho}_k\psi_0[z]$ represents a state analogous to the transverse magnetophonon state of a Wigner crystal[5] but its excitation does not vanish as $|k|\rightarrow 0$. instead it exhibits a "magnetoroton" minimum when $|k|$ is near the magnitude of the Wigner crystal's primitive reciprocal-lattice vector. In this article we study the broadening of this band of collective excitations due to the disorder potential which exists in the modulation-doped GaAs-Al_xGa_{1-x}As semiconductor heterojunction systems where the fractional quantum Hall effect is observed. An expression is derived relating the minimum magnetic field required to see the fractional Hall effect to the set-back distance of the doped Al_xGa_{1-x}As and the ideal system magnetoroton energy gap. Finally a qualitative connection is made with the activation energies seen in the magnetotransport coefficients.

II. SCREENING CONSIDERATIONS

Our theory for the magnetoroton broadening starts from an observation about the screening of the electron-ionized dopant interaction. In Fig. 1 we compare the dielectric functions describing the screening of a static external potential for the zero magnetic field and fractional quantum Hall cases. We have used the random phase approximation (RPA) for the zero magnetic field static polarizability. In the strong magnetic field limit we have used the theory of Ref. 1 for the contributions to the static polarizability from within the lowest Landau level.[6] Note that in the latter case the strong scattering from the long-wavelength part of the Coulomb interaction remains unscreened while larger angle scattering may be screened more strongly than at zero field. This conclusion applies only when the ground state is a fractional quantum Hall state and additional screening from localized quasiparticles, proportional to the departure of the filling factor from a fraction with an odd denominator, should occur as a fractional quantum Hall plateau is traversed. For high mobility samples with large set-back distances, the zero-field mobility may be limited by other scattering mechanisms, possibly by accidental doping in the GaAs or the

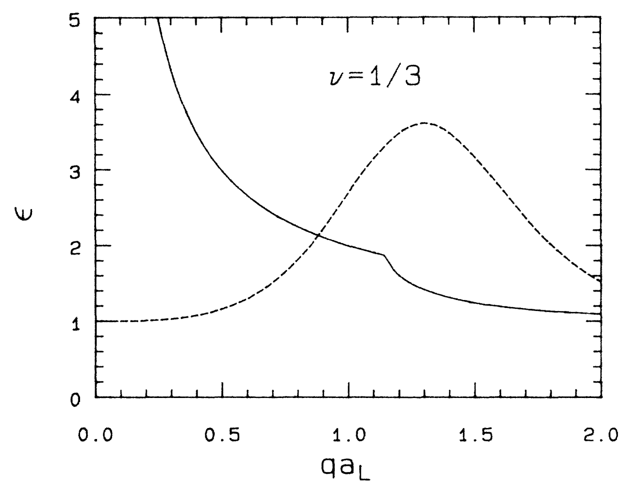

FIG. 1. Static dielectric function of a two-dimensional electron gas with $\nu=\frac{1}{3}$ vs qa_L (dashed line) compared with the random-phase-approximation static dielectric function in zero magnetic field at the same electron density (solid line: $k_F a_L=\sqrt{\nu}$).

nominally undoped $Al_xGa_{1-x}As$ spacer layer. Because of the absence of screening at long wavelength in the middle of a plateau, however, the dominant scattering mechanism in the fractional quantum Hall regime, except at extremely strong magnetic fields, is likely to continue to be Coulomb scattering from the remote ionized donors in the doped $Al_xGa_{1-x}As$. The theory outlined below is based on this assumption.

III. EXCITATIONS IN THE PRESENCE OF DISORDER

In the presence of disorder the magnetoroton states at different wave vectors are coupled. Expanding the external potential into its Fourier components gives

$$\langle \psi_{k'} | \sum_i V(z_i) | \psi_k \rangle = \int \frac{d^2q}{(2\pi)^2} V(q) \langle \psi_{k'} | \rho_q | \psi_k \rangle$$
$$= \int \frac{d^2q}{(2\pi)^2} V(q) \langle \psi_{k'} | \bar{\rho}_q | \psi_k \rangle$$
$$= V(k'-k) M(k',k) , \qquad (2)$$

where $M(k',k)$ is defined by

$$\frac{\langle \psi_0 | \bar{\rho}_{-k'} \bar{\rho}_q \bar{\rho}_k | \psi_0 \rangle}{[\bar{s}(k')\bar{s}(k)]^{1/2}} = (2\pi)^2 \delta^2(k'-q-k) M(k',k) \qquad (3)$$

and, as discussed below, is related to the three-point correlation function of the isotropic ground state. The factor in the denominator on the left-hand side of Eq. (3) comes from normalizing the magnetoroton wave functions

$$\bar{s}(k) = N^{-1} \langle \psi_0 | \bar{\rho}_{-k} \bar{\rho}_k | \psi_0 \rangle$$
$$= s(k) - 1 + \exp(-|k|^2/2) , \qquad (4)$$

where $s(k)$ is the static structure factor for the ground state;[1] $s(k)$ is related to the pair-distribution function, $g(r)$, by

$$s(k) = 1 + \frac{\nu \int d^2r}{2\pi} [g(r)-1] e^{-i\mathbf{k}\cdot\mathbf{r}}$$

and $\nu/2\pi$ is the electron density. The bare excitation energies of the magnetoroton states, $\Delta_0(k)$, are given as a functional of $\bar{s}(k)$ and the electron-electron interaction, $V_{ee}(q)$, in Ref. 1,

$$\Delta_0(k) = \int \frac{d^2q}{(2\pi)^2} V_{ee}(q) [e^{-|k|^2/2} (e^{(kq^*-k^*q)/2} - 1) \bar{s}(q)$$
$$+ (e^{\mathbf{k}\cdot\mathbf{q}} - e^{k^*q}) \bar{s}(k+q)] / \bar{s}(k) . \qquad (5)$$

Because of the extremum in $\Delta_0(k)$, the broadening of these modes must be treated self-consistently even for arbitrarily weak disorder. Since the excited states are labeled by wave vector the problem is formally identical to the broadening of single-particle states by disorder, except that the bare dispersion relation is different and the effective potential is nonlocal [see Eq. (2)]. We treat the broadening in a self-consistent Born approximation. Our approach, then, is entirely analogous to that adopted by Kallin and Halperin[7] in discussing the magnetoplasmon excitations which occur near $\hbar\omega_c$, and to facilitate comparison we adopt their notation. The Dyson equation for the configuration averaged magnetoroton propagator is

$$D^{-1}(k,\omega) = \omega - \Delta_0(k) - \Pi(k,\omega) , \qquad (6a)$$

where the magnetoroton self-energy is

$$\Pi(k,\omega) = \frac{1}{L^2} \sum_{\mathbf{q}} \langle V(-q) V(q) \rangle_c$$
$$\times |M(k+q,k)|^2 D(k+q,\omega) \qquad (6b)$$

and $\langle \cdots \rangle_c$ denotes a configuration average over the position of the ionized impurities in the $Al_xGa_{1-x}As$. Equation (6b) is readily solved once we prescribe a model for the ionized impurity potential and an approximate expression is derived for $M(k+q,k)$. We treat these two points in turn in the following sections.

IV. THE DISORDER POTENTIAL

The position of the ith ionized donor may be specified by its coordinate projected onto the plane of the two-dimensional electron gas ($\mathbf{R}_i$) and a set-back distance d_i. The two-dimensional Fourier transform of the effective interaction between an electron in the gas and a single-impurity is[8]

$$U_I^i(q) = \exp(-i\mathbf{q}\cdot\mathbf{R}_i) \frac{2\pi e^2}{\epsilon |q|} \frac{\exp(-|q| d_i)}{[1+(|q| z_0/3)]^3} , \qquad (7)$$

where ϵ is the dielectric function of the medium, which is taken to be uniform and z_0 is the average penetration of the interface electrons into the GaAs. If we assume that the ionized donors in the $Al_xGa_{1-x}As$ are set back by a minimum distance α and are distributed randomly in a volume of thickness t then

$$\langle U(-q)U(q)\rangle_c = n_I \left[\frac{2\pi e^2}{\epsilon |q|}\right]^2 \frac{(e^{-2|q|\alpha}-e^{-2|q|(\alpha+t)})}{(2tq)[1+(qz_0/3)]^6} , \quad (8)$$

where n_I is the areal density of the ionized donors. Actually there must be some correlation in the projected positions of the ionized donors, at least sufficient to prevent oscillations in their density on a macroscopic scale. These may be incorporated phenomenologically by multiplying Eq. (9) by a factor $S_I(q)$, where $S_I(q)$ is a two-dimensional static structure factor corresponding to the projection of ionized donor positions onto the x-y plane.

V. THREE-POINT CORRELATIONS

Finally we must consider the matrix element which defines $M(k',k)$:

$$\langle \psi_0 | \bar{\rho}_{-k-q}\bar{\rho}_q\bar{\rho}_k | \psi_0\rangle = N\Bigg[e^{-|k|^2/2}e^{-|k+q|^2/2}e^{(k+q)^* k/2} + e^{(k+q)^* k/2}e^{-|k+q|^2/2}h(k) + e^{-|k|^2/2}e^{(k+q)^* k/2}h(k+q) + e^{-(k+q)^* k/2}h(q) + \frac{1}{N}\langle\psi_0| \sum_{l,m,n}{}' e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}_l}e^{-i\mathbf{q}\cdot\mathbf{r}_m}e^{-i\mathbf{k}\cdot\mathbf{r}_n}|\psi_0\rangle\Bigg] . \quad (9)$$

Here $h(k)=s(k)-1$. Equation (9) follows from the definition for $\bar{\rho}_k$ implicit in Eq. (1) by separating terms where one, two, and three of the indices in the sums over particles are distinct. The prime on the sum in Eq. (9) means it is restricted to distinct points and this has allowed us to drop the projection onto the lowest Landau level. The first four terms on the right-hand side arise from the cases where either all the points or pairs of points are identical. These depend only on $s(k)$ and, at least for $\nu=1/m$ or $\nu=1-1/m$, where m is an odd integer, accurate approximations are available.[3,9] The last term is directly related to the ternary correlation function of the ground state, about which less is known. Fortunately, to describe the Coulomb scattering from remote ionized donors the exponential factor, $e^{-2|q|\alpha}$, in Eq. (8) allows us to use an approximation which need be valid only at long wavelength. Such an approximation is available from exact sum rules obeyed by the ternary correlation functions. As shown in the Appendix, to lowest order in $q\neq 0$

$$M(k+q,k)=i(\mathbf{q}\times\mathbf{k})_z \quad (10)$$

for any isotropic state formed entirely in the lowest Landau level. The conclusions we reach below follow directly from this simple result.

VI. RESULTS AND DISCUSSION

We have performed numerical calculations for the effect of remote ionized donor scattering on the collective excitations of the $\nu=\frac{1}{3}$ and $\nu=\frac{2}{3}$ fractional quantum Hall states using Eq. (10) for $M(k+q,q)$.[10] The Dyson equations [Eqs. (6)] were solved self-consistently for a variety of circumstances and compared with the approximate solutions which are obtained by setting $D(k+q,\omega)\simeq D(k,\omega)$ in Eq. (6b). The approximate solution may be written in the form

$$D(k,\omega)=\int \frac{d\omega'}{\pi}\frac{\rho(k,\omega')}{\omega-\omega'-i\eta} , \quad (11)$$

where $\rho(k,\omega)$, the spectral density of the magnetoroton propagator at wave vector k, is given by

$$\rho(k,\omega)=\left[1-\left[\frac{\omega-\Delta_0(k)}{\Gamma_\nu(k)}\right]^2\right]^{1/2} , \quad |\omega-\Delta_0(k)| \lesssim \Gamma_\nu(k) \quad (12a)$$

$$\Gamma_\nu^2(k)=\frac{4}{L^2}\sum_{\mathbf{q}}\langle U(-q)U(q)\rangle_c |M(k+q,k)|^2 . \quad (12b)$$

In all the cases we have studied Eq. (12) provide an excellent approximation to the numerical solutions to the Dyson equation. This was expected because of the importance of small-angle scattering and the remaining discussion will be in terms of these expressions. We should remark that our approach can be formally justified only in the weak scattering limit, i.e., only when $\Gamma_\nu(k) \ll \Delta(k)$. In examining its predictions for the strong scattering limit below, we do not expect numerical accuracy but rather a qualitative indication of the significance of set-back distances and magnetic fields for the occurrence of the fractional quantum Hall effect.

When $\Gamma_\nu(k)>\Delta_0(k)$ in Eqs. (12) $\rho(k,\omega)$ remains nonzero for $\omega<0$. This implies that the incompressible liquid ground state is no longer stable and the true ground state is probably of the nature of a Wigner glass.[11] Since the fractional quantum Hall effect will not occur in that regime the condition $\Gamma_\nu(k)<\Delta_0(k)$ is a necessary one for the occurrence of the fractional quantum Hall effect. From Eqs. (12), (10), and (8) we have that for $\alpha \gtrsim a_L$

$$\Gamma_\nu(k)=C\frac{ka_L}{\sqrt{2}}\left[\frac{a_L}{\alpha}\right] . \quad (13)$$

In Eq. (13) and below energies are in units of $e^2/\epsilon a_L$ except where noted and C is a correction factor, which must be smaller than one to account for the weakening of the scattering due to correlations in ion positions, finite donor-layer thickness, and finite electron-layer thickness. Since $ka_L \sim \sqrt{2\pi\nu}$ at the magnetoroton minimum ($k=k^*$), we have, by requiring $\omega_0(k^*)>\Gamma_\nu(k^*)$ the following condition for the occurrence of the fractional Quantum Hall effect

$$\frac{\alpha}{a_L}\sim\frac{C\sqrt{\pi\nu}}{\Delta(k^*)} . \quad (14)$$

For a given filling factor Eq. (14) may be interpreted as providing either a minimum set-back distance or, alternately a threshold magnetic field ($a_L = 256.6$ A$/\sqrt{H}$ for H expressed in tesla). Detailed calculations, assuming $z_0 \sim 100$ A, $t \sim 100$ A and a model for the density fluctuations of the remote donors typically give $C \sim \frac{1}{4}$. Using this value, taking as an example $\alpha = 300$ A and recalling that $\Delta(k^*)$ in units of $(e^2/\epsilon a_L)$ equals[1] 0.079, 0.018, and 0.006 gives threshold magnetic fields of 7.7, 89, and 570 T for $\nu = \frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{7}$, respectively. Thus the absence of plateaus near $\nu = \frac{1}{5}$ and $\nu = \frac{1}{7}$ at available fields would not necessarily imply a Wigner crystal ground state at these filling factors in the ideal system.[12,13]

It is interesting to consider what connections might exist between Eq. (12b) and the Landau-level broading parameters which appear in the self-consistent Born approximation theory of magnetotransport.[14] In that theory the maximum value of σ_{xx} as the chemical potential passes through the Nth Landau level is

$$\sigma_{xx}^N = \frac{e^2}{\pi^2 h}\left[\frac{\Gamma_N^{xx}}{\Gamma_N}\right]^2 , \tag{15a}$$

where, in our notation and for $N = 0$

$$(\Gamma_0^{xx})^2 = \frac{4}{L^2}\sum_{\mathbf{q}} \langle U(-q)U(q)\rangle_c q_y^2 \exp(-|q|^2/2) \tag{15b}$$

and Γ_0, which is directly related to the Landau-level width, is given by

$$(\Gamma_0)^2 = \frac{4}{L^2}\sum_{\mathbf{q}} \langle U(-q)U(q)\rangle_c \exp(-|q|^2/2) . \tag{15c}$$

It follows that for $\alpha \gtrsim 1$, $\Gamma_\nu(k) = k\Gamma_0^{xx}$. If there is a regime of temperature where the self-consistent Born approximation is valid and Γ_0 could be estimated independently,[15] Eq. (15a) could be combined with magnetotransport data to give an experimental estimate of $\Gamma_\nu(k)$. It would be useful to correlate such estimates with the occurrence of the fractional quantum Hall effect.

Finally, we make connection with the recent experiments of Boebinger *et al.*[16] on the temperature and magnetic field dependence of the fractional quantum Hall effect. These authors find a temperature dependence which, except at extremely low temperatures, indicates activated behavior. When the influence of impurities on the conductivity is treated is the lowest order of perturbation theory via a memory-function approach[17] one finds that the low-temperature conductivity is activated with an activation energy which is $\Delta(k^*) = \Delta_0(k^*) - \Gamma_\nu(k^*)$. Below, we associate the experimental activation energy with the minimum energy of the disorder broadened band of magnetoroton excitations. In this description the thermally populated gas of magnetorotons is providing a channel for dissipation. The results must be interpreted with some caution, however. For example, it has been suggested to us[18] that the magnetorotons may ultimately not provide a channel for dissipation but rather act in a manner similar to that of phonons in superfluid He. The resolution of this issue may well involve an understanding of some properties of the fractional quantum Hall states which has still not been achieved.

In Fig. 2 we have reproduced the experimental results from Ref. 16. (Note that their definition differs from ours by a factor of 2.) These data were taken on a series of $\nu = \frac{1}{3}$ and $\nu = \frac{2}{3}$ plateaus on similar samples where the magnetic field was varied without changing the disorder potential by applying a gate voltage. In temperature units, using parameters appropriate to GaAs,

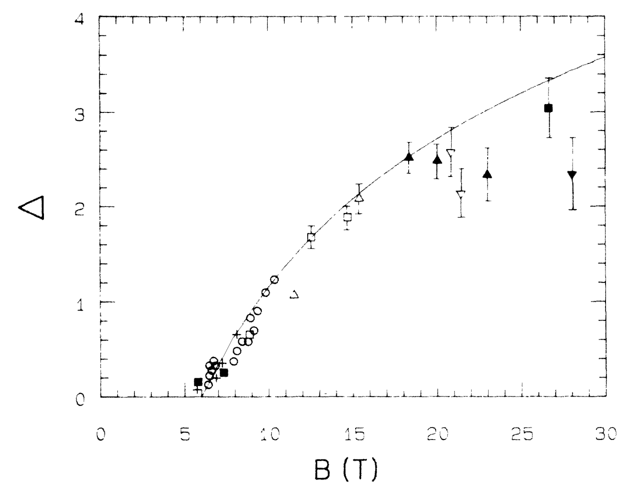

FIG. 2. Δ in temperature units versus B. The experimental data are from Ref. 16 and the solid line is the theory [Eq. (17)] which has been adjusted to give the correct threshold fold. This curve was calculated with $z_0 = 150$ Å.

$$\Delta = 51\sqrt{B}\,[\Delta_0(k^*) - \Gamma_\nu(k^*)] , \tag{16}$$

where $\Delta_0(k^*)$ and $\Gamma_\nu(k^*)$ are in units of $e^2/\epsilon a_L$ and B is in tesla. When the known parameters $(\alpha, t, \ldots)$ of the experimental samples are used the theory, which by particle-hole symmetry gives the same results for $\nu = \frac{1}{3}$, and $\nu = \frac{2}{3}$, gives $\Gamma_\nu(k^*) > \Delta_0(k^*)$ (i.e., no fractional quantum Hall effect) through most of the range of magnetic fields studied experimentally. We interpret this as indicating that the estimated values of Γ_ν should be reduced by about 40% as a result of corrections to our model potential and strong scattering effects. Indeed it would have been quite surprising if the theory were able to predict *a priori* detailed results for a given series of samples and we aim only to describe the magnetic field dependence Δ. The main result of our theory is that $\Gamma_\nu(k^*)$ should vary approximately as $B^{-1/2}$ [see Eq. (13)] as a consequence of Eq. (10). In fact $\Gamma_\nu(k^*)B^{1/2}$ will decrease slowly with B because of the increasing importance of a finite electron-layer thickness[19,20] but this may be compensated for if some short-range scatterers are present. [A dimensional analysis of Eq. (12b) suggests that $\Gamma_\nu(k^*) \sim B^{1/2}$ for short-range scatterers.] Taking a $B^{-1/2}$ dependence for $\Gamma_\nu(k^*)$ and adjusting the constant to agree with the experimental threshold field, H_t gives (in temperature units)

$$\Delta \simeq \Delta_{id}(H) - \Delta_{id}(H_t) , \tag{17}$$

where $\Delta_{id}(H)$ is evaluated including finite electron-layer thickness but neglecting disorder. We see in Fig. 2 that Eq. (17) is in remarkably good agreement with experiment.

VII. SUMMARY AND CONCLUSIONS

We have examined the influence of disorder on the collective excitations of a two-dimensional electron gas in the fractional quantum Hall regime. In the absence of disorder the collective excitation dispersion shows a deep minimum at a wave vector near the reciprocal-lattice vector of the Wigner crystal state of the electrons. The broadening of the collective excitation spectrum by disorder will cause this gap to collapse; correspondingly the ground state will become a Wigner glass[11] rather than an incompressible fluid[3] and the fractional quantum Hall effect will no longer occur. Based on some observations concerning screening in strong magnetic fields and on a moment sum rule for the three-point correlation function of the fluid state we have derived an approximate expression for the minimum ratio of the set-back distance to magnetic length required for the fractional Hall effect to occur. For example, for the $\nu=\frac{1}{3}$ case the remote ionized donors must be set back by a distance about 3 times greater than the magnetic length ($a_L=256.6A/\sqrt{B}$ with B in teslas).

At the moment, there is no complete theory of magnetotransport at finite temperatures in the fractional quantum Hall regime. We have presented values for the minimum excitation energy in a disordered system for comparison with activation energies extracted from magnetotransport experiments. When treated naively with a memory-function approach[17] these two quantities should be identical. Our results seem to demonstrate at least a qualitative connection but a final resolution must await a deeper understanding.

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APPENDIX

The last term in Eq. (9) is related to three-point correlations in the ground state. For q, k, and $k+q$ not equal to zero

$$\frac{1}{N}\sum_{l,m,n}{}' \langle \psi_o | e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}_l} e^{-i\mathbf{q}\cdot\mathbf{r}_m} e^{-i\mathbf{k}\cdot\mathbf{r}_n} | \psi_0\rangle = \tilde{h}^{(3)}(\mathbf{q},\mathbf{k}) = \nu^2 \int \frac{d^2 z_2}{2\pi} \int \frac{d^2 z_3}{2\pi} \exp(-i\mathbf{k}\cdot\mathbf{r}_2)\exp(-i\mathbf{q}\cdot\mathbf{r}_3) g(z_1=0,z_2,z_3) \,. \tag{A1}$$

In Eq. (A1) $g(z_1,z_2,z_3)$ is the three-point distribution function which may be conveniently expressed in the symmetric gauge occupation number representation

$$g(z_1=0,z_2,z_3) = \left[\frac{2\pi}{\nu}\right]^3 \sum_{i,j,k}{}' \langle \psi_0 | \delta^2(z_i-z_1)\delta^2(z_j-z_2)\delta^2(z_k-z_3) | \psi_0\rangle$$

$$= \nu^{-3} \sum_{s',t'=1}^{\infty} \sum_{s,t=1}^{\infty} \langle \psi_0 | c_{s'}^\dagger c_{t'}^\dagger c_t c_s n_0 | \psi_0\rangle \frac{(z_2^*)^{s'}(z_3^*)^{t'} z_2^s z_3^t \exp(-|z_2|^2/2)\exp(-|z_3|^2/2)}{2^{s+t}[s!t!(s')!(t')!]^{1/2}} \,. \tag{A2}$$

One possible approximation for $g(z_1,z_2,z_3)$ is the convolution approximation which has been useful in other contexts,[21]

$$h(z_1,z_2,z_3) = h(z_1,z_2)h(z_2,z_3) + h(z_2,z_3)h(z_3,z_1) + h(z_3,z_1)h(z_1,z_2) + \nu \int \frac{d^2 z_4}{2\pi} h(z_1,z_4)h(z_2,z_4)h(z_3,z_4) \,, \tag{A3}$$

where $h(z_1-z_2)=g(|z_1-z_2|)-1$ and $h(z_1,z_2,z_3)=g(z_1,z_2,z_3)-1-h(z_1,z_3)-h(z_1,z_2)-h(z_2,z_3)$ are, respectively, the two- and three-point correlation functions. This approximation must be used with caution in the present context, however, since it manifestly violates particle-hole symmetry in the lowest Landau level which plays an important role in the fractional quantum Hall effect.[22] For example, in the case of $\nu=1$, where $|\psi_0\rangle$ is the fully occupied state, we have from Eq. (A2) (Ref. 23)

$$h(z_1,z_2) = -e^{-|z_1-z_2|^2/2} \,,$$
$$h(z_1,z_2,z_3) = -e^{-|z_1|^2/2} e^{-|z_2|^2/2} e^{-|z_3|^2/2} \left(e^{(z_1 z_2^* + z_2 z_3^* + z_3 z_1^*)/2} + e^{(z_1^* z_2 + z_2^* z_3 + z_3^* z_1)/2}\right) \,, \tag{A4}$$

which can be Fourier transformed to give

$$\tilde{h}^{(3)}(\mathbf{q},\mathbf{k}) = e^{-|k|^2/2} e^{-|q|^2/2} \left(e^{-q^* k/2} + e^{-q k^*/2}\right) \,. \tag{A5}$$

The reciprocal space form of the convolution approximation takes the form

$$\tilde{h}^{(3)c}(\mathbf{q},\mathbf{k}) = h(|k|)h(|q+k) + h(|q+k|)h(|q|)$$
$$+ h(|q|)h(|k|)$$
$$+ h(|q+k|)h(|q|)h(|k|) \,, \tag{A6}$$

where

$$h(k)=\frac{\nu}{2\pi}\int d^2z_2 h(z_1=0,z_2)\exp(-i\mathbf{k}\cdot\mathbf{r}) .$$

Note that in the case of $\nu=1$ $[h(|k|)=-e^{-|k|^2/2}]$, $\tilde{h}^{(3)c}(q,k)$ when substituted into Eq. (9), fails to give the correct result that

$$\langle\psi_0|\bar{\rho}_{-k-q}\bar{\rho}_q\bar{\rho}_k|\psi_0\rangle=0 .$$

This is a serious deficiency in the current context and it is fortunate that, as mentioned in the text, we require an expression for $M(k+q,k)$ only for $q\ll 1$.

To consider the small-angle scattering limit of $M(k+q,k)$ we expand the factor

$$\exp(-i\mathbf{q}\cdot\mathbf{r}_3)=\exp\left[-\frac{i}{2}(q^*z_3+qz_3^*)\right]$$

in Eq. (A1) and use the following sum rule valid for any isotropic state in the lowest Landau level for $m=0$ or $m=1$:

$$\nu\int\frac{d^2z_3}{2\pi}z_3^m[g(z_1,z_2,z_3)-g(z_1,z_2)] =-g(z_1,z_2)(z_1^m+z_2^m) . \quad (A7)$$

Equation (A7) follows from Eq. (9) and Eq. (A1) for $m=0$ and $m=1$, respectively, using the facts that the number of electrons, $N=\sum_t c_t^\dagger c_t$, and the center of mass,

$$z_0=N^{-1}\sum_t c_{t+1}^{+}c_t\sqrt{2(t+1)} ,$$

are the constants of the motion. Actually, Eq. (A7) holds for all m when Laughlin's approximation is used for the ground state.[24] Using Eq. (A7) for $m=0,1,2$ and another sum rule[25] which follows from Eq. (9) and the fact that $L=\sum_t tc_t^\dagger c_t$ is a constant of the motion gives, up to second order in $|q|$,

$$\tilde{h}^{(3)}(\mathbf{q},\mathbf{k})=-2h(\mathbf{k})-\mathbf{q}\cdot\nabla_k h(|\mathbf{k}|)-\frac{1}{2}\sum_{i,j}q_iq_j\frac{\partial^2}{\partial k_i\partial k_j}h(k)-\tfrac{1}{4}|q|^2\mathbf{k}\cdot\nabla_{\mathbf{k}}h(|\mathbf{k}|)|+\frac{|q|^2}{2}h(k)+\cdots . \quad (A8)$$

Substituting Eq. (A8) into Eqs. (9) and (3) gives

$$M(k+q,q)=i(\mathbf{q}\times\mathbf{k})_z-\frac{(\mathbf{q}\times\mathbf{k})_z^2}{4}+\frac{(\mathbf{q}\cdot\mathbf{k})^2}{8}+\frac{i}{4}(\mathbf{q}\cdot\mathbf{k})(\mathbf{q}\times\mathbf{k})_z+\frac{1}{4|k|^2}\frac{|k\cdot\nabla_k\bar{s}(k)|}{\bar{s}(k)}[(\mathbf{q}\cdot\mathbf{k})^2-(\mathbf{q}\times\mathbf{k})_z^2]+\cdots . \quad (A9)$$

In Eqs. (A8) and (A9) the result given for the leading-order term is a general property of isotropic states in the lowest Landau level, but the second-order term applies only when ν^{-1} is an odd integer and depends on Laughlin's[3] approximate ground-state wave function.

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