Collective excitations of fractional Hall states and Wigner crystallization in higher Landau levels

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An expression has been derived for the collective-excitation dispersion for fractional Hall states which occur in higher orbital Landau levels in terms of the electron pair-correlation function in these states. Explicit results for the $n=1$ Landau level have been obtained at fractional filling factors $\nu=\frac{1}{3}$ and $\nu=\frac{1}{5}$ based on Laughlin's trial wave functions for the ground state. The results at $\nu=\frac{1}{3}$ are qualitatively different from those in the lowest Landau level and are consistent with a weak quantum Hall effect at this fraction for $n=1$. The results for $\nu=\frac{1}{5}$ are similar to those in the $n=0$ Landau level but the collective excitations have a higher energy. We associate this increase with a decrease in the fractional filling factor at which Wigner crystallization occurs. A moment sum rule is derived for pair-correlation functions in higher Landau levels.

Recently Girvin *et al.*[1] have suggested that a generalized single-mode approximation (SMA) ought to be valid in calculating the collective excitation energies of the fractional Hall states.[2] By fractional Hall states we mean the highly correlated incompressible quantum fluid states which occur in the lowest Landau level when the Landau-level filling factor, ν, is $1/m$ or $1-1/m$. In the SMA it is assumed that the part of the oscillator strength which is available within the lowest Landau level is nearly exhausted by a single excited state

$$|\psi_{\mathbf{k}}^0\rangle=\frac{\rho_{\mathbf{k}}^{0,0}|\psi_0^0\rangle}{(\langle\psi_0^0|\rho_{-\mathbf{k}}^{0,0}\rho_{\mathbf{k}}^{0,0}|\psi_0^0\rangle)^{1/2}}\ . \tag{1}$$

In Eq. (1) $|\psi_0^0\rangle$ is the ground state for one of the values of ν indicated above in the extreme strong magnetic field limit where Landau-level mixing can be ignored, and $\rho_{\mathbf{k}}^{0,0}$ is the Fourier component of the density operator projected onto the lowest Landau level. Comparisons with calculations based on direct numerical diagonalization of the Hamiltonian for systems with a small number of electrons[3] have indicated that this approach is extremely accurate. In this article we report on a calculation which generalizes these results to the fractional Hall states where higher orbital Landau levels are partially occupied. The existence of such states has been clearly established in magnetotransport experiments[4] and their ground-state properties have been discussed previously from a theoretical point of view.[5] More recent experimental[6] and theoretical[7,8] developments suggests that the sequence of fractional Hall states may be somewhat different in higher orbital Landau levels. In particular, the Laughlin states in higher orbital Landau levels[2,5] approach exact ground states as the range of a repulsive interaction is reduced only[7] when the inverse fractional filling factor, ν^{-1}, satisfies $\nu^{-1}\geq 3+2n$. (Here n is the orbital Landau level index.) For example, numerical calculations[8] for $\nu=\frac{1}{3}$ and $n=1$ suggest that with a Coulombic electron-electron interaction the ground state is not well approximated by Laughlin's state and may, in fact, not be incompressible.

It will be an important simplification in what follows that a plane-wave function of any electron coordinate, when expressed in terms of inter-Landau-level and intra-Landau-level ladder operators, can be factored as[9]

$$\exp(-i\mathbf{k}\cdot\mathbf{r}_i)=\exp\left[\frac{-ka_i^\dagger}{\sqrt{2}}\right]\exp\left[\frac{-k^*a_i}{\sqrt{2}}\right]\times\exp\left[\frac{-ikb_i}{\sqrt{2}}\right]\exp\left[\frac{-k^*b_i^\dagger}{\sqrt{2}}\right], \tag{2}$$

where a_i and $a_i^\dagger$ are the inter-Landau-level ladder operators and b_i and $b_i^\dagger$ are the intra-Landau-level operators and k is a complex number representation of the wave vector. All lengths will be expressed in units of $l\equiv(\hbar c/eB)^{1/2}$ and all energies in units of $e^2/\epsilon l$. Following earlier work,[5] we define a higher Landau-level generalization of the Laughlin states by

$$|\psi_0^n\rangle=\prod_{i=1}^{N}\frac{(a_i^\dagger)^n}{\sqrt{n!}}|\psi_0^0\rangle\ , \tag{3}$$

i.e., in the expansion of these states as sums over products of single-particle states the only change is to raise the Landau-level index of each single-particle state from 0 to n. Actually the approximate ground state is not $|\psi_0^n\rangle$, which only has states in the nth Landau level occupied but $|\phi_0^n\rangle$ which differs by having the lower Landau levels completely filled as well. Since the lower Landau levels are completely frozen out in the strong magnetic field limit which we consider here, this complication can be ignored in discussing the intra-Landau-level part of the ex-

citation spectrum but we shall return to it later in discussing the pair correlation function. Throughout we will use ν to refer to the fractional part of the Landau-level filling factors. For Laughlin states $\nu^{-1}=3,5,7,\ldots$ and as mentioned above, they become exact ground states in the limit of short-range repulsive interactions[7] for $\nu^{-1}\geq 3+2n$. Based on experience for $n=0$ we expect them to be accurate for realistic interactions whenever this inequality is satisfied. Their reliability for $\nu^{-1}<3+2n$ remains uncertain.

In analogy with Eq. (1) we approximate the many-body states describing collective excitations in higher Landau levels by

$$|\psi_k^n\rangle=\frac{\rho_k^{n,n}|\psi_0^n\rangle}{(\langle\psi_0^n|\rho_{-k}^{n,n}\rho_k^{n,n}|\psi_0^n\rangle)^{1/2}}\ , \tag{4}$$

where[9]

$$\begin{aligned}\rho_k^{n,n}&=L_n(|k|^2/2)\sum_i |n\rangle_{ii}\langle n|\exp\left[\frac{-ikb_i}{\sqrt{2}}\right]\\ &\quad\times\exp\left[\frac{-ik^*b_i^\dagger}{\sqrt{2}}\right]\\ &=L_n(|k|^2/2)\sum_i |n\rangle_{ii}\langle n|B_i(k)\end{aligned} \tag{5}$$

is the projection of the density operator onto the nth Landau level, $L_n(x)$ is the Laguerre polynomial, and $|n\rangle_{ii}\langle n|$ projects the ith particle onto the nth Landau level. Therefore,

$$\begin{aligned}\langle\psi_0^n|\rho_{-k}^{n,n}\rho_k^{n,n}|\psi_0^n\rangle&=[L_n(|k|^2/2)]^2\langle\psi_0^n|\bar\rho_{-k}\bar\rho_k|\psi_0^n\rangle\\ &=N[L_n(|k|^2/2)]^2\bar s(k)\ ,\end{aligned} \tag{6a}$$

where

$$\bar\rho_k\equiv\sum_i B_i(k)\ . \tag{6b}$$

Note than when Eq. (3) is used to estimate ground states in higher Landau levels, $\bar s(k)$ is independent of Landau level because $\bar\rho_k$ involves only intra-Landau-level operators.

Similarly

$$\langle\psi_k^n|H|\psi_k^n\rangle=\langle\psi_0^n|H|\psi_0^n\rangle+\Delta^n(k)\ , \tag{7a}$$

where

$$\begin{aligned}\Delta^n(k)&=\langle\psi_0^n|[\rho_{-k}^{n,n},[H,\rho_k^{n,n}]]|\psi_0^n\rangle/\langle\psi_0^n|\rho_{-k}^{n,n}\rho_k^{n,n}|\psi_0^n\rangle\\ &=\int\frac{d^2q}{(2\pi)^2}V_{ee}(q)[L_n(|q|^2/2)]^2[(e^{(q^*k-k^*q)/2}-1)\bar s(q)e^{-|k|^2/2}+(e^{k\cdot q}-e^{k^*q})\bar s(k+q)]/\bar s(k)\end{aligned} \tag{7b}$$

The second form of Eq. (7b) follows from the first by noting that, within the subspace associated with the nth Landau level,

$$[H,\rho_k^{n,n}]=\tfrac{1}{2}\int\frac{d^2q}{(2\pi)^2}V_{ee}(q)[L_n(|q|^2/2)]^2L_n(|k|^2/2)[\bar\rho_{-q}\bar\rho_q,\bar\rho_k] \tag{7c}$$

and using

$$[\bar\rho_{k_1},\bar\rho_{k_2}]=(e^{k_1^*k_2/2}-e^{k_1k_2^*/2})\bar\rho_{k_1+k_2}\ . \tag{7d}$$

[$V_{ee}(q)$ is the Fourier transform of the effective electron-electron interaction in the two-dimensional (2D) system.] The factor $[L_n(|k|^2/2)]^2$ cancels between the numerator and denominator of Eq. (7b) so that the expression for the excitation energy differs from the corresponding expression for $n=0$, which was derived in Ref. 1, only through the appearance of the factor $[L_n(|q|^2/2)]^2$ in the wave-vector integral.

In Fig. 1 we compare the results obtained previously for $\Delta^0(k)$ at $\nu=\frac{1}{3}$ with $\Delta^1(k)$ obtained by evaluating Eq. (7b) with the same $\bar s(k)$ but with $n=1$. Taking into account the spin degeneracy and particle-hole symmetry, we see that the latter curve describes the strong-field limit of the collective excitation spectrum for total filling factor $\nu_t=\frac{7}{3},\frac{8}{3},\frac{10}{3}$, and $\frac{11}{3}$. For $n=1$ we see that the collective excitation energies are smaller than for $n=0$. In fact, the small k region of this curve ($ka_L\lesssim 0.5$) is extremely sensitive to the behavior of $\bar s(k)$ at large k ($ka_L\gtrsim 4$), where its value for the Laughlin states is less accurately known. [$\bar s(k)$ is simply related to the Fourier transform of the electron pair-correlation function (see below).] Unlike the $n=0$ case, $\Delta^1(k)$ becomes negative for $ka_L\lesssim 0.5$ with very small changes in $\bar s(k)$. Recent experimental[6] and theoretical[7,8] work is consistent with the notion that this sensitivity reflects the fact that for $n=1$ the incompressible fluid state at $\nu=\frac{1}{3}$ is only weakly stable, if it is stable at all. The results at $\nu=\frac{1}{5}$ for the $n=1$ Landau level are entirely different as we see in Fig. 2. At this fractional filling factor the nature of the excitation spectrum is similar to that for the $n=0$ Landau level except that the excitation energies are increased. [The extreme sensitivity in the small k values of $\Delta^1(k)$ found for $\nu=\frac{1}{3}$ is *not* present at $\nu=\frac{1}{5}$.] We believe that the increase in the excitation energies reflects the fact that at $\nu=\frac{1}{5}$ we are farther from the Wigner crystallization transition for the higher Landau level.[10] We will see below that this idea is corroborated by comparisons of the ground-state energies of liquid and crystalline states.

From Eq. (6) it can be shown that for the higher

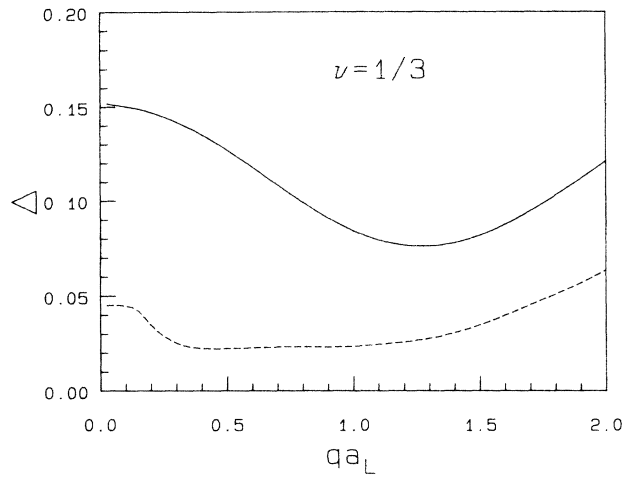

FIG. 1. Collective excitation dispersion for a fractional filling, $\nu=\frac{1}{3}$, of the $n=0$ Landau level (solid line) and the $n=1$ Landau level (dashed line). Δ is in units of $e^2\epsilon l_0$.

Landau-level Laughlin states[2,5]

$$h^n(k)=\{[L_n(|k|^2/2)]\}^2 h^0(k)\ , \tag{8a}$$

where

$$h^n(k)=\sigma\int d^2r\exp(-i\mathbf{k}\cdot\mathbf{r})[g^n(r)-1]\ . \tag{8b}$$

σ is the areal density of electrons and $g^n(r)$ is the pair-correlation function when the electrons are in the state $|\psi_0^n\rangle$. Thus the relation between pair-correlation functions in different Landau levels is much simpler in reciprocal space $[h^n(k)]$ than that in direct space $[h^n(r)=g^n(r)-1]$, which was discussed earlier.[5] Equation (8) suggests an alternate form for the direct space relationship. For every n

$$h^n(r)=\int\frac{d^2r'}{(2\pi)}\tilde{h}(r')\exp(-|\mathbf{r}-\mathbf{r}'|^2/2)\times[L_n(|\mathbf{r}-\mathbf{r}'|^2/2)]^2\ , \tag{9}$$

where $\tilde{h}(r')$ is a common auxiliary correlation function which is defined in terms of $h^0(r)$ by this equation. $\tilde{h}(r)$ may be interpreted as representing the correlation function of the centers of the Gaussian charge distributions which describe a localized electron in the nth Landau level. Equation (8) can also be used to simply evaluate the energy of $|\psi_0^n\rangle$ using

$$N^{-1}\langle\psi_0^n|H|\psi_0^n\rangle=\epsilon_0^n=\frac{e^2\sigma}{2}\int d^2r\,r^{-1}h^n(r)=\frac{e^2}{2}\int_0^\infty dq\,h^n(q)=\frac{e^2}{2}\int_0^\infty dq[L_n(q^2/2)]^2h^0(q)\ . \tag{10}$$

In Table I we list Laughlin state energies for $\nu^{-1}=3$, 5, 7, and 9 and $n=0$, 1, and 2. These were evaluated from Eq. (10) using pair-distribution functions calculated in a modified hypernetted chain (MHNC) approximation.[11,12] They differ by less than $0.002e^2/l_0$ from the values reported previously[5] for $\nu^{-1}=3$ and 5 calculated from Monte Carlo correlation functions[13] using a different approach. Also listed in Table I are charge-density-wave (CDW) state energies for partially occupied higher Landau levels calculated self-consistently in the Hartree-Fock approximation.[14] For $n=0$, the two states are quite close in energy indicating that the transition to a Wigner crystal state is likely to occur[15] for $\nu=\nu_w$, where $\nu_w^{-1}\sim 10$. For the higher Landau levels, however, there is no indication that the transition is near at the lowest values of ν considered. This is expected on physical grounds since Wigner crystallization should occur when the size of the most localized wave function available within a Landau level becomes small compared to the average separation between electrons. The most localized wave function in the nth Landau levels is (up to a translation and a corresponding phase factor) the $m=0$ symmetric gauge eigenfunction for which $\sigma\pi\langle r\rangle_n^2=\nu(n+1)$. Thus the n dependence of the critical filling factor for Wigner crystallization should be given approximately by $\nu_w^n=\nu_w^0/(n+1)$. For example if $\nu_w^0\sim\frac{1}{10}$ then $\nu_w^1\sim\frac{1}{20}$ and $\nu_w^2\sim\frac{1}{30}$. This suggests that it may be possible to observe the fractional Hall effect at smaller fractional filling factors in higher orbital Landau levels.[4,16] We also remark that for a given value of ν^{-1} the energy preference for the fluid state over the Wigner crystal state decreases where the inequality $\nu^{-1}\geq 3+2n$ is *not* satisfied. In fact, $\nu^{-1}=5$ and $n=2$ we find the CDW state to be lower in energy than Laughlin's state. This probably is an indication that in this case the Laughlin state is not a good approximation to the ground state.

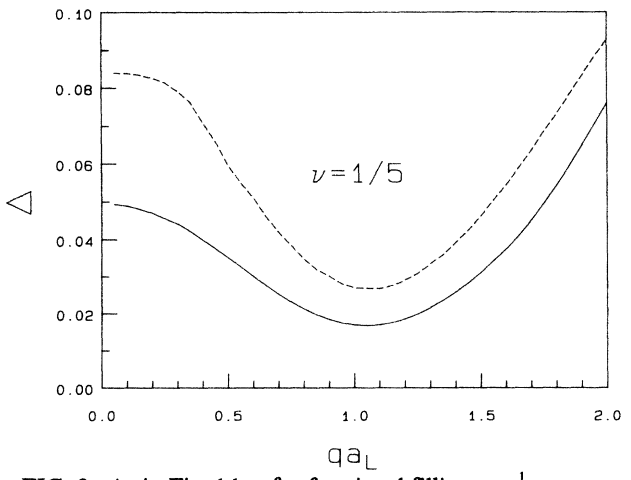

FIG. 2. As in Fig. 1 but for fractional filling $\nu=\frac{1}{5}$.

In closing we mention a moment sum rule for the pair-correlation function which holds for any electronic state of uniform density constructed entirely within the nth Landau level, $|\psi_n\rangle$. It is most easily established by using Eq. (8) to generalize its $n=0$ form,[1]

$$\int_0^\infty dr\,r^3h^0(r)=-2\nu^{-1} \tag{11}$$

to positive values of n. When combined with the charge neutrality sum rule,

$$\int_0^\infty dr\,rh^0(r)=-\nu^{-1}\ ,$$

Eq. (11) implies that

TABLE I. Energies per electron for Laughlin and charge-density-wave (CDW) states in nth Landau level at filling factor $\nu=1/m$. For $m=1$ both states reduce to the full Landau-level state. The energies are in units of e^2/l.

	$n=0$		$n=1$		$n=2$	
m	Laughlin	CDW	Laughlin	CDW	Laughlin	CDW
1	−0.627	−0.627	−0.470	−0.470	−0.401	−0.401
3	−0.409	−0.388	−0.325	−0.316	−0.265	−0.256
5	−0.327	−0.322	−0.294	−0.289	−0.247	−0.250
7	−0.280	−0.279	−0.264	−0.261	−0.252	−0.240
9	−0.250	−0.250	−0.244	−0.238	−0.233	−0.225

$$h^0(k)=-1+k^2/2+\cdots .$$

Expanding the Laquarre polynomial in Eq. (8) gives

$$h^n(k)=-1+(2n+1)k^2/2+\cdots ,$$

or in direct space

$$\int_0^\infty dr\, r^3 h^n(r)=-2(2n+1)\nu^{-1} . \tag{12}$$

The pair-correlation function for $|\phi_n\rangle$ which differs from $|\psi_n\rangle$ only by completely filling all the lower Landau levels is related to $h^n(r)$ by

$$h(r)=(n+\nu)^{-2}\left[-\sum_{s,s'=0}^{n-1} e^{-r^2/2}L_s(r^2/2)L_{s'}(r^2/2) -2e^{-r^2/2}\nu L_n(r^2/2)\sum_{s=0}^{n-1}L_s(r^2/2) +\nu^2 h^n(r)\right] . \tag{13}$$

Equation (13) follows from the expression for $h(k)$ in terms of density operators

$$h(k)\equiv N^{-1}\langle\psi_n|\rho_{-k}\rho_k|\psi_n\rangle-1, \quad k\neq 0 \tag{14}$$

and the expression for ρ_k given in Ref. 9.

$$\rho_k=\sum_{n',n}\sum_i |n'\rangle_{i\,i}\langle n|\left[\frac{n!}{(n')!}\right]^{1/2} \times\left[\frac{-ik}{\sqrt{2}}\right]^{n'-n}L_n^{n'-n}\left[\frac{k^2}{2}\right]B_i(k) . \tag{15}$$

Contributions to $h(k)$ from full Landau levels involve only terms where the particle indices in the ρ_k and ρ_{-k} factors are identical, i.e., there are no correlations involving full Landau levels. Using Eq. (13) we can verify that this full pair-correlation function which describes correlations among all the electrons when the nth Landau level is partially full obeys Eq. (11). This sum rule whose fundamental importance for the fractional Hall effect has been emphasized previously,[1] also holds for the classical two-dimensional one-component plasma[17] (2D OCP), where it reflects the inhibition of long-wavelength density fluctuations by the long-range $\ln(r)$ interaction. Thus it seems that the connection between the 2D OCP and the two-dimensional electron gas (2D EG) in a strong magnetic field, which was first made by Laughlin[2] for $\nu=1/m$ and later extended[12] to fractional values of ν on the basis of Eq. (11), persists for $\nu>1$.

To summarize, we have derived and evaluated an expression for the collective excitation dispersion of fractional quantum Hall states in higher orbital Landau levels. For $\nu=\frac{1}{3}$ and $n=1$ we find that: (i) the collective excitation energy is reduced compared to $n=0$, (ii) the pronounced minimum in the dispersion which occurs for wave vectors near the Wigner crystal's primitive reciprocal-lattice vector for $n=0$ is absent for $n=1$, and (iii) the dispersion at long wavelength, $qa_L\lesssim 0.5$, is extremely sensitive to the approximation used for the ground-state pair-correlation function. On the other hand, for $\nu=\frac{1}{5}$, we find that the collective excitation dispersion is qualitatively similar for $n=0$ and $n=1$ except that the excitation energies are higher for $n=1$ Based on comparisons of Wigner crystal and Laughlin state energies for $\nu^{-1}=3$, 5, 7, and 9 and $n=0$, $n=1$, and $n=2$ we conclude that these changes occur because (i) the transition to a Wigner crystal ground state occurs at smaller fractional filling factors in higher orbital Landau levels and (ii) the incompressible fluid ground state, which we approximate by Laughlin states, is only marginally stable, or possibly unstable, where the inequality $\nu^{-1}\geq 3+2n$ is not satisfied. This inequality is associated with a qualitative change is small distance correlations for uniform states which occurs[7] at $\nu^{-1}=3+2n$. The decrease of the excitation energy at $\nu=\frac{1}{3}$ reflects the marginal stability of the incompressible state at this fraction for $n=1$. The increase at $\nu=\frac{1}{5}$ on going from $n=0$ to $n=1$ reflects the decrease in the filling factor at which Wigner crystallization should occur. Both these changes are in accord with previous work on the energies of localized quasiparticle excitations[7] and the marginal stability of the $\nu=\frac{1}{3}$ state for $n=1$ is in accord with recent small system calculations.[8] We must emphasize, however, that all the theoretical results presented here refer to the strong magnetic field limit where Landau-level mixing can be ignored. For a given 2D electron density, higher orbital Landau levels are partially filled at lower magnetic field making the strong field limit more difficult to achieve ex-

perimentally. Furthermore, a higher 2D electron density is often achieved with a reduced set-back distance for the remote ionized donors, making disorder effects stronger and, at least partially, destroying the fractional quantum Hall effect. Our study suggests that some interesting physics can be probed if these obstacles can be overcome to allow the effect to be studied for $n=1$ in conditions comparable to those which have been achieved for $n=0$.

[1]S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. Lett. **54**, 581 (1985).

[2]R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[3]F. D. M. Haldane and E. H. Rezayi, Phys. Rev. Lett. **54**, 237 (1985).

[4]E. E. Mendez, L. L. Chang, M. Heiblum, L. Esaki, M. Naughton, K. Martin, and J. Brooks, Phys. Rev. B **30**, 7310 (1984); G. Ebert, K. von Klitzing, J. C. Maan, G. Remenyi, C. Probst, G. Weigmann, and W. Schlapp, J. Phys. C **17**, L775 (1984).

[5]A. H. MacDonald, Phys. Rev. B **30**, 3550 (1984).

[6]R. G. Clark, R. J. Nicholas, A. Usher, C. T. Foxon, and J. J. Harris, Surf. Sci. (to be published) and unpublished work.

[7]A. H. MacDonald and S. M. Girvin, Phys. Rev. B **33**, 4431 (1986).

[8]F. D. M. Haldane and E. H. Rezayi (private communication).

[9]A. H. MacDonald, H. C. A. Oji, and S. M. Girvin, Phys. Rev. Lett. **55**, 2208 (1985). Lengths are in units of $l_0=(\hbar c/eB)^{1/2}$.

[10]This point is discussed in detail by S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. B (to be published).

[11]J. P. Hansen and D. Levesque, J. Phys. C **14**, L603 (1981).

[12]A. H. MacDonald, G. C. Aers, and M. W. C. Dharmawardana, Phys. Rev. B **31**, 5529 (1985).

[13]D. Levesque, J. J. Weis, and A. H. MacDonald, Phys. Rev. B **30**, 1056 (1984).

[14]Expressions for the necessary Fock matrix elements are given by A. H. MacDonald, Phys. Rev. B **29**, 3057 (1984). The numerical methods used here were similar to those described in that article. For earlier work on the Hartree-Fock approximation for charge-density-wave states within the lowest Landau level see, H. Fukuyama, P. M. Platzman, and P. W. Anderson, Phys. Rev. B **19**, 5211 (1979); D. Yoshioka and H. Fukuyama, J. Phys. Soc. Jpn. **47**, 394 (1979); D. Yoshioka and P. A. Lee, Phys. Rev. B **27**, 4986 (1983), and references therein. Many of the entries in Table I for CDW state energies in higher Landau levels are different from those published earlier in Table I of Ref. 5. In the course of this work we came to realize that the earlier calculations did not always converge to the lowest energy solution of the Hartree-Fock equations for $n \geq 1$.

[15]The precise value of ν at which the transition should occur in the lowest Landau level is quite uncertain. The estimate $\nu_w^{-1} \sim 10$ was obtained in Ref. 10 by using Monte Carlo simulations to estimate the energy of Laughlin's state and the Hartree-Fock approximation to estimate the energy of the charge-density-wave state. Pui K. Lam and S. M. Girvin, Phys. Rev. B **30**, 473 (1984) have shown that correlation can lower the energy of the latter state by $\sim 0.04\nu^{3/2}e^2/l_0$. The amount by which additional correlations can lower the energy of a uniform density state below that of Laughlin's seems to be much smaller ($\sim 0.0002e^2/l_0$ at $\nu=\frac{1}{3}$ for a small sphere with six electrons according to the calculations of Ref. 3) so that a better estimate would be $\nu^{-1} \sim 7$. The possibility that a charge-density-wave state may be the ground state even at $\nu^{-1}=3$, was considered as a possible explanation for the fractional Hall effect by H. Fukuyama and P. M. Platzman, Phys. Rev. B **25**, 2934 (1983), and has recently been revived by S. T. Chui, Phys. Rev. B **32**, 1436 (1985).

[16]E. E. Mendez, H. Heiblum, L. L. Chang, and L. Esaki, Phys. Rev. B **28**, 4886 (1983).

[17]See J. M. Caillol, D. Levesque, J. J. Weis, and J. P. Hansen, J. Stat. Phys. **28**, 325 (1982), and references therein.