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Dynamics of the one-dimensional Potts model

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We consider the q-state Potts model in one dimension for the cases $q=2^m$, $m=1,2,3,\ldots$. The dynamical critical exponent is calculated with use of the real-space renormalization-group method. We find $z=2$ in the 4- and 8-state models, which agrees with the Ising case, and speculate that this generalizes to any m.

INTRODUCTION

Our understanding of the dynamics of systems near their critical points is very limited compared to the large body of knowledge that deals with the static properties. There has been much theoretical interest in recent years in extending the methods which have been developed for static phenomena to study the time-dependent properties.[1] Real-space renormalization-group techniques have been applied very successfully to determine the dynamical critical behavior of the two-dimensional Ising model[2] yielding values of the dynamical critical exponent that agree well with results of high-temperature expansions[3] and Monte Carlo methods.[4]

The use of the renormalization-group (RG) method to study the dynamics is very similar to its application to static phenomena. The RG transformation scales the microscopic length of the system by a factor b, exactly as in the statics. However, in the dynamics one must also scale the microscopic time by a factor b^z, where z is the dynamical critical exponent. z describes the critical slowing down of the system since it characterizes the dependence of the relaxation time on the correlation length $\tau=\xi^z$.

In this paper we apply the real-space renormalization-group method introduced by Achiam[2,5] to study the dynamics of the Potts model[6] in one dimension. It is natural to try to extend the treatment that works well in the two-state case directly to the model with more states. This proves to be much harder than one would at first expect since it is hard to construct an invariant subspace for the RG transformation, and the normalization of the probability is not preserved to lowest order as it is in the Ising case. We choose instead to analyze the q-state Potts model in terms of m-coupled Ising models, where $q=2^m$, and study the dynamics of the Ising variables. This avoids the difficulties mentioned above since the Ising dynamics is known to be well described by this method. We cannot, however, treat the cases $q=3,5,6,7,\ldots$ by the method presented in this paper, but we speculate that our results are valid in these cases also.

We describe the dynamics in terms of an empirical master equation which generalizes the original Glauber model[7] for Ising dynamics. In this model, the system relaxes to equilibrium from a slightly perturbed initial state via the interaction with a heat bath. The heat bath causes the spins at a single site to flip at each step τ_0, with a probability W. We choose to consider a model with no conservation laws. We must extend the usual model, which allows only single spin flips, by allowing both our Ising variables at the same site to flip in one time step. In this way, we include the effects of transitions between any pair of Potts states since, for example, we allow all the Ising variables at a given site to flip in our formulation. This is to be contrasted with the model described in Ref. 5, which allows only one kind of spin flip in each time interval.

The RG transformation used is the usual static decimation which eliminates half the spins. We find that $z=2$ for the 2-, 4-, and 8-state models, and we expect that this generalizes for the cases $q=2^m$, $m>3$. This agrees with the results due to Forgacs *et al.*,[8] who find $z=2$ for all one-dimensional Potts models with energylike perturbations. The terms energylike and magneticlike perturbations refer to the types of deviation from equilibrium that are considered in the relaxation of the system. We consider here only magneticlike perturbations in which the field controlling the time development appears as the coefficient of a magnetizationlike rather than an energylike term.

RENORMALIZATION OF THE MASTER EQUATION FOR $q=4$

We use the usual form for the reduced Hamiltonian for the Potts model

$$-\beta H=K\sum_{\langle ij\rangle}\delta_{s_i s_j}\,,$$

where $s_i=0,1,2,\ldots,q-1$, and consider first the case of a four-state model. For a discussion of the Potts model, see Wu.[9] We define two Ising models, σ and μ, and write

$$-\beta H=\tfrac{1}{4}K\sum_i(\sigma_i\sigma_{i+1}+\mu_i\mu_{i+1}+\sigma_i\sigma_{i+1}\mu_i\mu_{i+1}+1)\,, \tag{1}$$

and express the dynamics in terms of these Ising variables.

The master equation is

$$-\tau_0\frac{\partial}{\partial t}P(\sigma,\mu,t)=LP(\sigma,\mu,t)\,,$$

where

$$L=\sum_i[(1-P_i^\sigma)W_i^\sigma(\sigma,\mu)+(1-P_i^\mu)W_i^\mu(\sigma,\mu)+(1-P_i^\sigma P_i^\mu)W_i^{\sigma\mu}(\sigma,\mu)] . \quad (2)$$

We have introduced, for convenience, the spin-flip operator P_i^α for the spin of type α. Throughout this paper the symbol α is used to denote either σ or μ. The effect of this operator is to flip one spin at a given site:

$$P_i^\sigma f(\sigma_1,\mu_1,\ldots,\sigma_i,\mu_i,\ldots,\sigma_N,\mu_N)=f(\sigma_1,\mu_1,\ldots,-\sigma_i,\mu_i,\ldots,\sigma_N,\mu_N) \quad (3)$$

and $W_i^\alpha(\sigma,\mu)$ and $W_i^{\sigma\mu}(\sigma,\mu)$ are the transition probabilities. Note that the last term on the right-hand side of (2) is the term discussed above which flips both σ and μ.

We choose transition probabilities which satisfy the detailed balance condition to ensure that the system progresses to equilibrium at large times,[10]

$$\begin{aligned}&(1-P_i^\alpha)W_i^\alpha(\sigma,\mu)P_e(\sigma,\mu)=0 ,\\&(1-P_i^\sigma P_i^\mu)W_i^{\sigma\mu}(\sigma,\mu)P_e(\sigma,\mu)=0 ,\end{aligned} \quad (4)$$

where

$$P_e(\sigma,\mu)=P(\sigma,\mu,t=\infty) .$$

This does not, however, determine the transition rates uniquely, but we shall make the standard choice

$$W_i^\alpha(\sigma,\mu)=\left[\frac{P_i^\alpha P_e(\sigma,\mu)}{P_e(\sigma,\mu)}\right]^{1/2}$$

and

$$W_i^{\sigma\mu}(\sigma,\mu)=\left[\frac{P_i^\sigma P_i^\mu P_e(\sigma,\mu)}{P_e(\sigma,\mu)}\right]^{1/2} . \quad (5)$$

We now define a variable that characterizes the deviation of the system from equilibrium,

$$\phi(\sigma,\mu,t)=\frac{P(\sigma,\mu,t)}{P_e(\sigma,\mu)} ,$$

and write

$$-\tau_0\frac{\partial}{\partial t}P(\sigma,\mu,t)=L\phi(\sigma,\mu,t) , \quad (6)$$

where

$$L=\sum_i\sum_\alpha[P_e(\sigma,\mu)W_i^\alpha(\sigma,\mu)(1-P_i^\alpha)]+\sum_i[P_e(\sigma,\mu)W_i^{\sigma\mu}(\sigma,\mu)(1-P_i^\sigma P_i^\mu)] . \quad (7)$$

We consider only small perturbations and write

$$\phi(\sigma,\mu,t)=1+h_\sigma(t)\sum_i\sigma_i+h_\mu(t)\sum_i\mu_i . \quad (8)$$

The renormalization transformation $T(\sigma',\mu',\sigma,\mu)$ is applied to both sides of the master equation as follows:

$$-\tau_0\frac{\partial}{\partial t}\sum_{\sigma,\mu}T(\sigma',\mu',\sigma,\mu)P(\sigma,\mu,t)=\sum_{\sigma,\mu}T(\sigma',\mu',\sigma,\mu)L\phi(\sigma,\mu,t) , \quad (9)$$

where T is the static RG transformation

$$T(\sigma',\mu',\sigma,\mu)=\prod_i\delta(\sigma_i'-\sigma_{2i})\delta(\mu_i'-\mu_{2i}) , \quad (10)$$

which represents a decimation with $b=2$. The RG transformation is performed at a particular time, and so T commutes with $\partial/\partial t$ on the left-hand side of the master equation. The RG transformation is performed using the transfer matrix,[5]

$$\begin{aligned}\underline{M}_{n,n+1}(K)&=\exp[\tfrac{1}{4}K(\sigma_n\sigma_{n+1}+\mu_n\mu_{n+1}+\sigma_n\sigma_{n+1}\mu_n\mu_{n+1})]\\&=\exp[V(\sigma_n,\mu_n,\sigma_{n+1},\mu_{n+1})] ,\end{aligned} \quad (11)$$

where

$$-\beta H=\sum_n[V(\sigma_n,\mu_n,\sigma_{n+1},\mu_{n+1})+\tfrac{1}{4}K] .$$

We arrange both rows and columns in all matrices as

$$(\sigma,\mu)=(1,1);(-1,1);(1,-1);(-1,-1) .$$

For each configuration of neighboring spins, a particular element of $\underline{M}$ is selected which is precisely the appropriate term in the Boltzmann factor. To calculate the RG of the left-hand side of the master equation, we need to renormalize

$$P_e\left[1+h_\sigma\sum_i\sigma_i+h_\mu\sum_i\mu_i\right] .$$

The first term is easily calculated from

$$A(K)\underline{M}'_{n,n+1}(K')=\underline{M}_{2n,2n+1}(K)\cdot\underline{M}_{2n+1,2n+2}(K) , \quad (12)$$

where A is the contribution to the free energy from the decimation. We find the recursion relations

$$\begin{aligned}&A(x')^3=x^6+3x^{-2} ,\\&A(x')^{-1}=2x^2+2x^{-2} ,\end{aligned} \quad (13)$$

where $x=e^{K/4}$, with a zero-temperature fixed point at $x=\infty$.

We may evaluate the other terms similarly and write $R(P_e h_\alpha\sum_i\alpha_i)$, where R is the RG operation and α is σ or μ. If $i=2n$, then we get $P_e'\,\alpha_{2n}$ where P_e' is just the product of a factor $A\underline{M}'(K')$ for each nearest-neighbor pair. If $i=2n+1$, then we have to calculate

$$\underline{M}_{2n,2n+1}\cdot\underline{\alpha}_{2n+1}\cdot\underline{M}_{2n+1,2n+2} ,$$

where $\underline{\alpha}$ denotes either

$$\underline{\sigma}=\text{diag}(1,-1,1,-1) ,$$

or

$$\underline{\mu}=\text{diag}(1,1,-1,-1) .$$

We find to leading order

$$\underline{M}_{2n,2n+1}\cdot\underline{\alpha}_{2n+1}\cdot\underline{M}_{2n+1,2n+2}=A[\underline{M}'(K')]_{n,n+1}(\underline{\alpha}'_n+\underline{\alpha}'_{n+1})/2 , \quad (14)$$

which tells us that

$$R(P_e\phi)=P_e'\left[1+h_\sigma'\sum_i \sigma_i'+h_\mu'\sum_i \mu_i'\right], \quad (15)$$

where the rescaled field is $h_\alpha'=\lambda h_\alpha$ with $\lambda=2$.

The renormalization of the right-hand side proceeds similarly:

$$L\phi=\sum_i\sum_\alpha P^{(\alpha_i)}2h_\alpha\alpha_i+\sum_i P^{(\sigma_i\mu_i)}\sum_\alpha(2h_\alpha\alpha_i), \quad (16)$$

where

$$P^{(\alpha_i)}=P_e(\sigma,\mu)W_i^\alpha(\sigma,\mu)$$

and

$$P^{(\sigma_i,\mu_i)}=P_e(\sigma,\mu)W_i^{\sigma\mu}(\sigma,\mu).$$

Note that detailed balance tells us that $P^{(\alpha_i)}$ is independent of α_i and that $P^{(\sigma_i\mu_i)}$ depends only on the product $(\sigma_i\mu_i)$.

If $i=2n+1$, the contribution to the renormalized right-hand side vanishes. This is clear for the first term since

$$\mathrm{Tr}P^{(\alpha_{2n+1})}\alpha_{2n+1}=0. \quad (17)$$

In the second term the trace is performed over σ_{2n+1} and μ_{2n+1}. We can group the terms in this sum into pairs of terms

$$(\sigma,\mu)=[(1,1),(-1,-1)]$$

and

$$[(1,-1),(-1,1)],$$

where within each pair $P^{(\sigma_i\mu_i)}$ is the same, and so this term also vanishes. Thus,

$$\mathrm{Tr}P^{(\sigma_i\mu_i)}(h_\sigma\sigma_{2n+1}+h_\mu\mu_{2n+1})=0. \quad (18)$$

If $i=2n$, the first term is

$$R\sum_n\left[\sum_\alpha P^{(\alpha_{2n})}2h_\alpha\alpha_{2n}\right]=\sum_n\sum_\alpha 2h_\alpha\alpha_{2n}P_e'^{(2n-2,2n+2)}(\underline{M}_{2n-2,2n-1}\cdot\underline{M}_{2n-1,2n}^{(\alpha)})\otimes(\underline{M}_{2n,2n+1}^{(\alpha)}\cdot\underline{M}_{2n+1,2n+2}), \quad (19)$$

where

$$(\underline{M}_{2n-1,2n}^{(\alpha)}\otimes\underline{M}_{2n,2n+1}^{(\alpha)})=\exp(V_{2n-1,2n})W_{2n}^\alpha(\sigma,\mu)\exp(V_{2n,2n+1}), \quad (20)$$

and $P_e'^{(2n-2,2n+2)}$ is the renormalized equilibrium probability neglecting those spins from $2n-2$ to $2n+2$. The factor W_{2n}^α in $P^{(\alpha_{2n})}$ does not effect the renormalization of those spins outside the range $2n-2$ to $2n+2$, and these each give a factor $A\underline{M}(K')$. This accounts for the factor $P_e'^{(2n-2,2n+2)}$ in the above equation. The other terms include the effects of W_{2n}^α, which change the transfer matrix $\underline{M}_{2n-1,2n}$ into $\underline{M}_{2n-1,2n}^{(\alpha)}$.

The second term on the right-hand side of the master equation can be treated similarly:

$$R\sum_i\left[\sum_\alpha P^{(\sigma_i\mu_i)}2h_\alpha\alpha_i\right]=\sum_n\sum_\alpha(2h_\alpha\alpha_n')P_e'^{(2n-2,2n+2)}(\underline{M}_{2n-2,2n-1}\cdot\underline{M}_{2n-1,2n}^{(\sigma\mu)})\otimes(\underline{M}_{2n,2n+1}^{(\sigma\mu)}\cdot\underline{M}_{2n+1,2n+2}), \quad (21)$$

where

$$(\underline{M}_{2n-1,2n}^{(\sigma\mu)}\otimes\underline{M}_{2n,2n+1}^{(\sigma\mu)})=\exp(V_{2n-1,2n})W_{2n}^{\sigma\mu}(\sigma,\mu)\exp(V_{2n,2n+1}). \quad (22)$$

We obtain

$$\underline{M}^{(\mu)}=\begin{bmatrix}\underline{X} & \underline{X}\\ \underline{X} & \underline{X}\end{bmatrix}, \quad (23)$$

$$\underline{M}^{(\sigma)}=\begin{bmatrix}x\underline{U} & (1/x)\underline{U}\\ (1/x)\underline{U} & x\underline{U}\end{bmatrix}, \quad (24)$$

$$\underline{M}^{(\sigma\mu)}=\begin{bmatrix}\underline{X} & \underline{R}\\ \underline{R} & \underline{X}\end{bmatrix}, \quad (25)$$

where

$$\underline{X}=\begin{bmatrix}x & 1/x\\ 1/x & x\end{bmatrix},\quad \underline{U}=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix},\quad \underline{R}=\begin{bmatrix}1/x & x\\ x & 1/x\end{bmatrix}.$$

Near the fixed point we find

$$\underline{M}\cdot\underline{M}^{(\alpha)}=(1/\sqrt{2})A\underline{M}'^{(\alpha')} \quad (26)$$

and

$$\underline{M}\cdot\underline{M}^{(\sigma\mu)}=(1/\sqrt{2})A\underline{M}'^{(\sigma'\mu')}.$$

If we substitute (26) into (19) and (21), we find

$$R\left[\sum_i\sum_\alpha(P^{(\alpha_i)}+P^{(\sigma_i\mu_i)})2h_\alpha\alpha_i\right]=\tfrac{1}{2}\sum_n\sum_\alpha(P'^{(\alpha_n')}+P'^{(\sigma_n'\mu_n')})2h_\alpha\alpha_n', \quad (27)$$

where

$$P'^{(\alpha_n')}=A^2\underline{M}_{n-1,n}'^{(\alpha')}\otimes\underline{M}_{n,n+1}'^{(\alpha')}P_e'^{(2n-2,2n+2)} \quad (28)$$

and

$$P'^{(\sigma_n'\mu_n')}=A^2\underline{M}_{n-1,n}'^{(\sigma'\mu')}\otimes\underline{M}_{n,n+1}'^{(\sigma'\mu')}P_e'^{(2n-2,2n+2)}. \quad (29)$$

Note that $P'^{(\alpha_n')}$ is independent of α_n', and $P'^{(\sigma_n'\mu_n')}$ depends only on $(\sigma_n'\mu_n')$, as required. The factor of A is included so as not to disturb the normalization of the probability, see (12). From (27) we have

$$R(L\phi)=L'\left[1+\tfrac{1}{2}h_\sigma\sum_i\sigma_i'+\tfrac{1}{2}h_\mu\sum_i\mu_i'\right]$$

$$=L'\left[1+\tfrac{1}{4}h_\sigma'\sum_i\sigma_i'+\tfrac{1}{4}h_\mu'\sum_i\mu_i'\right] . \tag{30}$$

From (15) we now have

$$R(L\phi)=\tfrac{1}{4}L'\phi' , \tag{31}$$

and so the master equation becomes

$$-\tau_0\partial/\partial t P'(\sigma',\mu',t)=\tfrac{1}{4}L'\phi' . \tag{32}$$

We can now define the renormalized time scale $\tau_0'=4\tau_0=b^z\tau_0$, which yields a master equation of the same form. This gives us $z=2$ which agrees with the result in the usual Ising case.

GENERALIZATIONS OF THE METHOD

The calculation described above was also extended to the case of the eight-state model, for which we must add another Ising degree of freedom at each site. The reduced Hamiltonian is

$$-\beta H=\tfrac{1}{8}K\sum_n(\sigma_n\sigma_{n+1}+\mu_n\mu_{n+1}+\tau_n\tau_{n+1}+\sigma_n\sigma_{n+1}\mu_n\mu_{n+1}$$
$$+\mu_n\mu_{n+1}\tau_n\tau_{n+1}+\sigma_n\sigma_{n+1}\tau_n\tau_{n+1}+\sigma_n\sigma_{n+1}\mu_n\mu_{n+1}\tau_n\tau_{n+1}+1)$$
$$=\sum_n[V(\sigma_n,\mu_n,\tau_n,\sigma_{n+1},\mu_{n+1},\tau_{n+1})+\tfrac{1}{8}K] . \tag{33}$$

The operator L becomes

$$\sum_i\sum_\alpha P_e(\sigma,\mu,\tau)W_i^\alpha(\sigma,\mu,\tau)(1-P_i^\alpha)$$
$$+\sum_i\sum_{(\alpha,\beta)}P_e(\sigma,\mu,\tau)W_i^{\alpha\beta}(\sigma,\mu,\tau)(1-P_i^\alpha P_i^\beta)$$
$$+\sum_i P_e(\sigma,\mu,\tau)W_i^{\sigma\mu\tau}(\sigma,\mu,\tau)(1-P_i^\sigma P_i^\mu P_i^\tau) , \tag{34}$$

where α is σ, μ, or τ, and (α,β) denotes any of the three pairs of spins.

The detailed balance condition is

$$(1-P_i^\alpha)W_i^\alpha(\sigma,\mu,\tau)P_e(\sigma,\mu,\tau)=0 ,$$
$$(1-P_i^\alpha P_i^\beta)W_i^{\alpha\beta}(\sigma,\mu,\tau)P_e(\sigma,\mu,\tau)=0 , \tag{35}$$
$$(1-P_i^\sigma P_i^\mu P_i^\tau)W_i^{\sigma\mu\tau}(\sigma,\mu,\tau)P_e(\sigma,\mu,\tau)=0 ,$$

and we choose

$$W_i^{\sigma\mu\tau}=\left[\frac{P_i^\sigma P_i^\mu P_i^\tau P_e(\sigma,\mu,\tau)}{P_e(\sigma,\mu,\tau)}\right]^{1/2} . \tag{36}$$

ϕ is now

$$\left[1+\sum_\alpha h_\alpha(t)\sum_i\alpha_i\right] .$$

We use the same RG transformation and find the following recursion relations:

$$A(x')^7=x^{14}+7x^{-2} ,$$
$$A(x')^{-1}=2x^6+6x^{-2} , \tag{37}$$

where $x=e^{K/8}$.

To calculate $\underline{M}_{2n,2n+1}\cdot\underline{\alpha}_{2n+1}\cdot\underline{M}_{2n+1,2n+2}$, we make use of the symmetry under relabeling of the Ising spins. We thus choose to consider the $\underline{\alpha}$ with a matrix

$$\underline{\alpha}=\begin{bmatrix}\underline{I} & \underline{0}\\ \underline{0} & -\underline{I}\end{bmatrix}$$

where $\underline{I}$ is the 4×4 unit matrix. This gives

$$\underline{M}_{2n,2n+1}\cdot\underline{\alpha}_{2n+1}\cdot\underline{M}_{2n+1,2n+2}=A\underline{M}'_{n,n+1}(K')(\underline{\alpha}_n'+\alpha_{n+1}')/2 ,$$

and again we have $\lambda_\alpha=2$ for all α.

The right-hand side of the master equation is

$$L\phi=\sum_i\sum_\alpha\left[P^{(\alpha_i)}+P^{(\sigma_i\mu_i\tau_i)}+\sum_{\substack{(\alpha_i,\beta_i)\\ \beta_i\neq\alpha_i}}P^{(\alpha_i\beta_i)}\right]2h_\alpha(t)\alpha_i , \tag{38}$$

where

$$P^{(\alpha_i,\beta_i)}=P_e(\sigma,\mu,\tau)W_i^{\alpha\beta}(\sigma,\mu,\tau), \;\ldots .$$

If $i=2n+1$, the contribution is again zero, as pairs of terms with the same $P^{(\,)}$ factors cancel. The case of $i=2n$ proceeds as before with

$$\underline{M}^{(\sigma)}=\begin{bmatrix}\underline{W} & \underline{V}\\ \underline{V} & \underline{W}\end{bmatrix} , \tag{39}$$

$$\underline{M}^{(\sigma\mu)}=\begin{bmatrix}\underline{Y} & \underline{V}\\ \underline{V} & \underline{Y}\end{bmatrix} , \tag{40}$$

$$\underline{M}^{(\sigma\mu\tau)}=\begin{bmatrix}\underline{T} & \underline{S}\\ \underline{S} & \underline{T}\end{bmatrix} , \tag{41}$$

where

$$\underline{W}=\begin{bmatrix}x^3\underline{U} & (1/x)\underline{U}\\ (1/x)\underline{U} & x^3\underline{U}\end{bmatrix},\quad \underline{V}=(1/x)\begin{bmatrix}\underline{U} & \underline{U}\\ \underline{U} & \underline{U}\end{bmatrix} ,$$

$$\underline{Y}=\begin{bmatrix} x^3 & 1/x & 1/x & x^3 \\ 1/x & x^3 & x^3 & 1/x \\ 1/x & x^3 & x^3 & 1/x \\ x^3 & 1/x & 1/x & x^3 \end{bmatrix},$$

$$\underline{T}=\begin{bmatrix} x^3 & 1/x & 1/x & 1/x \\ 1/x & x^3 & 1/x & 1/x \\ 1/x & 1/x & x^3 & 1/x \\ 1/x & 1/x & 1/x & x^3 \end{bmatrix},$$

$$\underline{S}=\begin{bmatrix} 1/x & 1/x & 1/x & x^3 \\ 1/x & 1/x & x^3 & 1/x \\ 1/x & x^3 & 1/x & 1/x \\ x^3 & 1/x & 1/x & 1/x \end{bmatrix}.$$

Near the fixed point we again have

$$\underline{M}\cdot\underline{M}^{(\alpha)}=(1/\sqrt{2})A\underline{M}'^{(\alpha')} \tag{42}$$

for all of the $\underline{M}^{(\alpha)}$, $\underline{M}^{(\alpha\beta)}$, and $\underline{M}^{(\sigma\mu\tau)}$. Thus,

$$R(L\phi)=L'\left[1+\tfrac{1}{2}\sum_{\alpha}h_\alpha\sum_{i}\alpha_i'\right]=L'\left[1+\tfrac{1}{4}\sum_{\alpha}h_\alpha'\sum_{i}\alpha_i'\right] \tag{43}$$

and we find $z=2$ again.

We now consider extending the above arguments to $q=2^m$ with $m>3$. We have not yet been able to show that all of the above generalizes for arbitrary m. The λ for the fields can be easily shown to be 2, and the terms with $i=2n+1$ in the RG transformation of the right-hand side still vanish, but the lack of an explicit form for the $\underline{M}^{(\,)}$ makes it hard to show that in general $\underline{M}\cdot\underline{M}^{(\,)}=(1/\sqrt{2})A\underline{M}^{(\,)}$ for all the possible $\underline{M}^{(\,)}$

If we conjecture that this still holds, then it is clear that the method generalizes to any number of coupled Ising models. A recent paper by Forgacs *et al.*[8] analyzes the dynamics of the Potts model using a Migdal-type recursion method. They consider energylike perturbations from equilibrium and find $z=2$ independent of q.

The result $z=2$ for the one-dimensional Potts model is further supported by extending the domain-wall diffusion arguments of Cordery *et al.*[11] to the case of the Potts model. They regard the relaxation time as the time required to flip a cluster of spins of size ξ by the motion of a domain wall across the cluster. It is clear from the form of the Potts Hamiltonian that the flipping of the spin at a domain wall does not change the energy of the system and so it has a transition rate of order unity. The usual random-walk arguments imply that the number of steps required to move a domain wall a distance ξ is ξ^2. Since the time for each step is of order unity, we see that $\tau=\xi^2$ and we find $z=2$, which agrees with the result obtained by the RG arguments given above.

We also considered a slight generalization of the four-state model which allows the four-spin interaction term in the Hamiltonian to appear with a new field L, which may, in general, be different from K. We again find $z=2$, suggesting that this result may be valid for any one-dimensional model, provided the dynamics can be expressed in terms of Ising spin flips. We conclude, at least for the cases 2, 4, and 8, that the dynamical critical exponent for the Potts model in one dimension is 2 for both energylike and magneticlike perturbations.

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