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Thermodynamic Green's-function theory of quantum magnetic field effects in the dynamic, nonlocal longitudinal dielectric-response properties of a bounded solid-state plasma

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The analysis of dynamic nonlocal longitudinal dielectric response properties of a slab of quantum plasma in a magnetic field **H** (perpendicular to the plane slab faces) is carried out here with use of a thermodynamic Green's-function formulation of the random-phase approximation (RPA). The magnetic field Green's function for the slab incorporates magnetic field effects in terms of a closed-form integral representation, and the boundary condition of specular reflection is imposed in two alternative ways, in terms of (1) a partial eigenfunction expansion, and (2) an image series of infinite-space Green's functions. The RPA integral equation is solved for the *direct* slab dielectric function subject to Landau quantization, with results expressed in terms of the density perturbation response function $\mathscr{R}=\delta\rho/\delta V$ which depends on both z and z' because of the lack of translational invariance perpendicular to the slab faces (parallel to $\mathbf{H}=H\hat{\mathbf{z}}$). Correspondingly, $\mathscr{R}$ depends on two conjugate wave-vector transform variables q_z and q_z' which are interpreted as indices for rows and columns of $\mathscr{R}$ regarded as a matrix. The magnetic field dependencies and nonlocality of both the diagonal and nondiagonal elements of $\mathscr{R}$ are thoroughly examined here. Applications of this work to the Landau quantized nonlocal slab surface-plasmon dispersion relation are discussed.

I. INTRODUCTION

The substantial growth of research on solid surface properties in general has brought with it an intensified interest in dynamic solid-state plasma response phenomena near surfaces, and several serious efforts have been directed at determining the role of nonlocality in the theory of dynamic plasma response properties near solid surfaces. The object of this work is to determine the effects of Landau quantization of electron orbits due to an ambient magnetic field on the nonlocal dynamic longitudinal electrostatic dielectric response properties of a slab of quantum plasma of finite thickness, with the magnetic field perpendicular to the plane slab faces. A thermodynamic Green's-function[1] formulation of the random-phase approximation (RPA) is employed in the description of Landau-quantized plasma dynamics, subject to the boundary condition of specular reflection of the electrons at the slab surfaces. The Green's function developed for use here, which should have broad utility in other problems involving electron dynamics in a plasma slab subject to Landau quantization, incorporates magnetic field effects in terms of a closed-form integral representation which is useful for obtaining both low- and high-field limits, and it is presented in two alternative but equivalent forms which impose on the Green's function the boundary condition of specular reflection at the slab faces: (1) a partial eigenfunction expansion and (2) an image series of infinite-space Green's functions. The slab Green's function is used to determine the magnetic field dependence of the density of a thick semi-infinite quantum plasma as a function of chemical potential.

It is appropriate to observe that there are two distinct and different breakdowns of spatial translational invariance involved in the problem at hand. One is associated with the fact that the magnetic field drives the electrons into circular orbitals, thereby violating conservation of the direction of the momentum vector: Since the magnetic field is taken perpendicular to the slab faces here, this breakdown of spatial translational invariance occurs in the plane perpendicular to the field and parallel to the plane slab faces, and it is accompanied by the introduction of a characteristic magnetic field Green's-function[1(a)] factor $C(\bar{r},\bar{r}')$ [vectors are denoted as $\mathbf{r}\equiv(\bar{r},z)$, where the overbar $\bar{r}$ denotes the projection of the vector onto the plane of the slab faces perpendicular to the magnetic field],

$$C(\bar{r},\bar{r}')=\exp\{i[(e/2)\bar{r}\cdot\mathbf{H}\times\bar{r}'-\phi(\bar{r})+\phi(\bar{r}')]\}\ , \tag{1.1}$$

which embodies the $[\bar{r}+\bar{r}']$ and gauge (ϕ) dependencies of the Green's function $\bar{G}(\mathbf{r},t;\mathbf{r}'t')$, where

$$\bar{G}(\mathbf{r},t;\mathbf{r}',t')=C(\bar{r},\bar{r}')\bar{G}'(\bar{r}-\bar{r}',z,z';t-t')\ , \tag{1.2}$$

leaving the rest of the Green's function $\bar{G}'(\bar{r}-\bar{r}',z,z';t-t')$ effectively spatially translationally invariant in the plane perpendicular to the magnetic field (and parallel to the slab faces), since the magnitude of the associated momentum is conserved with a magnetic field as well as without ($\bar{G}'$ is independent of gauge ϕ as well). Since displacements along the field $\mathbf{H}=H\hat{\mathbf{z}}$ (such as occur in superposing images of the infinite-space Green's function to impose the boundary condition of specular reflection) do not affect the exponent $\bar{r}\cdot\mathbf{H}\times\bar{r}'\equiv\bar{r}\cdot\mathbf{H}\times(\bar{r}'+z\hat{\mathbf{z}})$, the function $C(\bar{r},\bar{r}')$ is unaffected by such displacements, and the same function $C(\bar{r},\bar{r}')$ characterizes the exact slab Green's function as well as the infinite-space Green's function, embodying all $\bar{r}+\bar{r}'$ dependence for both cases. Moreover, the ring diagrams[1] of the RPA involve $C(\bar{r},\bar{r}')$ in the form $C(\bar{r},\bar{r}')C(\bar{r}',\bar{r})\equiv 1$, so the dielectric properties are determined by a convolution integral of two $\bar{G}'(\bar{r}-\bar{r}',z,z';t-t')$ functions and they are effectively spatially translationally invariant in the plane perpendicular to the magnetic field and parallel to the slab faces; consequently the dielectric properties can be described in terms of a single wave-vector transform variable $\bar{q}$ conjugate to $[\bar{r}-\bar{r}']$, as one should expect physically in a spatially homogeneous magnetic field.

The other breakdown of spatial translational invariance is associated with the specular reflection of electrons at the slab faces, resulting in nonconservation of momentum perpendicular to the slab faces and parallel to the magnetic field. This breakdown of spatial translational invariance parallel to the field is an essential feature of the problem at hand and the dielectric properties of the slab correspondingly lack spatial translational invariance in the direction perpendicular to the slab faces and parallel to the field. Our analysis of the magnetic field dependence of the slab dielectric function is carried out in terms of the RPA density perturbation response function $\mathscr{R}=\delta\rho/\delta V$ employing transform techniques developed by Newns[2] and using the Landau-quantized slab Green's function discussed above. Since the slab dielectric function lacks spatial translational invariance along the field $\hat{z}$ direction, $\mathscr{R}$ depends on both z and z' in an essential way or, alternatively, it depends on two conjugate wave-number transform variables q_z and q_z': Viewing $\mathscr{R}$ as a matrix whose rows and columns are indexed by q_z and q_z', respectively, the nondiagonal elements are generally *nontrivial* and are evaluated in detail here along with the diagonal elements. Following Newns's[2] designation of the diagonal and nondiagonal parts of $\mathscr{R}$ by D and $-A$, respectively, we write

$$\mathscr{R}(\bar{q},q_z,q_z';\nu)=D(\bar{q},q_z;\nu)\delta_{q_zq_z'}-A(\bar{q},q_z,q_z';\nu)\,,$$

and construct closed-form convolution-integral representations for both D and $-A$ in terms of Green's functions.[3] Special attention is given to D in the thick semi-infinite slab limit, where it is seen to be simply related to the magnetic-field-dependent RPA bulk infinite-space longitudinal dielectric function $\epsilon^{\infty}_{\mathrm{RPA}}(\mathbf{q},\omega)$. Very extensive evaluations[3] of the nondiagonal elements $-A$ are carried out here, and the relation of these elements to two-dimensional[4] density perturbation response is carefully examined. It should be noted that these detailed evaluations of nondiagonal elements can be employed to enrich the body of information we have developed concerning the diagonal elements of a slab of finite thickness by using the identity

$$D(\bar{q},q_z;\nu)=\sum_{q_z'}A(\bar{q},q_z,q_z';\nu)\,,$$

which Newns observed to be the condition that the density perturbation vanish at the slab surface.

Two brief conference papers[5,6] were presented earlier to summarize the results of our studies of the slab Green's function[5] in a magnetic field, and of the dynamic nonlocal and inhomogeneous longitudinal dielectric response function[6] of a slab of quantum magnetoplasma. Our intention is to set forth here a much more detailed and thorough exposition of these studies. It should be noted that the presence of slab boundaries adds considerable complication to the solution of the RPA integral equation for the inverse dielectric function and we have discussed this in another paper:[30] Our intention here is directed at the construction of a closed-form solution for the *direct* dielectric function of the slab in a normal magnetic field rather than its inverse. The inversion procedure involved in formulating the slab surface-plasmon dispersion relation in a magnetic field will also be reviewed, with emphasis on Newns's "diagonal" approximation in which nondiagonal elements $-A(\bar{q},q_z,q_z';\nu)$ are neglected, although our detailed evaluation of such nondiagonal elements provides the basis for a more refined and accurate analysis of the roles of the magnetic field, nonlocality, and spatial inhomogeneity in the surface-plasmon spectrum. It also provides the means to analyze dynamic, nonlocal inhomogenous surface interactions of the slab magnetoplasma, as well as its correlation and exchange phenomena, and we shall discuss these applications in later papers.

II. UNCORRELATED ONE-ELECTRON GREEN'S FUNCTION FOR A SLAB OF QUANTUM PLASMA IN MAGNETIC FIELD

In order to incorporate the magnetic field in an analytically tractable manner, we shall formulate the longitudinal dielectric response function of a slab of Landau-quantized plasma in terms of thermodynamic Green's functions.[1] The uncorrelated one-electron thermodynamic Green's function for a slab (in magnetic field) is developed in this section in terms of a partial eigenfunction expansion and an image-series representation, and it is applied to the evaluation of magnetic field effects on the uncorrelated density of a semi-infinite (thick) slab. The magnetic field is perpendicular to the plane slab faces throughout this work (Fig. 1).

A. Partial eigenfunction expansion of the Green's function for a slab plasma in magnetic field

The calculation of the uncorrelated one-electron Green's function for a slab of quantum plasma in magnetic field may proceed from the differential equation for the spectral weight[1] function $\overline{\mathscr{A}}'(\bar{R},z,z';T)$ of

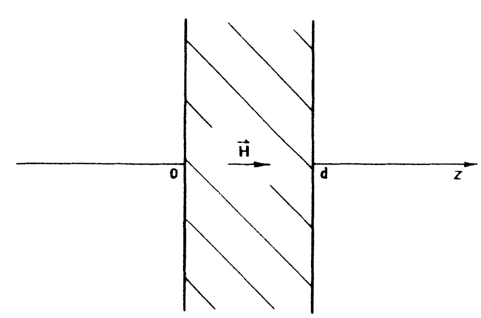

FIG. 1. Plane slab of solid-state plasma in a magnetic field perpendicular to the slab surfaces.

$\overline{G}'(\overline{R},z,z';t-t')$ (where $\overline{R}=\overline{r}-\overline{r}'$, $T=t-t'$), which was developed in Ref. 1(a). It should be noted that dependencies on gauge and on $\overline{r}+\overline{r}'$, insofar as it is associated with the magnetic field, have already been eliminated, so the use of Ref. 1(a) for $\mathbf{H}=H\hat{\mathbf{z}}$ perpendicular to the slab faces yields (we take $\hbar\to 1$ throughout this paper, except where otherwise indicated)

$$\left[\frac{\nabla_{\overline{R}}^2}{2m}+\frac{1}{2m}\frac{\partial^2}{\partial z^2}-\frac{m\omega_c^2}{8}(X^2+Y^2)-\mu_0 H\sigma_3+\zeta+i\frac{\partial}{\partial T}\right]\overline{\mathscr{A}}'(\overline{R},z,z',T)=0\ , \tag{2.1}$$

and the differential equation above involves $z+z'$ only as it is associated with the presence of the slab boundaries. Of course, we still have the sum rule

$$\overline{\mathscr{A}}'(\overline{R},z,z';T=0)=\delta(\overline{R})\delta(z-z')\ .$$

[The notation of Refs. 1 and 4–6 will generally be maintained here, except where otherwise noted. The magnetic field is perpendicular to the planar slab boundaries, with $\mathbf{H}=H\hat{\mathbf{z}}$ and the plane of the boundaries being the (x,y) plane. 2D position vectors in this plane are denoted as $\overline{r}=(x,y)$, whereas 3D position vectors are denoted as $\mathbf{r}=(\overline{r},z)=(x,y,z)$. For wave vectors in 3D we use $\mathbf{q}=(\overline{q},q_z)$, and for 2D we use $\overline{q}=(q_x,q_y)$, etc. Ω and ν are frequency variables, whereas T, t_1, and t_2 are time variables $(T=t_1-t_2)$. We take Landau-level energy separation $(\hbar\to 1)$ to match spin splitting in most of this work. Cyclotron frequency $\omega_c=eH/mc$, $\mu_0=e\hbar/2mc$, H is the magnetic field strength, m is the effective mass, the Fermi-Dirac distribution function $f_0(\omega)=(1+e^{\beta(\omega-\zeta)})^{-1}$, ζ is the chemical potential, thermal energy $\beta^{-1}=k_BT_0$ (k_B is the Boltzmann constant, T_0 is absolute temperature in degrees Kelvin), $\tau=-i\beta$, the unit step function $\eta_+(x)=1$ for $x>0$, and $=0$ for $x<0$, and the Pauli spin matrix $\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$. Other notation is defined in the text or in Refs. 1 and 4–6 (generally, we employ standard notation).]

This differential equation may be Fourier-transformed with respect to $\overline{R}$ as was done in the infinite-plasma case. However, the slab plasma boundary conditions, which we take to represent specular reflection of electrons at the slab boundaries ($z=0$ and d; $z'=0$ and d), introduce essential spatial inhomogeneity into the problem through the requirement that the electron wave functions and Green's function shall all vanish at the slab boundaries. The concomitant dependence of the Green's function on $z+z'$ will first be represented here through a partial eigenfunction expansion with respect to the direction parallel to the magnetic field. Since the Green's function must vanish at the slab boundaries, we also must have

$$\overline{\mathscr{A}}'(\overline{R},z,z';T)=0 \quad \text{for } z=0,d, \text{ and } z'=0,d\ .$$

In order to expand $\overline{\mathscr{A}}'$ in the complete set of eigenfunctions referring to the direction parallel to the field, we employ eigenfunctions of the Hermitian operator $\partial^2/\partial z^2$, which vanish at the boundaries, setting

$$\overline{\mathscr{A}}'(\overline{R},z,z';T)=\sum_n \sin(p_{zn}z)\phi_n(z')\overline{\mathscr{A}}'(p_{zn},\overline{R};T)\ .$$

This satisfies the boundary conditions if $p_{zn}=(n\pi/d)$ $(n=1,2,3,\ldots)$, and if $\phi_n(z')=0$ for $z'=0$ and $z'=d$. Since the eigenfunctions satisfy

$$\frac{\partial^2}{\partial z^2}\sin(p_{zn}z)=-p_{zn}^2\sin(p_{zn}z)\ ,$$

it is clear that $\overline{\mathscr{A}}'(p_{zn},\overline{R};T)$ satisfies the following equation, which is identical in form to the corresponding infinite-space equation:

$$\left[\frac{\nabla_{\overline{R}}^2}{2m}-\frac{p_{zn}^2}{2m}-\frac{m\omega_c^2}{8}(X^2+Y^2)-\mu_0 H\sigma_3+\zeta+i\frac{\partial}{\partial T}\right]\overline{\mathscr{A}}'(p_{zn},\overline{R};T)=0\ . \tag{2.2}$$

The solution of this infinite-space equation for $\overline{\mathscr{A}}'(p_{zn},\overline{R};T)$ is given in momentum space $\overline{R}\to\overline{P}$ [see Ref. 1(a)] as if there were no boundaries, so we may use the known-infinite-space result

$$\overline{\mathscr{A}}'(p_{zn},\overline{P};T)=K\exp[i(\zeta-\mu_0H\sigma_3-p_{zn}^2/2m)T]\times\sec\left[\frac{\omega_cT}{2}\right]\exp\left[\frac{-i\overline{p}^{\,2}}{m\omega_c}\tan\left[\frac{\omega_cT}{2}\right]\right]\ . \tag{2.3}$$

Thus, going back to position representation, the spectral weight for the slab Green's function is given by

$$\overline{\mathscr{A}}'(\overline{R},z,z';T)=K\sum_n\sin(p_{zn}z)\phi_n(z')\int\frac{d^2\overline{P}}{(2\pi)^2}e^{i\overline{P}\cdot\overline{R}}\exp\left[i\left[\zeta-\mu_0H\sigma_3-\frac{p_{zn}^2}{2m}\right]T\right]\times\sec\left[\frac{\omega_cT}{2}\right]\exp\left[-i\frac{\overline{P}^{\,2}}{m\omega_c}\tan\left[\frac{\omega_cT}{2}\right]\right]\ . \tag{2.4}$$

In the limit $T\to 0$ we have

$$\overline{\mathscr{A}}'(\overline{R},z,z';T=0)=K\sum_n\sin(p_{zn}z)\phi_n(z')\int\frac{d^2\overline{P}}{(2\pi)^2}\exp[i(\overline{P}\cdot\overline{R})]=K\sum_n\sin(p_{zn}z)\phi_n(z')\delta(\overline{R})\ ,$$

where $\delta(\overline{R})$ is a two-dimensional delta function. Since the sum rule dictates that

$$\overline{\mathscr{A}}'(\overline{R},z,z';T=0)=\delta(\mathbf{R})=\delta(\overline{R})\delta(z-z')\ ,$$

we have

$$\delta(z-z')=K\sum_{n=1}^{\infty}\sin(p_{zn}z)\phi_n(z')\ ,$$

whence $\phi_n(z')\to\sin(p_{zn}z')$ (by orthogonality), which appropriately vanishes at $z'=0$, and $z'=d$. Note that K must be so chosen to give the eigenfunctions proper normalization to unity, so that $K=2/d$. Thus, we then obtain the spectral weight for the slab Green's function in magnetic field as

$$\overline{\mathscr{A}}'(\overline{R},z,z';T)=\frac{2}{d}\sum_{n=1}^{\infty}\sin(p_{zn}z)\sin(p_{zn}z')$$
$$\times\left\{\int\frac{d^2\overline{P}}{(2\pi)^2}e^{i\overline{P}\cdot\overline{R}}\sec\left[\frac{\omega_c T}{2}\right]\exp\left[i\left[\zeta-\mu_0 H\sigma_3-\frac{p_{zn}^2}{2m}\right]T\right]\exp\left[-i\frac{\overline{P}^2}{m\omega_c}\tan\left[\frac{\omega_c T}{2}\right]\right]\right\}. \tag{2.5}$$

Note that the integral $\int[d^2\overline{P}/(2\pi)^2](\cdots)$ is just the corresponding infinite-plasma integral [see Ref. 1(a)], and we may write it as $\overline{\mathscr{A}}'_0(p_{zn},\overline{R};T)$ in the notation of Ref. 1(a). Hence, we have

$$\overline{\mathscr{A}}'(\overline{R},z,z';T)=\frac{2}{d}\sum_{n=1}^{\infty}\sin(p_{zn}z)\sin(p_{zn}z')\overline{\mathscr{A}}'_0(p_{zn},\overline{R};T)\ .$$

Since $\overline{\mathscr{A}}'_0(p_{zn},\overline{R};T)$ denotes the corresponding infinite-plasma spectral weight function with no boundaries, it is clear that the slab Green's functions $\overline{G}'_{1>}$ and $\overline{G}'_{1<}$ are given by the partial eigenfunction expansion

$$\left.\begin{matrix}\overline{G}'_{1>}(\overline{R},z,z';T)\\ \overline{G}'_{1<}(\overline{R},z,z';T)\end{matrix}\right\}=\frac{2}{d}\sum_{n=1}^{\infty}\sin(p_{zn}z)\sin(p_{zn}z')\left\{\begin{matrix}\overline{G}'_{10>}(p_{zn},\overline{R};T)\ ,\\ \overline{G}'_{10<}(p_{zn},\overline{R};T)\ ,\end{matrix}\right. \tag{2.6}$$

where $\overline{G}'_{10>}(p_{zn},\overline{R};T)$ and $\overline{G}'_{10<}(p_{zn},\overline{R};T)$ are the infinite-space Green's functions, as if no boundaries were present, and are given by [Ref. 1(a)]

$$\left.\begin{matrix}\overline{G}'_{10>}(p_{zn},\overline{R};T)\\ \overline{G}'_{10<}(p_{zn},\overline{R};T)\end{matrix}\right\}=\exp(i\zeta T)\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\left\{\begin{matrix}-i[1-f_0(\omega)]\\ if_0(\omega)\end{matrix}\right\}e^{-i\omega T}$$
$$\times\int_{-\infty}^{+\infty}dT'\,e^{i\omega T'}\int\frac{d^2\overline{P}}{(2\pi)^2}e^{i\overline{P}\cdot\overline{R}}\exp\left[-i\left[\mu_0 H\sigma_3+\frac{p_{zn}^2}{2m}\right]T'\right]$$
$$\times\sec\left[\frac{\omega_c T'}{2}\right]\exp\left[-i\frac{\overline{P}^2}{m\omega_c}\tan\left[\frac{\omega_c T'}{2}\right]\right]\ , \tag{2.7}$$

or, alternatively, one may execute the $\overline{P}$ integral as the Fourier transform of a Gaussian to obtain

$$\left.\begin{matrix}\overline{G}'_{10>}(p_{zn},\overline{R};T)\\ \overline{G}'_{10<}(p_{zn},\overline{R};T)\end{matrix}\right\}=e^{i\zeta T}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\left\{\begin{matrix}-i[1-f_0(\omega)]\\ if_0(\omega)\end{matrix}\right\}e^{-i\omega T}$$
$$\times\int_{-\infty}^{\infty}dT'\,e^{i\omega T'}\exp\left[-i\left[\mu_0 H\sigma_3+\frac{p_{zn}^2}{2m}\right]T'\right]\frac{m\omega_c}{4\pi i\sin(\omega_c T'/2)}\exp\left[\frac{im\omega_c\overline{R}^2}{4\tan(\omega_c T'/2)}\right]. \tag{2.8}$$

In all of the T' integrals the contour must be understood as being slightly displaced off the real axis such that it becomes the standard inverse Laplace-transform contour upon rotation through 90°.

B. Image-series representation of the Green's function for the slab plasma in magnetic field

In this section the uncorrelated one-electron Green's function for the magnetic field problem is developed using an image-series technique appropriate to slab boundaries. This is an equivalent alternative representation to the partial eigenfunction expansion given above. This image-series representation of the Green's function is particularly useful in cases where the slab is thick, when only a small number of images are significant in the calculation of the Green's function.

The basic infinite-space Green's function for a bulk plasma in magnetic field without boundaries, which will be employed in the construction of the image Green's function for the slab plasma, is known to be given in position representation by $\overline{G}'_{10}$ [Ref. 1(a), $\hbar=1$]

$$\left.\begin{matrix} \overline{G}'_{10>}(\overline{R},z,z';T) \\ \overline{G}'_{10<}(\overline{R},z,z';T) \end{matrix}\right\} = \exp(i\zeta T)\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\begin{Bmatrix} i[f_0(\omega)-1] \\ if_0(\omega) \end{Bmatrix} e^{-i\omega T}$$

$$\times \int_{-\infty}^{+\infty} dT'\, e^{i\omega T'}\left[\frac{\sqrt{\pi}}{2\pi}\right]^3\left[\frac{2m}{iT'}\right]^{1/2}\frac{m\omega_c}{i\sin(\omega_c T'/2)}\, e^{-i\mu_0 H\sigma_3 T'}$$

$$\times \exp\left[\frac{im\omega_c \overline{R}^{\,2}}{4}\cot(\omega_c T'/2)\right]\exp\left[\frac{im}{2T'}(z-z')^2\right]$$

$$= \begin{cases} \overline{G}'_{10>}(\overline{R},|z-z'|;T) \\ \overline{G}'_{10<}(\overline{R},|z-z'|;T)\,. \end{cases} \tag{2.9}$$

The above infinite-space Green's function can be employed to obtain the finite slab Green's function by superposing a series of such infinite-space Green's functions where the source points are distributed at appropriate image-source locations in accordance with finite-slab boundary conditions (specular reflection) which dictate the vanishing of the Green's function at the faces of the slab (with **H** perpendicular to the slab faces).

The appropriate superposition of infinite-space Green's functions to achieve vanishing of the resulting slab Green's function at the slab faces is given by

$$\overline{G}'_1(\overline{R},z,z';T) = \sum_{n=-\infty}^{+\infty}[\overline{G}'_{10}(\overline{R},z,2nd+z';T) - \overline{G}'_{10}(\overline{R},z,2nd-z';T)]\,. \tag{2.9a}$$

This satisfies the Green's-function equation in $z\rightarrow[0,d]$ since only one source point $n=0$ occurs in this fundamental interval at z'. Moreover, it is manifestly periodic in z,z' with period $2d$. Since $\overline{G}'_{10}$ is the infinite-space Green's function (no boundaries), we have

$$\overline{G}'_1(\overline{R},z,z';T) = \sum_{n=-\infty}^{+\infty}\overline{G}'_{10}(\overline{R},|z-2nd-z'|;T) - \overline{G}'_{10}(\overline{R},|z-2nd+z'|;T)\,. \tag{2.9b}$$

To show that $\overline{G}'_1(\overline{R},z,z';T)$ as given by this image series does indeed satisfy the boundary conditions at the slab faces, we note the following.

(a) $\overline{G}'_1(\overline{R},z,z';T)$ is odd under $z\rightarrow -z$, and therefore $\overline{G}'_1(\overline{R},z=0,z';T)$ vanishes at $z=0$ (To prove this, set $z\rightarrow -z$ and $n\rightarrow -n$.)

(b) $\overline{G}'_1(\overline{R},z=d,z';T)$ vanishes at $z=d$, since we have $\overline{G}'_1(\overline{R},z=d,z';T)$ given as

$$\sum_{n=-\infty}^{+\infty}\overline{G}'_{10}(\overline{R},|d-2nd-z'|;T) - \overline{G}'_{10}(\overline{R},|d-2nd+z'|;T) = \sum_{n=-\infty}^{+\infty}\overline{G}'_{10}(\overline{R},|z'+(2n-1)d|;T) - \sum_{n=-\infty}^{+\infty}\overline{G}'_{10}(\overline{R},|z'-(2n-1)d|;T)\,.$$

If we now put $n\rightarrow -n$ in the second series on the right-hand side, and recall that the entire Green's function has period $2d$, then it is clear that the second term on the right-hand side just cancels the first term, so that the Green's function vanishes at $z=d$, as it should.

(c) Having proved in (a) and (b) that $\overline{G}'_1(\overline{R},z,z';T)$ vanishes at $z=0,d$ above, one may invoke similar arguments to prove that it similarly vanishes at $z'=0,d$ (one can probably prove the latter by reciprocity as well).

The equivalence of this image-series representation of the Green's function with the partial eigenfunction expansion given above for the Green's function may be established using the Poisson sum formula. The image series is especially useful for the case of a very thick slab, for in this case only the first few terms of the image series are needed for an accurate approximation to the Green's function.

C. Evaluation of density for a semi-infinite medium in magnetic field

The density may be calculated from the Green's function using

$$N=-i\,\mathrm{Tr}\int d^3r \lim_{T\to 0}\lim_{z'\to z}\lim_{\bar r'\to\bar r}\bar G'_{1<}(\bar r,z,\bar r',z';T)\,, \tag{2.10a}$$

and an uncorrelated evaluation of the density will be obtained using the uncorrelated Green's function above. In the semi-infinite thick-slab limit ($d\to\infty$), only the $n=0$ terms of the image series [Eq. (2.9b)] contribute, whence

$$\bar G'_1(\bar R,z,z';T)\to\bar G'_{10}(\bar R,|z-z'|;T)-\bar G'_{10}(\bar R,|z+z'|;T)\,.$$

The first term just corresponds to the infinite-bulk-plasma expression for density in quantizing magnetic field, and has been evaluated earlier.[1(a)] The second term provides the contribution of the only active image for the semi-infinite plasma, and it represents the effect of the boundary on the expression for density. Thus the density ρ is given by

$$\rho=\rho_1+\rho_2=-i\lim_{T\to 0}\lim_{z\to z'}\lim_{\bar r\to\bar r'}\mathrm{Tr}\times[\bar G'_{10<}(\bar r-\bar r',|z-z'|;T)-\bar G'_{10<}(\bar r-\bar r',|z+z'|;T)]\,, \tag{2.10b}$$

where the infinite-bulk-plasma expression for density ρ_1 has been evaluated in Ref. 1(a) as (restore $\hbar$ here)

$$\rho_1=2\int\frac{d\omega}{\hbar^3}f_0(\omega)\eta_+(\omega)\int\frac{dx}{2\pi}e^{i\omega x}\int\frac{dp_z}{2\pi}e^{-ip_z^2(x/2m)}\int\frac{d^2\bar p}{(2\pi)^2}\exp\left[\frac{-i\bar p^2}{m\hbar\omega_c}\tan\left[\frac{\hbar\omega_c x}{2}\right]\right],$$

and evaluation of the Gaussian momentum integrals yields

$$\rho_1=2\int\frac{d\omega}{\hbar^3}f_0(\omega)\eta_+(\omega)\int\frac{dx}{2\pi}e^{i\omega x}\left[\frac{\sqrt{\pi}}{2\pi}\right]^3\left[\frac{2m}{ix}\right]^{1/2}\frac{m\hbar\omega_c}{i\tan(\hbar\omega_c x/2)}\,. \tag{2.11a}$$

The second term ρ_2, which arises from the image and represents the effect of the boundary, may be similarly evaluated as[3]

$$\rho_2=-2\int\frac{d\omega}{\hbar^3}f_0(\omega)\eta_+(\omega)\int\frac{dx}{2\pi}e^{i\omega x}\left[\frac{\sqrt{\pi}}{2\pi}\right]^3\left[\frac{2m}{ix}\right]^{1/2}\frac{m\hbar\omega_c}{i\tan(\hbar\omega_c x/2)}\exp\left[\frac{-2mz^2}{\hbar^2 ix}\right]. \tag{2.11b}$$

Thus the semi-infinite slab density ρ in magnetic field is given by

$$\rho=2\int_0^\infty d\omega\frac{f_0(\omega)}{\hbar^3}\eta_+(\omega)\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}e^{\omega s}\left[\frac{\sqrt{\pi}}{2\pi}\right]^3\left[\frac{2m}{s}\right]^{1/2}\frac{m\hbar\omega_c}{\tanh(\hbar\omega_c s/2)}\left[1-\exp\left[\frac{-2mz^2}{\hbar^2 s}\right]\right]. \tag{2.11c}$$

In the nondegenerate case, we may put $f_0(\omega)\to e^{\zeta\beta}e^{-\omega\beta}$, so that the ω and s integrations are Laplace transform and inverse. Then one immediately obtains

$$\rho=\frac{2\exp(\zeta\beta)}{\hbar^3}\left[\frac{\sqrt{\pi}}{2\pi}\right]^3\left[\frac{2m}{\beta}\right]^{1/2}\times\frac{m\hbar\omega_c}{\tanh(\hbar\omega_c\beta/2)}(1-e^{-2mz^2/\beta\hbar^2})\,. \tag{2.12}$$

This nondegenerate evaluation of ρ has the requisite properties of vanishing at the bounding surface $z=0$ and approaching the infinite bulk value deep in the medium as $z\to\infty$.

The evaluation of ρ for the semi-infinite medium in the degenerate case involves the evaluation of the s integral of Eq. (2.11c), which is, in fact, a prototype of many integrals which occur in this work. The s integral is an inverse Laplace transform whose integrand is responsible for two distinct types of contributions associated with the two distinct types of singularities of the s integrand, namely (a) a branch cut along the negative real axis with a branch point at the origin, and (b) isolated singularities evenly spaced along the imaginary s axis at $s=s_n=\pm i2\pi n/\hbar\omega_c$. These two types of singularities result in contributions which are (a) monotonic in magnetic field dependence and (b) oscillatory in the de Haas–van Alphen (dHvA) sense, respectively. We will first consider an evaluation appropriate to low magnetic field $\hbar\omega_c/\zeta\ll 1$, and later discuss an alternative evaluation procedure appropriate to higher-field strength.

For low-field strength ($\hbar\omega_c\ll\zeta$) in the degenerate limit, we separate the s integral into a branch-cut contribution denoted by the subscript Γ and isolated (pole) singularity contributions denoted by the subscript C_n. At low field the branch-cut contribution may be approximated by replacing $1/\tanh(\hbar\omega_c s/2)$ by the leading term of its Laurent expansion [but one can not go much further with this Laurent expansion in the branch-cut integral for reasons discussed in Ref. 1(a), p. 57], obtaining $\rho_\Gamma=\rho_{1\Gamma}+\rho_{2\Gamma}$, where

$$\rho_{1\Gamma}=2\int_0^{\zeta}\frac{d\omega}{\hbar^3}\int_\Gamma \frac{ds}{2\pi i}e^{\omega s}\left[\frac{\sqrt{\pi}}{2\pi}\right]^3\left[\frac{2m}{s}\right]^{3/2}$$

$$=\left[\frac{m}{2\pi}\right]^{3/2}\frac{2}{\Gamma(\frac{5}{2})}\frac{\zeta^{3/2}}{\hbar^3}, \tag{2.13a}$$

$$\rho_{2\Gamma}=-2\int_0^{\zeta}\frac{d\omega}{\hbar^3}\int_\Gamma \frac{ds}{2\pi i}e^{\omega s}\left[\frac{\sqrt{\pi}}{2\pi}\right]^3\left[\frac{2m}{s}\right]^{3/2}e^{-2mz^2/\hbar^2 s}$$

$$=\left[\frac{m}{2\pi}\right]^{3/2}\frac{2}{\hbar^3}\left[-\frac{\hbar^3\sin(2z\sqrt{2m\zeta}/\hbar)}{2\sqrt{\pi}(2m)^{3/2}z^3}+\frac{\hbar^2\zeta^{1/2}\cos(2z\sqrt{2m\zeta}/\hbar)}{2\sqrt{\pi}mz^2}\right]. \tag{2.13b}$$

The details of integrating $\rho_{1\Gamma}$ have been discussed in Ref. 1(a) (Appendix I), and the integration of $\rho_{2\Gamma}$ is similar except for the fact that here the s integral represents a Bessel function of half-integer order,[7] which is an elementary function [see Ref. 7(b), Vol. I, p. 245, Eqs. (33) and (35), and Ref. 7(a), Vol. II, p. 15, Eq. (5) and p. 79, Eq. (14)]. This low-field approximation is just the zero-field limit quoted by Newns[2] as follows ($\rho_{\rm bulk}\equiv\rho_{1\Gamma}$):

$$\rho_\Gamma=\rho_{\rm bulk}\left[1-\frac{3\sin(2p_Fz)}{(2p_Fz)^3}+\frac{3\cos(2p_Fz)}{(2p_Fz)^2}\right] \tag{2.14}$$

(except that one must bear in mind that $p_F=\sqrt{2m\zeta}$ involves magnetic field corrections through the implicit dependence of ζ on applied field), which has the requisite properties of vanishing at the bounding surface $z=0$ and approaching the infinite bulk value deep in the medium as $z\to\infty$ by way of a Friedel-Kohn "wiggle."

The de Haas–van Alphen oscillatory terms arising from the isolated singularities of the s integrand are more interesting for our purposes. Integrating by parts on ω, these terms are given by $\rho_{\rm dHvA}=\sum_n \rho_{C_n}$, where

$$\rho_{C_n}=-\left[\frac{m}{2\pi}\right]^{3/2}\hbar\omega_c\int_0^\infty\frac{d\omega}{\hbar^3}\frac{df_0(\omega)}{d\omega}\oint_{C_n}\frac{ds}{2\pi i}\frac{\exp(s\omega)}{s^{3/2}\tanh(\hbar\omega_c s/2)}(1-e^{-2mz^2/\hbar^2 s}). \tag{2.15}$$

The contour denoted by C_n is a small circle about the nth isolated singularity at $s=s_n=\pm i2\pi n/\hbar\omega_c$ ($n=1,2,\ldots,\infty$). Since the s integrand has a simple pole at s_n, the integral is readily evaluated by residues as

$$\oint_{C_n}\frac{ds}{2\pi i}(\cdots)=\frac{\exp[\pm i(2\pi n\omega/\hbar\omega_c)]}{(\pm i2\pi n/\hbar\omega_c)^{3/2}}\frac{2}{\hbar\omega_c}\left[1-\exp\left[\mp\frac{\hbar\omega_c}{i2\pi n}\frac{2mz^2}{\hbar^2}\right]\right],$$

and the ensuing ω integral may be performed for finite temperature as indicated in Ref. 1(a) (Appendix I) with the result

$$\rho_{\rm dHvA}=\sum_n\rho_{C_n}=\frac{m^{3/2}(\hbar\omega_c)^{1/2}}{\pi\beta\hbar^3}\sum_{n=1}^{\infty}\frac{\cos\left[\frac{2\pi n\zeta}{\hbar\omega_c}-\frac{3\pi}{4}\right]-\cos\left[\frac{2\pi n\zeta}{\hbar\omega_c}-\frac{3\pi}{4}+\frac{m\omega_c z^2}{\pi n\hbar}\right]}{n^{1/2}\sinh(2\pi^2 n/\hbar\omega_c\beta)}, \tag{2.16}$$

which has the requisite properties of vanishing at the bounding surface $z=0$ and approaching the infinite bulk value deep in the medium as $z\to\infty$. It should be noted that the effective dHvA oscillation phase is dependent on distance z from the bounding surface.

Considering an alternative evaluation of ρ appropriate to higher-magnetic-field strength in the degenerate case, we rewrite the integrand of (2.11c) by introducing the expansion

$$\left[\tanh\left[\frac{\hbar\omega_c s}{2}\right]\right]^{-1}=\sum_{\pm}\sum_{r=0}^{\infty}\exp\left[(\pm1-1-2r)\frac{\hbar\omega_c s}{2}\right].$$

The evaluation of ρ_1 is then straightforward, and the detailed result obtained in Ref. 1(b) is given by (zero temperature)

$$\rho_1=\frac{m^{3/2}\omega_c}{2^{1/2}\pi^2\hbar^2}\sum_{\pm}\sum_{r=0}^{\infty}[\zeta\pm\tfrac{1}{2}\hbar\omega_c-(r+\tfrac{1}{2})\hbar\omega_c]^{1/2}\eta_+[\zeta\pm\tfrac{1}{2}\hbar\omega_c-(r+\tfrac{1}{2})\hbar\omega_c]. \tag{2.17a}$$

For ρ_2, the associated s integral represents a Bessel function of half-integer order [Ref. 7(a), Vol. II, p. 15, Eq. (5)] which is elementary, $\sim J_{-1/2}(z)=\sqrt{2/\pi}\cos z/z^{1/2}$. The ensuing ω integral at zero temperature is elementary[7] and it yields the result

$$\rho_2=-\frac{m^{3/2}\omega_c}{2^{1/2}\pi^2\hbar^2}\sum_{\pm}\sum_{r=0}^{\infty}\frac{\hbar}{2\sqrt{2m}z}\sin\left[\frac{2\sqrt{2m}z}{\hbar}[\zeta\pm\tfrac{1}{2}\hbar\omega_c-(r+\tfrac{1}{2})\hbar\omega_c)]^{1/2}\right]\eta_+[\zeta\pm\tfrac{1}{2}\hbar\omega_c-(r+\tfrac{1}{2})\hbar\omega_c]. \tag{2.17b}$$

It should be noted that ρ_2 cancels ρ_1 at $z=0$, and ρ_2 vanishes at $z\to\infty$, as one should expect. Moreover, the low-field limit of this expression in which $\sum_r \to \int$ (associated with the close packing of Landau eigenstates) correctly yields the zero-field result for ρ_2. The quantum strong-field limit ($\hbar\omega_c/\zeta > 1$, all electrons in the lowest Landau state) is given by the leading terms in ρ_1 and ρ_2, namely

$$\rho=\rho_{\text{bulk}}\left[1-\frac{1}{2p_F z}\sin(2p_F z)\right], \tag{2.18}$$

where $\rho_{\text{bulk}}=(m^{3/2}\omega_c\zeta^{1/2})/(2^{1/2}\pi^2\hbar^2)$ is the bulk density expression in the quantum strong-field limit. [The bulk density expression here in the quantum strong-field limit has a different functional form than does $\rho_{1\Gamma}$ for low magnetic field in Eq. (2.14) due to differing functional dependences of number density on chemical potential in the two cases. Nonetheless, both represent the same physical quantity, bulk density. (As elsewhere in this paper, the chemical potential referred to is understood to be the one appropriate to the ambient-magnetic-field strength in the case considered.).] It is clear that there is Friedel-Kohn "wiggle" behavior here, albeit one dimensional (parallel to the field) due to the extremely high field.

III. LONGITUDINAL DIELECTRIC RESPONSE OF A SLAB OF QUANTUM PLASMA IN A MAGNETIC FIELD PERPENDICULAR TO THE SLAB

This study of the longitudinal dielectric response properties of a slab of quantum plasma in a magnetic field (perpendicular to the slab faces) is undertaken within the framework of the random-phase approximation (RPA). Using a Green's-function formulation of the longitudinal dielectric function ϵ, we generalize Newns's description of longitudinal response properties of a slab in terms of the density perturbation response matrix[2] $\mathscr{R}=\delta\rho/\delta V$ to include magnetic field effects associated with Landau quantization. In the thick semi-infinite limit the diagonal part of $\mathscr{R}$ is seen to be determined by the bulk infinite-space plasma dielectric function in the presence of an ambient magnetic field, as was found to be the case in the zero-field limit.[2] This result is valid for degenerate solid-state magnetoplasmas as well as for nondegenerate gaseous magnetized plasmas.[8]

A. Formulation in terms of the density perturbation response matrix

The description of longitudinal dielectric response properties of a slab of quantum plasma in magnetic field in the random-phase approximation devolves upon the integral equation[1(a)] for the inverse dielectric function $K(1,2)=\delta V(1)/\delta U(2)$,

$$K(1,2)=\delta(1-2)-i\int d(3)\int d(4)v(1-3)\times\bar{G}'_1(3,4)\bar{G}'_1(4,3^+)K(4,2) \tag{3.1}$$

[$v(1-3)$ is the interparticle Coulomb potential], where $\bar{G}'_1$ is the uncorrelated Green's function in the presence of slab boundaries and magnetic field perpendicular to the slab which has been evaluated in terms of a partial eigenfunction expansion as well as an image-series representation in Sec. II. The presence of the slab boundaries and the concomitant lack of spatial translational invariance result in substantial difficulty in solving for $K(1,2)$, and our efforts at solving the integral equation exactly for $K(1,2)$ are discussed elsewhere.[30] It is simpler to write the solution for the direct dielectric function $\epsilon(1,2)=\delta U(1)/\delta V(2)$, which is inverse to $K(1,2)$ in the sense that

$$\int d(3)K(1,3)\epsilon(3,2)=\int d(3)\epsilon(1,3)K(3,2)=\delta(1-2). \tag{3.2}$$

The solution of the RPA integral equation for the direct dielectric function $\epsilon(1,2)$ is given explicitly by

$$\epsilon(1,2)=\delta(1-2)+i\int d(3)v(1-3)\bar{G}'_1(3,2)\bar{G}'_1(2,3^+), \tag{3.3}$$

and this may be expressed[1-3] in terms of the density perturbation response function

$$\mathscr{R}(1,2)\equiv\delta\rho(1)/\delta V(2)=-i\bar{G}'_1(1,2)\bar{G}'_1(2,1^+)$$

as follows:

$$\epsilon(1,2)=\delta(1-2)-\int d(3)v(1-3)\mathscr{R}(3,2). \tag{3.4}$$

[It should be noted that $\mathscr{R}=\delta\rho/\delta V$ as defined here differs from the corresponding density perturbation response function $\mathscr{R}$ defined by Newns[2] by a minus sign. This is a consequence of the fact that our potentials are just the negatives of the potentials defined by Newns, since we define potentials in accordance with the usual Poisson equation ∇^2(potential) $=-4\pi$(charge density), whereas Newns's potentials are defined to satisfy ∇^2(potential) $=+4\pi$(charge density) and are thus clearly just the negatives of our potentials. The concomitant difference in sign between our $\mathscr{R}$ response function and Newns's counterpart is clearly evident in the $\mathscr{R}$ term of Eq. (3.4), which carries a minus sign explicitly, whereas the corresponding term of Newns's Eq. (46) carries a plus sign instead. This difference in sign between our $\mathscr{R}$ response function and Newns's counterpart applies to both the diagonal part D and the nondiagonal part $-A$ to be defined below by $\mathscr{R}(\bar{q},q_z,q'_z;\nu)=D(\bar{q},q_z;\nu)\delta_{q_zq'_z}-A(\bar{q},q_z,q'_z;\nu)$.] The polarizability $4\pi\alpha_0(1,2)$ may be recognized as

$$4\pi\alpha_0(1,2)=-\int d(3)v(1-3)\mathscr{R}(3,2). \tag{3.5}$$

The thermodynamic Green's function and dielectric response functions (both inverse and direct) which we have been dealing with here are antiperiodic and periodic, respectively, in regard to time with period $\tau=-i\beta=-i/k_BT_0$, and, of course, time integrals are extended over the fundamental interval $[0,\tau]$. In particular, $\epsilon(1,2)$ is periodic and may be represented by a Fourier series

$$\epsilon(1,2)=\frac{1}{\tau}\sum_{\nu\text{ even}}\exp\left\{i\left[\frac{\pi\nu}{\tau}(t_1-t_2)\right]\right\}\bar{\epsilon}(\mathbf{r}_1,\mathbf{r}_2,\nu), \tag{3.6}$$

where the Fourier-series coefficient is given by

$$\bar{\epsilon}(\mathbf{r}_1,\mathbf{r}_2;\nu)=\int_0^\tau dT \exp\left[-i\frac{\pi\nu}{\tau}T\right]\epsilon(\mathbf{r}_1,\mathbf{r}_2;T)\,, \tag{3.7}$$

($T=t_1-t_2$) or, alternatively,

$$\bar{\epsilon}(\mathbf{r}_1,\mathbf{r}_2;\nu)=\delta^3(\mathbf{r}_1-\mathbf{r}_2)-\int d\mathbf{r}_3 v(\mathbf{r}_1-\mathbf{r}_3)\mathscr{R}(\mathbf{r}_3,\mathbf{r}_2,\nu)\,,$$

where

$$\begin{aligned}\mathscr{R}(\mathbf{r}_3,\mathbf{r}_2;\nu)&=\int_0^\tau dT \exp\left[-i\pi\nu\frac{T}{\tau}\right]\frac{\delta\rho(3)}{\delta V(2)}\\ &=-i\int_0^\tau dT\, e^{-i\pi\nu T/\tau}\bar{G}'_1(\mathbf{r}_3,\mathbf{r}_2;T)\\ &\qquad\times\bar{G}'_1(\mathbf{r}_2,\mathbf{r}_3;-T)\,.\end{aligned} \tag{3.8}$$

[Equations (3.6) and (3.7) here, along with Eq. (I.30ff) of Ref. 1(a) and Eq. (1) of Ref. 4, employ a convention that differs from the usual one given by Eq. (I.11) of Ref. 1(a) and Eq. (5.19) of Ref. 1(c) in that *the sign of frequency ν is reversed.* This reversal of the sign of frequency does not affect the real part of the response function which is an even function of frequency. However, this sign reversal negates the imaginary part of the response function since it is an odd function of frequency. These comments should be borne in mind in interpreting the results of this work. The even (odd) properties of the real (imaginary) parts of the response function are discussed in Appendix C.] This Fourier-series coefficient is discontinuous across the real frequency axis, and that discontinuity may be used to obtain the corresponding spectral weight function of the Martin-Schwinger spectral representation[1(c)] of the periodic response function. However, we may alternatively go directly to the corresponding retarded physical response function by noting that its Fourier transform is given by the Fourier-series coefficient evaluated above the real frequency axis [Ref. 1(a), p. 10, footnote 6], involving the replacement $\pi\nu/\tau\rightarrow\Omega+i\varepsilon$. Thus, the Fourier transform of the retarded physical response function $\epsilon_{\text{phys}}(\mathbf{r}_1,\mathbf{r}_2;\Omega)$ is given by

$$\begin{aligned}\epsilon_{\text{phys}}(\mathbf{r}_1,\mathbf{r}_2;\Omega)&=\delta^3(\mathbf{r}_1-\mathbf{r}_2)\\ &\quad-\int d\mathbf{r}_3\, v(\mathbf{r}_1-\mathbf{r}_3)\\ &\qquad\times\mathscr{R}(\mathbf{r}_3,\mathbf{r}_2;\nu\rightarrow(\tau/\pi)[\Omega+i\varepsilon])\,,\end{aligned} \tag{3.9}$$

where $\mathscr{R}(\mathbf{r}_3,\mathbf{r}_2;\nu\rightarrow(\tau/\pi)[\Omega+i\varepsilon])$ is the retarded physical response function representing the density perturbation. We now explicitly construct the density perturbation $\mathscr{R}(\bar{r},z,\bar{r}',z';\nu)$ by substituting Eq. (2.6) into Eq. (3.8), and obtain [note that all Green's functions appearing below are the $\bar{G}'_{10}$ Green's functions appearing in (2.6) and (2.7), and they refer to the uncorrelated infinite-space Green's function as if no boundaries were present; for notational convenience we hereafter drop the subscript "10" and drop the prime, and write $\bar{G}'_{10}\rightarrow\bar{G}$]

$$\begin{aligned}\mathscr{R}(\bar{r},z,\bar{r}',z';\nu)=\frac{4}{d^2}\int\frac{d^2\bar{k}}{(2\pi)^2}\int\frac{d^2\bar{k}'}{(2\pi)^2}\exp[i(\bar{k}-\bar{k}')\cdot(\bar{r}-\bar{r}')]\\ \times\sum_{k_z>0}\sum_{k_z'>0}\sin(k_z z)\sin(k_z z')\sin(k_z' z)\sin(k_z' z')F(\bar{k},k_z;\bar{k}',k_z';\nu)\,,\end{aligned} \tag{3.10}$$

where k_z,k_z' now play the roles of $p_{zn},p_{zn}'\rightarrow n\pi/d$ of Eq. (2.6), and

$$F(\bar{k},k_z;\bar{k}',k_z';\nu)=-i\int_0^\tau dt\, e^{-i\pi\nu t/\tau}\bar{G}_<(\bar{k},k_z;-t)\bar{G}_>(\bar{k}'k_z';t)\,. \tag{3.11}$$

In accordance with the prescription discussed above to obtain the retarded physical response function we must now put $\nu\rightarrow(\tau/\pi)[\Omega+i\varepsilon]$, and for convenience we write this as $\nu\rightarrow\nu+i\delta$ [with $\delta=(\tau/\pi)\varepsilon$]. It is also useful to express the t integral $\int_0^\tau dt(\cdots)$ in terms of half-time axis integrals as was done in Ref. 1(a). Thus, for the integral $F(\bar{k},k_z;\bar{k}',k_z';\nu+i\delta)$ we may set $\int_0^\tau dt=\int_0^{-\infty}dt+\int_{-\infty}^{\tau}dt$, and be assured of convergence,

$$\begin{aligned}iF(\bar{k},k_z;\bar{k}',k_z';\nu+i\delta)=&\int_0^{-\infty}dt\exp\left[-i\frac{\pi}{\tau}(\nu+i\delta)t\right]\bar{G}_<(\bar{k},k_z;-t)\bar{G}_>(\bar{k}',k_z';t)\\ &+\int_{-\infty}^{\tau}dt\exp\left[-i\frac{\pi}{\tau}(\nu+i\delta)t\right]\bar{G}_<(\bar{k},k_z;-t)\bar{G}_>(\bar{k}',k_z';t)\,.\end{aligned} \tag{3.12}$$

Following the procedures of Ref. 1(a), this may be rewritten as

$$\begin{aligned}iF(\bar{k},k_z;\bar{k}',k_z';\nu+i\delta)=&-\left[\int_0^{\infty}dt\exp\left[\frac{-i\pi}{\tau}(\nu-i\delta)t\right]\bar{G}_<(\bar{k},k_z;-t)\bar{G}_>(\bar{k}',k_z';t)\right]^*\\ &+\int_{-\infty}^{0}dt\exp\left[\frac{-i\pi}{\tau}(\nu+i\delta)t\right]\bar{G}_>(\bar{k},k_z;-t)\bar{G}_<(\bar{k}',k_z';t)\,,\end{aligned} \tag{3.13}$$

where we have translated the second t-integration variable through τ (to get to the origin as a limit), and we have used the antiperiodicity properties of $\overline{G}_>$ and $\overline{G}_<$.

The physical response function for the slab of quantum plasma in magnetic field is therefore given in the $(\overline{r},z)$ representation by

$$\mathscr{R}(\overline{r},z;\overline{r}',z';\Omega+i\varepsilon)=\frac{-i4}{d^2}\int\frac{d^2\overline{k}}{(2\pi)^2}\int\frac{d^2\overline{k}'}{(2\pi)^2}e^{i(\overline{k}-\overline{k}')\cdot(\overline{r}-\overline{r}')}\times\sum_{k_z}\sum_{k_z'}\sin(k_zz)\sin(k_zz')\sin(k_z'z)\sin(k_z'z')\times\left[\left[-\int_0^\infty dt\,e^{-it(\Omega-i\varepsilon)}\overline{G}_<(\overline{k},k_z;-t)\overline{G}_>(\overline{k}',k_z';t)\right]^*+\left[\int_{-\infty}^0 dte^{-it(\Omega+i\varepsilon)}\overline{G}_>(\overline{k},k_z;-t)\overline{G}_<(\overline{k}',k_z';t)\right]\right]. \quad (3.14)$$

Newns[2] has pointed out that it is advantageous to employ the spatial Fourier representation defined by

$$f(\overline{r},z)=\frac{2}{d}\sum_{q_z>0}\int\frac{d^2\overline{q}}{(2\pi)^2}e^{-i\overline{q}\cdot\overline{r}}\cos(q_zz)f(\overline{q},q_z)\;, \quad (3.15a)$$

where

$$f(\overline{q},q_z)=\int_0^d dz\int d^2\overline{r}\,e^{i\overline{q}\cdot\overline{r}}\cos(q_zz)f(\overline{r},z)\;, \quad (3.15b)$$

and $q_z=(2n+1)\pi/d$ ($n=0,1,2,\ldots$) for functions which are odd (antisymmetric) across the slab in the sense that $f(\overline{r},z)=-f(\overline{r},d-z)$, describing antisymmetric potentials and antisymmetric surface-plasmon modes. [A similar development for even (symmetric) functions $f(\overline{r},z)=f(\overline{r},d-z)$ and symmetric surface-plasmon modes is given by Newns (Ref. 2, p. 3313) and Yildiz (Ref. 3, pp. 215 and 216). For even symmetric functions $f(z)=+f(d-z)$, one should replace Eq. (3.15a) by

$$f(\overline{r},z)=\frac{2}{d}\sum_{q_z\geq 0}\eta_{q_z}\int\frac{d^2\overline{q}}{(2\pi)^2}e^{-i\overline{q}\cdot\overline{r}}\cos(q_zz)f(\overline{q}z)\;,$$

where $q_z=2\pi n/d$, $n=0,1,2,\ldots$, and $\eta_{q_z}=1$ for $q_z>0$ with $\eta_{q_z}=\frac{1}{2}$ for $q_z=0$.] We will discuss symmetric and antisymmetric potentials again in Sec. V below. It should be noted that in the semi-infinite "thick" limit $d\to\infty$, the parity of the potential does not matter and both the antisymmetric and symmetric surface-plasmon modes have the same characteristic frequency. Confining our attention to odd (antisymmetric) functions, we apply this spatial Fourier representation doubly to the z and z' dependences of $\mathscr{R}(\overline{r},z,\overline{r}',z';\nu)$.

Recognizing that there is effective spatial translational invariance in the plane perpendicular to the magnetic field, so that $\mathscr{R}(\overline{r},z,\overline{r}',z';\nu)$ depends on $\overline{r}-\overline{r}'$ to the exclusion of $\overline{r}+\overline{r}'$, we further introduce the infinite Fourier transform with respect to the variable $(\overline{r}-\overline{r}')$. Thus we obtain

$$\mathscr{R}(\overline{q},q_z,q_z';\nu)=\int d^2(\overline{r}-\overline{r}')\int_0^d dz\int_0^d dz'\,e^{i\overline{q}\cdot(\overline{r}-\overline{r}')}\cos(q_zz)\cos(q_z'z')\mathscr{R}(\overline{r},z;\overline{r}',z';\nu)\;. \quad (3.16)$$

We shall employ Eq. (3.10) here in a slightly modified form, by relaxing the summation restrictions on k_z and k_z', thus extending these sums over negative as well as positive k_z and k_z'. The effect of this is to spuriously double each series, and since two such doublings are involved we must compensate for the spurious quadrupling by dividing by 4. (There will be no change in the restrictions $q_z>0$, $q_z'>0$.) Thus we use

$$\mathscr{R}(\overline{r},z;\overline{r}',z';\nu)=\frac{4}{d^2}\int\frac{d^2\overline{k}}{(2\pi)^2}\int\frac{d^2\overline{k}'}{(2\pi)^2}e^{i(\overline{k}-\overline{k}')\cdot(\overline{r}-\overline{r}')}\times\frac{1}{4}\sum_{\text{all }k_z}\sum_{\text{all }k_z'}\sin(k_zz)\sin(k_zz')\sin(k_z'z)\sin(k_z'z')F(\overline{k},k_z;\overline{k}',k_z';\nu)\;, \quad (3.17)$$

and substitution into (3.16) involves the integrals $\Gamma(k_z,k_z',q_z)$ evaluated below

$$\Gamma(k_z,k_z',q_z)=\int_0^d dz\sin(k_z'z)\sin(k_zz)\cos(q_zz)=\frac{d}{4}(\delta_{k_z',k_z+q_z}+\delta_{k_z',k_z-q_z}-\delta_{k_z',-k_z-q_z}-\delta_{k_z',-k_z+q_z})\;. \quad (3.18)$$

Thus we obtain the result

$$\mathscr{R}(\bar{q},q_z,q_z';\nu)=\sum_{\text{all } k_z}\sum_{\text{all } k_z'}\tfrac{1}{16}\int\frac{d^2\bar{k}}{(2\pi)^2}F(\bar{k},k_z,\bar{k}+\bar{q},k_z';\nu)\Big[(\delta_{k_z',k_z+q_z}-\delta_{k_z',-k_z-q_z}-\delta_{k_z',q_z-k_z}+\delta_{k_z',k_z-q_z})\times(\delta_{k_z',k_z+q_z'}-\delta_{k_z',-k_z-q_z'}-\delta_{k_z',q_z'-k_z}+\delta_{k_z',k_z-q_z'})\Big]. \quad (3.19)$$

On using the property that the function F depends only on the modulii of its arguments, (3.19) may be simplified as

$$\mathscr{R}(\bar{q},q_z,q_z';\nu)=\tfrac{1}{4}\int\frac{d^2\bar{k}}{(2\pi)^2}\left[\sum_{k_z=-\infty}^{+\infty}F(\bar{k},k_z,\bar{k}+\bar{q},k_z+q_z;\nu)\delta_{q_zq_z'}-F(\bar{k},\alpha,\bar{k}+\bar{q},\gamma;\nu)-F(\bar{k},\gamma,\bar{k}+\bar{q},\alpha;\nu)\right], \quad (3.20)$$

where $\alpha=\frac{1}{2}(q_z'+q_z)$ and $\gamma=\frac{1}{2}(q_z'-q_z)$. (Here we have noted that the restrictions $q_z>0,q_z'>0$ eliminate certain terms.) Equation (3.20) together with the expression given above for $F(\bar{k},k_z,\bar{k}',k_z';\nu)$ in terms of the magnetic field Green's function serve to generalize the development of Newns[2] to include effects of Landau quantization in the case when the magnetic field is perpendicular to the slab surface. Considering $\mathscr{R}(\bar{q},q_z,q_z';\nu)$ as a matrix in the indices q_z and q_z', Newns[2] identifies the "diagonal" part $D(\bar{q},q_z;\nu)$ and "nondiagonal" part $A(\bar{q},q_z,q_z';\nu)$ as

$$\mathscr{R}(\bar{q},q_z,q_z';\nu)=D(\bar{q},q_z;\nu)\delta_{q_zq_z'}-A(\bar{q},q_z,q_z';\nu), \quad (3.21)$$

where

$$D(\bar{q},q_z;\nu)=\frac{e^2}{4}\int\frac{d^2\bar{k}}{(2\pi)^2}\sum_{k_z=-\infty}^{+\infty}F(\bar{k},k_z,\bar{k}+\bar{q},k_z+q_z;\nu) \quad (3.22)$$

and

$$A(\bar{q},q_z,q_z';\nu)=\frac{e^2}{4}\int\frac{d^2\bar{k}}{(2\pi)^2}[F(\bar{k},\alpha,\bar{k}+\bar{q},\gamma;\nu)+F(\bar{k},\gamma,\bar{k}+\bar{q},\alpha;\nu)]. \quad (3.23)$$

The characterization[2] of $A(\bar{q},q_z,q_z';\nu)$ as being "nondiagonal" is somewhat misleading since its diagonal elements are, in fact, generally nonzero, $A(\bar{q},q_z,q_z;\nu)\neq 0$. A factor e^2 has been inserted into both parts of $\mathscr{R}(\bar{q},q_z,q_z';\nu)$ in accordance with the recognition that a description of dielectric response properties requires the *charge*-density perturbation, which may be obtained from the density perturbation by multiplying by e^2 (since the former is given by the charge-density—charge-density correlation function, whereas the latter is given by the density-density correlation function). It should be noted that in the semi-infinite limit $d\rightarrow\infty$, we have the diagonal term as

$$\int\frac{d^2\bar{k}}{(2\pi)^2}\sum_{k_z=-\infty}^{+\infty}F(\bar{k},k_z,\bar{k}+\bar{q},k_z+q_z;\nu)\delta_{q_zq_z'}\rightarrow 4\pi\delta(q_z-q_z')\int\frac{d^3k}{(2\pi)^3}F(\mathbf{k},\mathbf{k}+\mathbf{q};\nu), \quad (3.24)$$

since $k_z=n\pi/d$ with

$$\sum_{k_z}\rightarrow\frac{d}{\pi}\int dk_z$$

and also $q_z=(2n+1)\pi/d$ with

$$\sum_{q_z>0}\rightarrow\frac{d}{2\pi}\int_0^\infty dq_z$$

and

$$\delta_{q_zq_z'}\rightarrow\frac{2\pi}{d}\delta(q_z-q_z').$$

Now we have

$$\int\frac{d^3k}{(2\pi)^3}F(\mathbf{k},\mathbf{k}+\mathbf{q};\nu)=-i\int\frac{d^3k}{(2\pi)^3}\int_0^\tau dt\,e^{-i(\pi\nu/\tau)t}\times\bar{G}_<(\mathbf{k},-t)\bar{G}_>(\mathbf{k}+\mathbf{q};t)=-iI(-\mathbf{q},\nu)=-iI(\mathbf{q}\nu), \quad (3.25)$$

where $I(\mathbf{q},\nu)$ is the corresponding quantity for an *infinite* quantum plasma in magnetic field as given in Ref. 1(a), p. 11, Eq. (I.33). Thus in the semi-infinite limit $d\rightarrow\infty$, we have

$$\int\frac{d^2\bar{k}}{(2\pi)^2}\sum_{k_z=-\infty}^{+\infty}F(\bar{k},k_z,\bar{k}+\bar{q},k_z+q_z;\nu)\delta_{q_zq_z'}\rightarrow -i4\pi\delta(q_z-q_z')I(\mathbf{q},\nu),$$

and the semi-infinite-medium result can finally be written as

$$\mathscr{R}(\bar{q},q_z,q_z';\nu)=D(\bar{q},q_z;\nu)\delta(q_z-q_z')-A(\bar{q},q_z,q_z';\nu), \quad (3.26)$$

where

$$D(\bar{q},q_z;\nu)=-i\pi e^2I(\mathbf{q};\nu) \quad (3.27)$$

and

$$A(\bar{q},q_z,q_z';\nu)=\frac{e^2}{4}\int\frac{d^2\bar{k}}{(2\pi)^2}[F(\bar{k},\alpha,\bar{k}+\bar{q},\gamma;\nu)+F(\bar{k},\gamma,\bar{k}+\bar{q},\alpha;\nu)], \quad (3.28)$$

where $\alpha=\frac{1}{2}(q_z'+q_z)$ and $\gamma=\frac{1}{2}(q_z'-q_z)$. We shall henceforth adopt Newns's notation, which involves dropping the factor $-\pi$ in (3.27) so that

$$\mathscr{R}^{\text{diag}}(\bar{q},q_z,q_z';\nu)=D(\bar{q},q_z;\nu)\delta_{q_zq_z'} , \tag{3.29}$$

where

$$D(\bar{q},q_z;\nu)=ie^2I(\mathbf{q};\nu) \tag{3.30}$$

for the semi-infinite medium. (In order to make contact with the notation of Newns,[2] we note that the diagonal part $\mathscr{R}^{\text{diag}}(\bar{q},q_z,q_z';\nu)$ may be written in the semi-infinite limit as

$$\mathscr{R}^{\text{diag}}(\bar{q},q_z,q_z';\nu)=D(\bar{q},q_z;\nu)\delta(q_z-q_z')$$

or, alternatively,

$$\begin{aligned}\mathscr{R}^{\text{diag}}(\bar{q},q_z,q_z';\nu)&=D(\bar{q},q_z;\nu)\frac{d}{2\pi}\delta_{q_zq_z'}\\&=-i\pi e^2I(\mathbf{q};\nu)\frac{d}{2\pi}\delta_{q_z,q_z'} ,\end{aligned}$$

where we have used the equivalence of $\delta(q_z-q_z')$ with $(d/2\pi)\delta_{q_zq_z'}$. But Newns introduces a factor of convenience $2/d$ in all his transforms, so that if we follow his notation we must replace $(d/2\pi)\delta_{q_zq_z'}$ by $(1/\pi)\delta_{q_zq_z'}$, whence $\mathscr{R}^{\text{diag}}(\bar{q},q_z,q_z';\nu)\rightarrow -ie^2I(\mathbf{q};\nu)\delta_{q_zq_z'}$. It must also be borne in mind that Newns's $\mathscr{R}$ function is the negative of ours, so that in his notation we finally have $\mathscr{R}^{\text{diag}}(\bar{q},q_z,q_z';\nu)=ie^2I(\mathbf{q};\nu)\delta_{q_zq_z'}$. [See the note following Eq. (3.4).] Now Newns identifies $D(q,q_z;\nu)$ as the coefficient of $\delta_{q_zq_z'}$, so that according to his notation

$$\mathscr{R}^{\text{diag}}(\bar{q},q_z,q_z';\nu)\rightarrow\mathscr{D}(\bar{q},q_z;\nu)\delta_{q_zq_z'} ,$$

whence Newns's identification for $\mathscr{D}(\bar{q},q_z;\nu)$ is given by

$$\mathscr{D}(\bar{q},q_z;\nu)\rightarrow ie^2I(\mathbf{q};\nu)$$

(which differs from our identification by a factor of $-\pi$).)

This important result may be restated in terms of the bulk infinite-space RPA longitudinal dielectric function in magnetic field $\epsilon^{\infty}_{\text{RPA}}(\mathbf{q};\nu)$, since its relation to $I(\mathbf{q},\nu)$ is given in Ref. 1(a) as

$$\epsilon^{\infty}_{\text{RPA}}(\mathbf{q};\nu)=1+\frac{i4\pi e^2}{q^2}I(\mathbf{q};\nu) , \tag{3.31}$$

whence

$$D(\bar{q},q_z;\nu)=\frac{q^2}{4\pi}[\epsilon^{\infty}(\mathbf{q};\nu)-1] \tag{3.32}$$

for the semi-infinite medium in magnetic field.

In accordance with our discussion above for obtaining the corresponding retarded physical response function $\mathscr{R}(\bar{q},q_z,q_z';\nu+i\delta)$ in terms of half-time axis integrals, we may now identify $D(\bar{q},q_z;\nu+i\delta)$ in the semi-infinite limit as (using Newns's notation)

$$\begin{aligned}D(\bar{q},q_z;\nu+i\delta)&=ie^2I(\mathbf{q};(\tau/\pi)[\Omega+i\varepsilon])\\&=ie^2(-\mathscr{I}^*_>+\mathscr{I}_<) ,\end{aligned} \tag{3.33}$$

where [in the notation of Ref. 1(a)],

$$\mathscr{I}_>=\int_0^{\infty}dt\,e^{-i(\Omega-i\varepsilon)t}\int\frac{d^3k}{(2\pi)^3}\bar{G}_<(\mathbf{k};-t)\bar{G}_>(\mathbf{k}-\mathbf{q};t) \tag{3.34a}$$

and

$$\mathscr{I}_<=\int_{-\infty}^0 dt\,e^{-i(\Omega+i\varepsilon)t}\int\frac{d^3k}{(2\pi)^3}\bar{G}_>(\mathbf{k};-t)\bar{G}_<(\mathbf{k}-\mathbf{q};t) . \tag{3.34b}$$

Similarly, the nondiagonal part $A(\bar{q},q_z,q_z';\nu+i\delta)$ of the retarded physical response function may be written as (Newns's notation)

$$A(\bar{q},q_z,q_z';\nu+i\delta)=-\frac{e^2}{4}\int\frac{d^2\bar{k}}{(2\pi)^2}[F(\bar{k},\alpha,\bar{k}+\bar{q},\gamma;\nu+i\delta)+F(\bar{k},\gamma,\bar{k}+\bar{q},\alpha;\nu+i\delta)] , \tag{3.35}$$

where

$$\begin{aligned}iF(\bar{k},k_z,\bar{k}',k_z';\nu+i\delta)=&-\left[\int_0^{\infty}dt\,e^{-i(\Omega-i\varepsilon)t}\bar{G}_<(\bar{k},k_z;-t)\bar{G}_>(\bar{k}',k_z';t)\right]^*\\&+\int_{-\infty}^0 dt\,e^{-i(\Omega+i\varepsilon)t}\bar{G}_>(\bar{k},k_z;-t)\bar{G}_<(\bar{k}',k_z';t)\end{aligned} \tag{3.36}$$

and

$$\alpha=\tfrac{1}{2}(q_z'+q_z),\quad \gamma=\tfrac{1}{2}(q_z'-q_z) . \tag{3.37}$$

It should be noted that we have made the correspondence $(\pi/\tau)(\nu+i\delta)\leftrightarrow\Omega+i\varepsilon$. This completes the identification of both the diagonal and nondiagonal parts of the retarded physical response matrix in terms of Green's functions in the case when a magnetic field is present.

B. The structure of the diagonal matrix elements of the physical density perturbation response matrix for a semi-infinite medium [evaluation of $D(\bar{q},q_z;\nu+i\delta)$]

In accordance with the discussion in the preceding section, we may identify the diagonal part of the physical response matrix $D(\bar{q},q_z;\nu+i\delta)$ in terms of half-time-axis integrals in the semi-infinite limit as (using Newns's notation)

$$D(\bar{q},q_z;\nu+i\delta)=ie^2 I(\bar{q},q_z;(\tau/\pi)[\Omega+i\varepsilon]) \\ =ie^2(-\mathscr{I}^*_{>}+\mathscr{I}_{<})\ , \tag{3.38a}$$

with $\nu+i\delta\to(\tau/\pi)[\Omega+i\varepsilon]$. Here, $I(\bar{q},q_z;(\tau/\pi)[\Omega+i\varepsilon])$ is the corresponding quantity for an *infinite* quantum plasma in magnetic field which is related to the bulk infinite-space RPA longitudinal dielectric function in magnetic field $\epsilon^{\infty}_{\mathrm{RPA}}(\mathbf{q};\Omega)$ by Eq. (3.31), and this quantity has been analyzed thoroughly [see Refs. 1(a) and 1(b)] both in regard to its mathematical representations and also in regard to the associated physical implications of its structure concerning dynamic and static plasma screening phenomena for the infinite-bulk Landau-quantized plasma. We shall not recount here the associated bulk-infinite-plasma physical properties in a magnetic field, which may be found in Refs. 1(a) and 1(b). However, we shall draw on these references for a few of the principal mathematical representations of $I(\bar{q},q_z;\nu+i\delta)$ already developed therein, and refer the interested reader to these references for further detailed discussion of the representations and their physical significance. It should be noted that $D(\bar{q},q_z;\nu+i\delta)$ consists of real and imaginary parts as follows:

$$D(\bar{q},q_z;\nu+i\delta)=D_1(\bar{q},q_z;\nu+i\delta)+iD_2(\bar{q},q_z;\nu+i\delta)\ , \tag{3.38b}$$

where

$$D_1(\bar{q},q_z;\nu+i\delta)=-e^2\,\mathrm{Im}I(\bar{q},q_z;(\tau/\pi)[\Omega+i\varepsilon]) \tag{3.38c}$$

and

$$D_2(\bar{q},q_z;\nu+i\delta)=e^2\,\mathrm{Re}I(\bar{q},q_z;(\tau/\pi)[\Omega+i\varepsilon])\ . \tag{3.38d}$$

In the notation of Ref. 1(a), one may alternatively write

$$D_1(\bar{q},q_z;\nu+i\delta)\to-\left[\frac{4\pi}{q^2}\right]^{-1}\frac{\Omega(\mathbf{q};\Omega)}{\Omega}\ , \tag{3.39a}$$

which is directly proportional to the bulk free-electron polarizability for an infinite plasma, and

$$D_2(\bar{q},q_z;\nu+i\delta)\to\left[\frac{8\pi}{q^2}\right]^{-1}\Gamma(\mathbf{q};\Omega)\ , \tag{3.39b}$$

which is directly proportional to the imaginary part of the bulk dielectric function for an infinite plasma. Explicit formulas for $\Omega(\mathbf{q};\Omega)/\Omega$ and $\Gamma(\mathbf{q};\Omega)$ are given in Ref. 1(a), Eqs. (I.37) and (I.38), and (II.40), (II.41), and (II.42).

It should be noted that D_1 and ImI are even functions of Ω, whereas D_2 and ReI are odd functions of Ω. The integrals involved in $I(\mathbf{q};(\tau/\pi)[\Omega+i\varepsilon])$ have been explored in Ref. 1(a), Eqs. (II.40a), (II.40b), (II.41), and (II.42) with the results (P means principal part below)

$$\frac{1}{\hbar^3}\mathrm{Im}I(\hbar\mathbf{q};(\tau/\pi)[\hbar\Omega+i\varepsilon])=\mathrm{P}\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}\frac{\omega'}{\Omega^2-\omega'^2}\frac{f_0(\omega)}{\hbar^3}R(\omega,\hbar\omega';\hbar\mathbf{q})\ , \tag{3.40a}$$

or

$$\frac{1}{\hbar^3}\mathrm{Im}I(\hbar\mathbf{q};(\tau/\pi)[\hbar\Omega+i\varepsilon])=\frac{1}{2i}\mathrm{P}\int_L dT\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}(e^{-i(\Omega+\omega')T}-e^{-i(\Omega-\omega')T})\frac{f_0(\omega)}{\hbar^3}R(\omega,\hbar\omega';\hbar\mathbf{q}) \tag{3.40b}$$

and

$$\frac{1}{\hbar^3}\mathrm{Re}I(\hbar\mathbf{q};(\tau/\pi)[\hbar\Omega+i\varepsilon])=\tfrac{1}{2}\int\frac{d\omega}{2\pi}\frac{f_0(\omega)}{\hbar^3}R(\omega,\hbar\Omega;\hbar\mathbf{q})\ , \tag{3.41}$$

where

$$R(\omega,\hbar\omega';\hbar\mathbf{q})=\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}dy\,e^{i\omega x}\frac{2i\sin(\hbar\omega' x/2)}{\hbar}e^{-i(\omega' y/2)}\frac{\pi^{3/2}}{(2\pi)^3}\left[\frac{2m}{ix}\right]^{1/2}\frac{m\hbar\omega_c}{i\tan(\hbar\omega_c x/2)} \\ \times\exp\left[-i\frac{q_z^2}{2m}\frac{\hbar^2x^2-y^2}{4x}\right]\exp\left[-i\frac{\bar{q}^2}{2m}\frac{\cos(\omega_c y/2)-\cos(\hbar\omega_c x/2)}{(\omega_c/\hbar)\sin(\hbar\omega_c x/2)}\right]\ . \tag{3.42}$$

Corresponding to this one may develop exact expressions [Ref. 1(a), Eqs. (III.1), (III.34), (III.35), and (IV.1)] for both $D_1(\mathbf{q};\nu+i\delta)$ and $D_2(\mathbf{q};\nu+i\delta)$ as follows:

$$D_1(\bar{q},q_z;(\tau/\pi)[\Omega+i\varepsilon])=-e^2\int_0^{\infty}d\omega\frac{f_0(\omega)}{\hbar^3}\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}e^{s\omega}\frac{\pi^{3/2}}{(2\pi)^3}\left[\frac{2m}{s}\right]^{1/2}\frac{m\hbar\omega_c}{\tanh(\hbar\omega_c s/2)}\aleph\ , \tag{3.43}$$

where $\aleph$ is expressed in terms of half-time-axis integrals as [Ref. 1(a), p. 30, Eqs. (III.34) and (III.35)]

$$\varkappa=\frac{2i}{\hbar}\int_0^\infty dT\exp(-i\Omega T)$$
$$\times\left\{\begin{array}{l}\exp\left[\frac{-q_z^2}{8ms}[(2T-i\hbar s)^2+\hbar^2s^2]\right]\exp\left[\frac{\hbar\bar{q}^{\,2}}{2m\omega_c}\frac{\cos[(\omega_c/2)(2T-i\hbar s)]-\cosh(\hbar\omega_c s/2)}{\sinh(\hbar\omega_c s/2)}\right]\\ -\exp\left[\frac{-q_z^2}{8ms}[(2T+i\hbar s)^2+\hbar^2s^2]\right]\exp\left[\frac{\hbar\bar{q}^{2}}{2m\omega_c}\frac{\cos[(\omega_c/2)(2T+i\hbar s)]-\cosh(\hbar\omega_c s/2)}{\sinh(\hbar\omega_c s/2)}\right]\end{array}\right\}, \tag{3.44}$$

and, alternatively [Ref. 1(a), p. 23, Eq. (III.1)],

$$\begin{aligned}D_1(\bar{q},q_z;(\tau/\pi)[\Omega+i\varepsilon])=&-e^2\mathrm{P}\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}\frac{\omega'}{\Omega^2-(\omega')^2}\frac{f_0(\omega)}{\hbar^3}\\&\times\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\,e^{i\omega x}\frac{2i\sin(\hbar\omega'x/2)}{\hbar}e^{-i\omega'y/2}\frac{\pi^{3/2}}{(2\pi)^3}\left[\frac{2m}{ix}\right]^{1/2}\\&\times\frac{m\hbar\omega_c}{i\tan(\hbar\omega_c x/2)}\exp\left[\frac{-iq_z^2}{2m}\frac{\hbar^2x^2-y^2}{4x}\right]\\&\times\exp\left[\frac{-i\bar{q}^{\,2}\hbar}{2m\omega_c}\frac{\cos(\omega_c y/2)-\cos(\hbar\omega_c x/2)}{\sin(\hbar\omega_c x/2)}\right].\end{aligned} \tag{3.45}$$

The corresponding exact expression for $D_2(\bar{q},q_z;(\tau/\pi)[\Omega+i\varepsilon])$ is given by [Ref. 1(a), p. 38, Eq. (IV.1)]

$$\begin{aligned}D_2(\bar{q},q_z;(\tau/\pi)[\Omega+i\varepsilon])=&\frac{e^2}{2}\omega_c\left[\frac{m}{2\pi}\right]^{3/2}\int_{-\infty}^{\infty}dy\,e^{-i\Omega y/2}\\&\times\int_0^\infty\frac{d\omega}{\hbar^3}f_0(\omega)\int_{\delta-i\infty}^{\delta+i\infty}\frac{ds}{2\pi i}e^{\omega s}\frac{1}{\sqrt{s}}\frac{\sinh(\hbar\Omega s/2)}{\tanh(\hbar\omega_c s/2)}\exp\left[\frac{-q_z^2}{8ms}(y^2+\hbar^2s^2)\right]\\&\times\exp\left[\frac{-\hbar\bar{q}^{\,2}}{2m\omega_c}\frac{\cosh(\hbar\omega_c s/2)-\cos(\omega_c y/2)}{\sinh(\hbar\omega_c s/2)}\right].\end{aligned} \tag{3.46}$$

It is very useful to have available a low wave number power-series expansion of $D_1(\bar{q},q_z;(\tau/\pi)[\Omega+i\varepsilon])$, which may be obtained from Ref. 1(a), pp. 23 and 24, Eq. (III.4). The first few terms of the low-wave-number power expansion of D_1 are given by

$$\begin{aligned}D_1(\bar{q},q_z;(\tau/\pi)[\Omega+i\varepsilon])=&-q_z^2\left[\frac{e^2\rho^\infty}{m\Omega^2}\right]-\bar{q}^{\,2}\left[\frac{e^2\rho^\infty}{m(\Omega^2-\omega_c^2)}\right]-\frac{q_z^4 3e^2\alpha^\infty}{m^2\Omega^4}\\&-\frac{\bar{q}^{\,4}e^2\sigma^\infty}{m^2\omega_c^2}\left[\frac{1}{\Omega^2-(2\omega_c)^2}-\frac{1}{\Omega^2-\omega_c^2}\right]+\frac{q_z^2\bar{q}^{\,2}e^2\sigma^\infty}{m^2\omega_c^2\Omega^2}-\frac{q_z^2\bar{q}^{\,2}e^2\sigma^\infty}{m^2\omega_c^2}\frac{\Omega^2+\omega_c^2}{(\Omega^2-\omega_c^2)^2}\\&-\frac{q_z^2\bar{q}^{\,2}e^2\alpha^\infty}{m^2}\frac{3\Omega^2+\omega_c^2}{(\Omega^2-\omega_c^2)^3},\end{aligned} \tag{3.47}$$

where $\rho^\infty,\sigma^\infty,\alpha^\infty$ are defined by

$$\rho^\infty=2\int_0^\infty d\omega\frac{f_0(\omega)}{\hbar^3}\eta_+(\omega)\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}e^{\omega s}\frac{\pi^{3/2}}{(2\pi)^3}\left[\frac{2m}{s}\right]^{1/2}\frac{m\hbar\omega_c}{\tanh(\hbar\omega_c s/2)}, \tag{3.48a}$$

$$\sigma^\infty=\int_0^\infty d\omega\frac{f_0(\omega)}{\hbar^3}\eta_+(\omega)\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}e^{\omega s}\frac{\pi^{3/2}}{(2\pi)^3}\left[\frac{2m}{s}\right]^{1/2}\frac{m(\hbar\omega_c)^2}{[\tanh(\hbar\omega_c s/2)]^2}, \tag{3.48b}$$

$$\alpha^\infty=\int_0^\infty d\omega\frac{f_0(\omega)}{\hbar^3}\eta_+(\omega)\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}e^{\omega s}\frac{\pi^{3/2}}{(2\pi)^3}\left[\frac{2m}{s}\right]^{1/2}\frac{m\hbar\omega_c}{\tanh(\hbar\omega_c s/2)}\frac{2}{s}. \tag{3.48c}$$

[It should be noted that ρ^∞ has the significance of the bulk unperturbed density in the absence of any surface for the infinite plasma in a quantizing magnetic field; the parameter α^∞ here should not be confused with $\alpha=\frac{1}{2}(q_z'+q_z)$ in the nondiagonal elements.] These integrals may be evaluated in the degenerate case [see Appendix I of Ref. 1(a)] in terms of a branch-line contribution (denoted by the subscript Γ), and isolated pole contributions (denoted by the subscript C_n), with the results [$\Gamma(x)$ is the gamma function of argument x]

$$\rho_\Gamma^\infty \cong \left[\frac{m}{2\pi}\right]^{3/2} \frac{2}{\Gamma(\frac{5}{2})} \frac{\zeta^{3/2}}{\hbar^3} ,$$

$$\sigma_\Gamma^\infty \cong \left[\frac{m}{2\pi}\right]^{3/2} \frac{2}{\Gamma(\frac{7}{2})} \frac{\zeta^{5/2}}{\hbar^3} + \left[\frac{m}{2\pi}\right]^{3/2} \frac{1}{3\Gamma(\frac{3}{2})} \frac{(\hbar\omega_c)^2\zeta^{1/2}}{\hbar^3} ,$$

$$\alpha_\Gamma^\infty \cong \left[\frac{m}{2\pi}\right]^{3/2} \frac{2}{\Gamma(\frac{7}{2})} \frac{\zeta^{5/2}}{\hbar^3} + \left[\frac{m}{2\pi}\right]^{3/2} \frac{(\hbar\omega_c)^2\zeta^{1/2}}{6\Gamma(\frac{3}{2})\hbar^3} ,$$

and

$$\sum_n \rho_{C_n}^\infty = \frac{m^{3/2}(\hbar\omega_c)^{1/2}}{\pi\beta\hbar^3} \sum_{n=1}^{\infty} \frac{\cos[(2\pi n/\hbar\omega_c)\zeta - 3\pi/4]}{\sqrt{n}\,\sinh(2\pi^2 n/\hbar\omega_c\beta)} ,$$

$$\sum_n \sigma_{C_n}^\infty = \frac{m^{3/2}(\hbar\omega_c)^{3/2}}{2\pi^2\beta\hbar^3} \sum_{n=1}^{\infty} \frac{\cos[(2\pi n/\hbar\omega_c)\zeta - 5\pi/4]}{n^{3/2}\sinh(2\pi^2 n/\hbar\omega_c\beta)} \left[1-\frac{2\pi^2 n/\hbar\omega_c\beta}{\tanh(2\pi^2 n/\hbar\omega_c\beta)}\right]$$
$$+\frac{m^{3/2}(\hbar\omega_c)^{1/2}\zeta}{\pi\beta\hbar^3} \sum_{n=1}^{\infty} \left[\frac{\cos[(2\pi n/\hbar\omega_c)\zeta - 3\pi/4]}{\sqrt{n}\,\sinh(2\pi^2 n/\hbar\omega_c\beta)} - \frac{3\hbar\omega_c}{4\pi\zeta} \frac{\cos[(2\pi n/\hbar\omega_c)\zeta - 5\pi/4]}{n^{3/2}\sinh(2\pi^2 n/\hbar\omega_c\beta)}\right] ,$$

$$\sum_n \alpha_{C_n}^\infty = \frac{m^{3/2}(\hbar\omega_c)^{3/2}}{2\pi^2\beta\hbar^3} \sum_{n=1}^{\infty} \frac{\cos[(2\pi n/\hbar\omega_c)\zeta - 5\pi/4]}{n^{3/2}\sinh(2\pi^2 n/\hbar\omega_c\beta)} .$$

The corresponding evaluation of $\rho^\infty,\sigma^\infty,\alpha^\infty$ in the nondegenerate case yields [Ref. 1(a), Appendix I]

$$\rho^\infty = \frac{e^{\zeta\beta}}{\hbar^3} \frac{2\pi^{3/2}}{(2\pi)^3} \left[\frac{2m}{\beta}\right]^{1/2} \frac{m\hbar\omega_c}{\tanh(\hbar\omega_c\beta/2)} ,$$

$$\sigma^\infty = \frac{\hbar\omega_c/2}{\tanh(\hbar\omega_c\beta/2)}\rho^\infty ,$$

$$\alpha^\infty = \frac{1}{\beta}\rho^\infty .$$

One can readily obtain other useful exact expressions for $D_1(\overline{q},q_z(\tau/\pi)[\Omega+i\varepsilon])$ and $D_2(\overline{q},q_z;(\tau/\pi)[\Omega+i\varepsilon])$ in terms of a Bessel-function representation and a Landau-series representation, as well as evaluations in the quantum strong-field limit, nondegenerate limit, semiclassical limit, and classical limit, from work already carried out in Refs. 1(a) and 1(b), etc. Thus the quantum magnetic field effects in the diagonal part of the response matrix $D(\overline{q},q_z;(\tau/\pi)[\Omega+i\varepsilon])$ have been exhaustively evaluated, and we turn our attention to the evaluation of the nondiagonal part of the response matrix, $A(\overline{q},q_z,q_z';\nu+i\delta)$, next.

IV. EVALUATION OF THE NONDIAGONAL PART OF THE DENSITY PERTURBATION RESPONSE MATRIX $A(\overline{q},q_z,q_z';\nu+i\delta)$

It is clear from the considerations of Sec. III that boundary-induced changes of the longitudinal dielectric response properties of a finite medium are in part transmitted through the nondiagonal elements of the density perturbation response matrix $A(\overline{q},q_z,q_z';\nu+i\delta)$. In the semi-infinite limit, where the diagonal part $D(\overline{q},q_z;\nu+i\delta)$ has been seen to assume its bulk infinite-space form, the "nondiagonal" elements play a central role in describing the boundary induced loss of spatial translational invariance and its impact in changing the longitudinal dielectric properties of the medium. In this section we develop an evaluation of $A(\overline{q},q_z,q_z';\nu+i\delta)$ in closed form and examine its close relation to the two-dimensional[4] density perturbation response function in magnetic field. Specific results are presented for A in a low-wave-number power expansion, and for higher wave numbers we develop expansions of A in terms of (a) a modified Bessel function series and also (b) a Landau series. The role of quantum magnetic field effects in A is carefully accounted for at every stage of our considerations. The zero-field limit is also discussed.

A. Closed-form expression for $A(\overline{q},q_z,q_z';\nu+i\delta)$ at arbitrary field strength and its relation to two-dimensional response

It is very useful to connect the nondiagonal part of the density perturbation response matrix A to the *two-dimensional* density perturbation response function in the limiting case when $q_z\to0$ and $q_z'\to0$. Considering A as given by Eq. (3.23) (we take $e^2\to1$ here as well as $\hbar\to1$),

$$A(\overline{q},q_z,q_z';\nu)=\frac{1}{4}\int\frac{d^2\overline{k}}{(2\pi)^2}[F(\overline{k},\alpha,\overline{k}+\overline{q},\gamma;\nu) + F(\overline{k},\gamma,\overline{k}+\overline{q},\alpha;\nu)] , \tag{4.1}$$

where

$$\alpha=\tfrac{1}{2}(q_z'+q_z) \text{ and } \gamma=\tfrac{1}{2}(q_z'-q_z) \tag{4.2}$$

and [Eq. (3.11)]

$$F(\bar{k},k_z;\bar{k}',k_z';\nu)=-i\int_0^\tau dt \exp\left[-\frac{i\pi\nu t}{\tau}\right] \times \bar{G}_<(\bar{k},k_z;-t)\bar{G}_>(\bar{k}',k_z';t) , \tag{4.3}$$

we note that when $q_z=q_z'=\alpha=\gamma=0$, one has

$$A(\bar{q},0,0;\nu)=-\frac{i}{2}\int\frac{d^2\bar{k}}{(2\pi)^2}\int_0^\tau dt \exp\left[-\frac{i\pi\nu t}{\tau}\right] \times \bar{G}_<(\bar{k},0;-t) \times \bar{G}_>(\bar{k}+\bar{q},0;t) . \tag{4.4}$$

Recognizing that the infinite-space Green's function in momentum space [as given by Eq. (2.7) with no boundary] becomes the two-dimensional Green's function when momentum along the field vanishes [as given in Ref. 4, Eq. (10)], we may introduce the "ring diagram" integral $I^{2D}(\bar{q};\nu)$ of the two-dimensional density perturbation response function [Ref. 4, Eq. (5)] to make the identification

$$A(\bar{q},0,0;\nu)=-(i/2)I^{2D}(\bar{q};\nu) . \tag{4.5}$$

This connection provides a very valuable check of calculations of $\lim_{q_z\to 0}\lim_{q_z'\to 0} A(\bar{q},q_z,q_z';\nu)$ (for both real and imaginary parts when $\nu\to\nu+i\delta$) in a variety of representations and approximations, and this is exploited throughout our work in this section. This connection of $A(\bar{q},0,0;\nu)$ with two-dimensional response is in fact just one aspect of a broader relationship between $A(\bar{q},q_z,q_z';\nu)$ and two-dimensional response properties, as we shall soon see.

The explicit construction of $A(\bar{q},q_z,q_z';\nu+i\delta)$ will now be undertaken using Eqs. (3.35) and (3.36),

$$A(\bar{q},q_z,q_z';\nu+i\delta)=\frac{i}{4}\left[\int_0^\infty dt\, e^{-i(\Omega-i\varepsilon)t}\int\frac{d^2\bar{k}}{(2\pi)^2}\bar{G}_<(\bar{k},\alpha;-t)\bar{G}_>(\bar{k}+\bar{q},\gamma;t)\right]^* -\frac{i}{4}\left[\int_{-\infty}^0 dt\, e^{-i(\Omega+i\varepsilon)t}\int\frac{d^2\bar{k}}{(2\pi)^2}\bar{G}_>(\bar{k},\alpha;-t)\bar{G}_<(\bar{k}+\bar{q},\gamma;t)\right]+(\alpha\leftrightarrow\gamma) . \tag{4.6}$$

[Here, $(\alpha\leftrightarrow\gamma)$ means that one should add terms of the same form as the preceding ones, but with the roles of α and γ interchanged. It is clear that A is symmetric in α and γ as well as being symmetric as a matrix in the indices q_z and q_z'.] The Green's function involved here may be written as [Eq. (2.7)]

$$\left.\begin{matrix}\bar{G}_>(\bar{k},k_z;t)\\ \bar{G}_<(\bar{k},k_z;t)\end{matrix}\right\}=e^{i\zeta t}\int\frac{d\omega}{2\pi}\left\{\begin{matrix}-i(1-f_0(\omega))\\ if_0(\omega)\end{matrix}\right\}e^{-i\omega t} \times\left[\int_{-\infty}^{+\infty}dt' e^{i\omega t'}\exp[-i(\mu_0 H\sigma_3+k_z^2/2m)t']\sec(\omega_c t'/2)\exp[-(i\bar{k}^2/m\omega_c)\tan(\omega_c t'/2)]\right] . \tag{4.7}$$

Defining the integrals $\mathscr{I}_>$ and $\mathscr{I}_<$,

$$\mathscr{I}_>=\int_0^\infty dt\, e^{-i(\Omega-i\varepsilon)t}\int\frac{d^2\bar{k}}{(2\pi)^2}\bar{G}_<(\bar{k},\alpha;-t)\bar{G}_>(\bar{k}+\bar{q},\gamma;t) ,$$

$$\mathscr{I}_<=\int_{-\infty}^0 dt\, e^{-i(\Omega+i\varepsilon)t}\int\frac{d^2\bar{k}}{(2\pi)^2}\bar{G}_>(\bar{k};\alpha;-t)\bar{G}_<(\bar{k}+\bar{q},\gamma;t) ,$$

it is readily seen that

$$A(\bar{q},q_z,q_z';\nu+i\delta)=\frac{i}{4}(\mathscr{I}_>^*-\mathscr{I}_<)+(\alpha\leftrightarrow\gamma) . \tag{4.8}$$

It should be noted that $\mathscr{I}_>$ and $\mathscr{I}_<$ as defined here differ from their three-dimensional infinite-space counterparts discussed in Ref. 1(a) by deleting $\int dk_z/2\pi$ and setting $k_z\to\alpha$ in the first G function and setting $k_z\to\gamma$ in the second G function. Following the procedures of Ref. 1(a), p. 18, we may evaluate $\mathscr{I}_>$ and $\mathscr{I}_<$ as defined here as follows (note that $t\to T$ as a change of a "dummy" integration variable):

$$\mathscr{I}_>=\int_0^\infty dT\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}e^{-i(\Omega+\omega'-\omega-i\varepsilon)T}f_0(\omega)[1-f_0(\omega')]Q(\omega,\omega';\bar{q},\alpha,\gamma) , \tag{4.9a}$$

$$\mathscr{I}_<=\int_{-\infty}^0 dT\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}e^{-i(\Omega+\omega'-\omega+i\varepsilon)T}f_0(\omega')[1-f_0(\omega)]Q(\omega,\omega';\bar{q},\alpha,\gamma) , \tag{4.9b}$$

where

$$Q(\omega,\omega';\bar{q},\alpha,\gamma)=\int_{-\infty}^{\infty}dT\int_{-\infty}^{\infty}dT'\,e^{i\omega T}e^{i\omega' T'}P(T,T';\bar{q},\alpha,\gamma)\ , \tag{4.10}$$

and

$$P(T,T';\bar{q},\alpha,\gamma)=e^{-i(\alpha^2/2m)T}e^{-(i\gamma^2/2m)T'}P^{2\mathrm{D}}(T,T',\bar{q})\ , \tag{4.11}$$

and

$$P^{2\mathrm{D}}(T,T';\bar{q})=\mathrm{Tr}\int\frac{d^2\bar{k}}{(2\pi)^2}e^{-i\mu_0 H\sigma_3(T+T')}\sec\left[\frac{\omega_c T}{2}\right]\sec\left[\frac{\omega_c T'}{2}\right]\times\exp\left[\frac{-i\bar{k}^2}{m\omega_c}\tan\left[\frac{\omega_c T}{2}\right]\right]\exp\left[\frac{-i(\bar{k}+\bar{q})^2}{m\omega_c}\tan\left[\frac{\omega_c}{2}T'\right]\right]\ . \tag{4.12}$$

The quantity $P^{2\mathrm{D}}(T,T';\bar{q})$ is in fact just the corresponding quantity involved in the density perturbation response function of the *two-dimensional* plasma (as our notation is intended to suggest), and it is discussed and evaluated in Ref. 4, Eq. (13). This establishes a broad relationship between $A(\bar{q},q_z,q_z';\nu+i\delta)$ and two-dimensional response. The existence of such a relationship should not really come as a surprise, since the correspondence of $\mathcal{I}_>$ and $\mathcal{I}_<$ as defined here with their three-dimensional infinite-space counterparts was seen to involve the deletion of $\int dk_z/2\pi$ (along with selection of special values for k_z as indicated above); such a mathematical manipulation clearly reduces the dimensionality of the problem by 1 (with appropriate qualifications), leaving a problem which is essentially two dimensional, as reflected in the occurrence of $P^{2\mathrm{D}}(T,T';\bar{q})$ here. The final evaluation of $P^{2\mathrm{D}}(T,T';\bar{q})$ is carried out in Ref. 3, Eqs. (I.3-16)—(I.3-21), with the result

$$P^{2\mathrm{D}}(T,T';\bar{q})=2\exp\left[\frac{-i\bar{q}^2}{m\omega_c}\frac{\sin(\omega_c T/2)\sin(\omega_c T'/2)}{\sin[(\omega_c/2)(T+T')]}\right]\frac{\pi}{(2\pi)^2}\frac{m\omega_c}{i\tan[(\omega_c/2)(T+T')]}\ . \tag{4.13}$$

Setting $T+T'=x$ and $T-T'=y$, we have

$$Q(\omega,\omega';\bar{q},\alpha,\gamma)=\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}dy\exp\left[i\left[\omega-\frac{\alpha^2}{2m}+\omega'-\frac{\gamma^2}{2m}\right]\frac{x}{2}\right]\exp\left[i\left[\omega-\frac{\alpha^2}{2m}-\omega'+\frac{\gamma^2}{2m}\right]\frac{y}{2}\right]\times\frac{\pi}{(2\pi)^2}\frac{m\omega_c}{i\tan(\omega_c x/2)}\exp\left[\frac{-i\bar{q}^2}{m\omega_c}\frac{\cos(\omega_c y/2)-\cos(\omega_c x/2)}{2\sin(\omega_c x/2)}\right]\ . \tag{4.14}$$

Comparing this with the corresponding quantity $Q^{2\mathrm{D}}(\omega,\omega';\bar{q})$ of the *two-dimensional* density perturbation response function [as given in Ref. 4, Eq. (15)], we find an alternative description of the broad relationship between the nondiagonal elements under analysis here and two-dimensional response in the form

$$Q(\omega,\omega';\bar{q},\alpha,\gamma)=Q^{2\mathrm{D}}(\omega-\alpha^2/2m,\omega'-\gamma^2/2m,\bar{q})=\mathrm{real}\ . \tag{4.15}$$

The construction of $A=A(\bar{q},q_z,q_z';\nu+i\delta)$ may now proceed with the use of Eqs. (4.8), (4.9), and (4.15). The detailed manipulations are fully explained in Ref. 3 and the result may be stated as follows:

$$-4iA=\int_0^{\infty}dT\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}e^{iT(\Omega+\omega'-\omega+i\varepsilon)}[f_0(\omega)-f_0(\omega')]\times[Q^{2\mathrm{D}}(\omega-\alpha^2/2m,\omega'-\gamma^2/2m,\bar{q})+Q^{2\mathrm{D}}(\omega-\gamma^2/2m,\omega'-\alpha^2/2m,\bar{q})]\ . \tag{4.16}$$

Introducing Eqs. (4.14) and (4.15) and further following the manipulations detailed in Ref. 3, we treat two distinct terms in (4.16), the first one involving $[f_0(\omega)]$ and the second involving $[-f_0(\omega')]$. The ω' integral of the first term yields a $\delta(T-x/2-y/2)$ function so that the y integral may be carried out immediately, and the second term may be treated in a similar manner, with the result

$$\begin{aligned}-4iA(\bar{q},q_z,q_z';\nu+i\delta)=\;&2\int d\omega\,f_0(\omega)\int\frac{dx}{2\pi}e^{i(\omega-\alpha^2/2m)x}\times\int_0^{\infty}dT\exp\left[iT\left[\Omega-\frac{\alpha^2}{2m}+\frac{\gamma^2}{2m}+i\varepsilon\right]\right]\mathcal{F}(x,x+2T)\\&-2\int d\omega\,f_0(\omega)\int\frac{dx}{2\pi}e^{i(\omega-\gamma^2/2m)x}\times\int_0^{\infty}dT\exp\left[iT\left[\Omega-\frac{\alpha^2}{2m}+\frac{\gamma^2}{2m}+i\varepsilon\right]\right]\mathcal{F}(x,x-2T)+(\alpha\leftrightarrow\gamma)\ ,\end{aligned} \tag{4.17}$$

where

$$\mathcal{F}(x,y)=\frac{m\omega_c}{4\pi i\tan(\omega_c x/2)}\exp\left[\frac{-i\bar{q}^2}{2m\omega_c}\frac{\cos(\omega_c y/2)-\cos(\omega_c x/2)}{\sin(\omega_c x/2)}\right] \tag{4.18}$$

This closed-form result for $A(\bar{q},q_z,q_z';\nu+i\delta)$ is valid for arbitrary wave number, frequency, magnetic field strength, and statistical regime (temperature). It serves as our basis for all further expansions of A in special circumstances.

The zero-field limit of A is readily obtained by putting $\omega_c\to 0$ in $\mathcal{F}(x,y)$, whence

$$\mathcal{F}(x,x\pm 2T)=\frac{m}{2\pi i x}\exp\left[\frac{i\bar{q}^2}{2m}\left[\pm T+\frac{T^2}{x}\right]\right],$$

and then the T integral of Eq. (4.17) is seen to yield the Erfc function.[7] Redefining the x integration variable by $s=ix$, this integral then yields the inverse-square-root function [Ref. 7(b)I, p. 267, Eq. (14), and also p. 233, Eq. (4)]. The multiple valuedness of this function requires careful consideration, and the details of the selection of branch are fully explained in Ref. 3. The result for ReA at zero magnetic field is given by

$$\mathrm{Re}A=\frac{m}{8\pi}\left[\frac{2m}{\bar{q}^2}\right]^{1/2}\int d\omega\, f_0(\omega)\left[\frac{\eta_+[\alpha^2/2m+(m/2\bar{q}^2)(\Omega_{\alpha\gamma}+\bar{q}^2/2m)^2-\omega]}{[\alpha^2/2m+(m/2\bar{q}^2)(\Omega_{\alpha\gamma}+\bar{q}^2/2m)^2-\omega]^{1/2}}\eta_+(\omega-\alpha^2/2m)+(\Omega\leftrightarrow-\Omega)\right]+(\alpha\leftrightarrow\gamma)\,, \tag{4.19}$$

where $\Omega_{\alpha\gamma}$ is the shifted frequency

$$\Omega_{\alpha\gamma}=\Omega-\alpha^2/2m+\gamma^2/2m\,, \tag{4.20}$$

and $\eta_+(x)=1$ for $x>0$ and $\eta_+(x)=0$ for $x<0$. Evaluating this in the zero-temperature degenerate case, we have

$$\mathrm{Re}A=-\frac{m}{4\pi}\left[\frac{2m}{\bar{q}^2}\right]^{1/2}\left[\left\{\left[\frac{\alpha^2}{2m}+\frac{m}{2\bar{q}^2}\left[\Omega_{\alpha\gamma}+\frac{\bar{q}^2}{2m}\right]^2-\zeta\right]^{1/2}\eta_+\left[\frac{\alpha^2}{2m}+\frac{m}{2\bar{q}^2}\left[\Omega_{\alpha\gamma}+\frac{\bar{q}^2}{2m}\right]^2-\zeta\right]\right.\right.$$
$$\left.\left.-\left[\frac{m}{2\bar{q}^2}\right]^{1/2}\left|\Omega_{\alpha\gamma}+\frac{\bar{q}^2}{2m}\right|\right\}+(\Omega\leftrightarrow-\Omega)\right]\eta_+(\zeta-\alpha^2/2m)+(\alpha\leftrightarrow\gamma)\,, \tag{4.21}$$

which is consistent with the zero-field result of Newns.[2] A detailed comparison with Newns's result is given in Ref. 3, Appendix III. (In comparing Eq. (4.21) with corresponding zero-field results of D. E. Beck [Phys. Rev. B, **4**, 1555 (1971), Eq. (8a)], we find Beck's result to closely resemble the structure of Eq. (4.21), but with slight discrepancies which may be due to a misprint.)

The correctness of our choice of branch for the square-root function may be verified by noting that Eq. (4.5) requires that

$$\mathrm{Im}I^{2D}(\bar{q},\Omega+i\varepsilon)=2\,\mathrm{Re}A(\bar{q},0,0;\Omega+i\varepsilon)\,, \tag{4.22}$$

with $q_z=q_z'=\alpha=\gamma=0$, and this is further related to the two-dimensional plasmon dispersion relation through[4]

$$1=\frac{2\pi e^2}{\bar{q}}\mathrm{Im}I^{2D}(\bar{q},\Omega+i\varepsilon)=\frac{4\pi e^2}{\bar{q}}\mathrm{Re}A(\bar{q},0,0;\Omega+i\varepsilon)\,. \tag{4.23}$$

For low wave numbers ($\bar{q}^2/2m\Omega\ll 1$) our choice of branch yields

$$\mathrm{Re}A(\bar{q},0,0;\Omega+i\varepsilon)\to\zeta\bar{q}^2/2\pi\Omega^2\,, \tag{4.24}$$

and the corresponding 2D plasmon dispersion relation is given by

$$1=2e^2\zeta\bar{q}/\Omega^2\,, \tag{4.25}$$

or, noting that 2D density is given by (Ref. 4) $\rho^{2D}=m\zeta/\pi$ ($\hbar\to 1$), our choice of branch confirms the known 2D plasmon frequency[4] $\Omega^2=2\pi e^2\rho^{2D}\bar{q}/m$. Such confirmation would have been spoiled by making an incorrect choice of branch.

B. Low-wave-number power expansion of $A(\bar{q},q_z,q_z';\nu+i\delta)$.

The nondiagonal part of the density perturbation response matrix $A(\bar{q},q_z,q_z';\nu+i\delta)$ may be developed in a low-wave-number power series in $(\hbar\bar{q}^2/m\omega_c)^n$ by expanding the exponential factor of $\mathcal{F}(x,y)$ of Eq. (4.18). Such an expansion renders the T integration of Eq. (4.17) elementary, and produces a result of the form

$$A(\bar{q},q_z,q_z';\nu+i\delta)=\sum_{n=0}^{\infty}A(n)\quad\text{where } A(n)\propto(\bar{q}^2)^n\,, \tag{4.26}$$

in which the terms $A(n)$ are classified in accordance with their dependence on integral powers of $\bar{q}^2$. It is straightforward to develop a general expression for $A(n)$ in terms of an ω- and x-integral representation, but it is tedious.

Furthermore, the identification of these particular integral representations in terms of those integrals which occur in the low-wave-number description of the two-dimensional density perturbation response function requires substantial familiarity with the latter. For these reasons the reader is advised to consult Refs. 3 and 4 for the fully detailed evaluation of $A(n)$, and we will just state the results for $A(0)$, $A(1)$, and $A(2)$ below, for arbitrary magnetic field strength. For $A(0)$ we find

$$-4iA(0)=\frac{i}{\Omega_{\alpha\gamma}+i\varepsilon}[\rho^{2D}(\zeta_\alpha)-\rho^{2D}(\zeta_\gamma)]+(\alpha\leftrightarrow\gamma)\ , \tag{4.27}$$

where $\rho^{2D}(\zeta)$ is the two-dimensional density as a function of chemical potential, and $\zeta_x=\zeta-x^2/2m$ is a shifted chemical potential. With $s=ix$ and $\hbar\rightarrow 1$ we have

$$\rho^{2D}(\zeta)=2\int d\omega f_0(\omega-\zeta) \times\int_{-i\infty+\delta}^{+i\infty+\delta}\frac{ds}{2\pi i}e^{s\omega}\frac{m\omega_c}{4\pi\tanh(\omega_c s/2)}\ . \tag{4.28}$$

For $A(1)$ we find

$$-4iA(1)=-\frac{\bar{q}^2}{m\omega_c^2}\frac{i}{\Omega_{\alpha\gamma}+i\varepsilon}[\sigma^{2D}(\zeta_\alpha)-\sigma^{2D}(\zeta_\gamma)]+\frac{\bar{q}^2}{2m\omega_c}\frac{i}{\Omega_{\alpha\gamma}+\omega_c+i\varepsilon}\left[\frac{\sigma^{2D}(\zeta_\alpha)}{\omega_c}-\frac{\sigma^{2D}(\zeta_\gamma)}{\omega_c}+\frac{\rho^{2D}(\zeta_\alpha)}{2}+\frac{\rho^{2D}(\zeta_\gamma)}{2}\right]$$
$$+\frac{\bar{q}^2}{2m\omega_c}\frac{i}{\Omega_{\alpha\gamma}-\omega_c+i\varepsilon}\left[\frac{\sigma^{2D}(\zeta_\alpha)}{\omega_c}-\frac{\sigma^{2D}(\zeta_\gamma)}{\omega_c}-\frac{\rho^{2D}(\zeta_\alpha)}{2}-\frac{\rho^{2D}(\zeta_\gamma)}{2}\right]+(\alpha\leftrightarrow\gamma)\ , \tag{4.29}$$

where $\sigma^{2D}(\zeta)$ is proportional to the two-dimensional average Landau orbital energy (including spin) as a function of chemical potential, and ζ_x is again a shifted chemical potential,

$$\sigma^{2D}(\zeta)=\int d\omega f_0(\omega-\zeta)\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}e^{s\omega}\frac{m\omega_c^2}{4\pi[\tanh(\omega_c s/2)]^2}\ . \tag{4.30}$$

For $A(2)$ we find

$$-4iA(2)=\left[\frac{\bar{q}^2}{2m\omega_c}\right]^2\frac{i}{\Omega_{\alpha\gamma}+i\varepsilon}[\mu^{2D}(\zeta_\alpha)-\mu^{2D}(\zeta_\gamma)+\tfrac{1}{2}\chi^{2D}(\zeta_\alpha)-\tfrac{1}{2}\chi^{2D}(\zeta_\gamma)]$$
$$-\left[\frac{\bar{q}^2}{2m\omega_c}\right]^2\frac{i}{\Omega_{\alpha\gamma}+\omega_c+i\varepsilon}\left[\mu^{2D}(\zeta_\alpha)+\frac{\sigma^{2D}(\zeta_\alpha)}{\omega_c}-\mu^{2D}(\zeta_\gamma)+\frac{\sigma^{2D}(\zeta_\gamma)}{\omega_c}\right]$$
$$+\left[\frac{\bar{q}^2}{2m\omega_c}\right]^2\frac{i}{\Omega_{\alpha\gamma}-\omega_c+i\varepsilon}\left[\mu^{2D}(\zeta_\gamma)+\frac{\sigma^{2D}(\zeta_\gamma)}{\omega_c}-\mu^{2D}(\zeta_\alpha)+\frac{\sigma^{2D}(\zeta_\alpha)}{\omega_c}\right]$$
$$+\frac{1}{4}\left[\frac{\bar{q}^2}{2m\omega_c}\right]^2\frac{i}{\Omega_{\alpha\gamma}+2\omega_c+i\varepsilon}[\mu^{2D}(\zeta_\alpha)+2\sigma^{2D}(\zeta_\alpha)/\omega_c+\rho^{2D}(\zeta_\alpha)/2-\mu^{2D}(\zeta_\gamma)+2\sigma^{2D}(\zeta_\gamma)/\omega_c-\rho^{2D}(\zeta_\gamma)/2]$$
$$-\frac{1}{4}\left[\frac{\bar{q}^2}{2m\omega_c}\right]^2\frac{i}{\Omega_{\alpha\gamma}-2\omega_c+i\varepsilon}[\mu^{2D}(\zeta_\gamma)+2\sigma^{2D}(\zeta_\gamma)/\omega_c+\rho^{2D}(\zeta_\gamma)/2$$
$$-\mu^{2D}(\zeta_\alpha)+2\sigma^{2D}(\zeta_\alpha)/\omega_c-\rho^{2D}(\zeta_\alpha)/2]+(\alpha\leftrightarrow\gamma)\ , \tag{4.31}$$

where we again have shifted chemical potentials, and the two-dimensional integrals $\mu^{2D}(\zeta)$ and $\chi^{2D}(\zeta)$ are defined as

$$\mu^{2D}(\zeta)=\int d\omega f_0(\omega-\zeta)\int_{-i\infty+\delta}^{+i\infty+\delta}\frac{ds}{2\pi i}e^{s\omega}\frac{m\omega_c}{4\pi}\frac{[\cosh(\omega_c s/2)]^3}{[\sinh(\omega_c s/2)]^3}\ , \tag{4.32}$$

and

$$\chi^{2D}(\zeta)=\int d\omega f_0(\omega-\zeta)\int_{-\infty+\delta}^{+i\infty+\delta}\frac{ds}{2\pi i}e^{s\omega}\frac{m\omega_c}{4\pi}\frac{\cosh(\omega_c s/2)}{[\sinh(\omega_c s/2)]^3}\ , \tag{4.33}$$

and they are related by

$$\mu^{2D}(\zeta)=\chi^{2D}(\zeta)+\tfrac{1}{2}\rho^{2D}(\zeta)\ . \tag{4.34}$$

The two-dimensional integrals $\rho^{2D}(\zeta)$ and $\sigma^{2D}(\zeta)$ have already been evaluated in Refs. 4 and 3, and the results are tabulated here in Appendix A along with the evaluation of $\mu^{2D}(\zeta)$ and $\chi^{2D}(\zeta)$. [It should be noted that the real and imaginary parts of $A(n)$ are readily separated in accordance with the prescription $1/(x\pm i\varepsilon)=P(1/x)\mp i\pi\delta(x)$.]

C. Expansions of $A(\bar{q},q_z,q_z';\nu+i\delta)$ in terms of (a) a modified Bessel-function series and (b) a Landau series

In order to determine the structure of $A(\bar{q},q_z,q_z';\nu+i\delta)$ for arbitrary wave numbers, we expand the exponential factor of $\mathcal{F}(x,y)$ of Eq. (4.18) in a modified Bessel-function series using the identity

$$\exp[x\cos(\omega_c y/2)]=\sum_{n=-\infty}^{+\infty} I_n(x)e^{in\omega_c y/2} .$$

This expansion renders the T integration of Eq. (4.17) elementary, and we obtain the result

$$A(\bar{q},q_z,q_z';\nu+i\delta)=-\frac{m\omega_c}{8\pi}\sum_{n=-\infty}^{+\infty}\frac{1}{\Omega_{\alpha\gamma}+n\omega_c+i\varepsilon}[Z^{(n)}(\zeta_\alpha)+\mathfrak{z}^{(n)}(\zeta_\alpha)]$$
$$+\frac{m\omega_c}{8\pi}\sum_{n=-\infty}^{+\infty}\frac{1}{\Omega_{\alpha\gamma}-n\omega_c+i\varepsilon}[Z^{(n)}(\zeta_\gamma)+\mathfrak{z}^{(n)}(\zeta_\gamma)]+(\alpha\leftrightarrow\gamma) , \tag{4.35}$$

where we again have the shifted frequency $\Omega_{\alpha\gamma}$ and also the shifted chemical potentials ζ_α, ζ_γ, and $Z^{(n)}(\zeta)$ and $\mathfrak{z}^{(n)}(\zeta)$ are defined by the two-dimensional integrals

$$Z^{(n)}(\zeta)=\int d\omega\, f_0(\omega-\zeta)\int_{-i\infty+\delta}^{+i\infty+\delta}\frac{ds}{2\pi i}e^{s\omega}\frac{\sinh(n\omega_c s/2)}{\tanh(\omega_c s/2)}\exp\left[\frac{-\bar{q}^2}{2m\omega_c\tanh(\omega_c s/2)}\right]I_n(\bar{q}^2/[2m\omega_c\sinh(\omega_c s/2)]) , \tag{4.36}$$

$$\mathfrak{z}^{(n)}(\zeta)=\int d\omega\, f_0(\omega-\zeta)\int_{-i\infty+\delta}^{+i\infty+\delta}\frac{ds}{2\pi i}e^{s\omega}\frac{\cosh(n\omega_c s/2)}{\tanh(\omega_c s/2)}\exp\left[\frac{-\bar{q}^2}{2m\omega_c\tanh(\omega_c s/2)}\right]I_n(\bar{q}^2/[2m\omega_c\sinh(\omega_c s/2)]) . \tag{4.37}$$

The real and imaginary parts of $A(\bar{q},q_z,q_z';\nu+i\delta)$ may be separated in accordance with the prescription $1/(x\pm i\varepsilon)=P(1/x)\mp\pi i\delta(x)$.

For low- and/or intermediate-magnetic-field strengths (when many Landau levels are populated, $\hbar\omega_c\ll\zeta$) the zero-temperature degenerate limit of $Z^{(n)}(\zeta)$ has been evaluated in Ref. 4 and it consists of two parts: the first term $Z^{(n)}_{\rm semi}(\zeta)$ represents a semiclassical limit of $Z^{(n)}(\zeta)$ in which quantum magnetic field effects are completely neglected (but classical magnetic field effects are nevertheless present), and the second term $Z^{(n)}_{\rm dHvA}(\zeta)$ represents low- and/or intermediate-field dHvA oscillatory corrections,

$$Z^{(n)}(\zeta)=Z^{(n)}_{\rm semi}(\zeta)+Z^{(n)}_{\rm dHvA}(\zeta) . \tag{4.38}$$

Recounting the results of Ref. 4 ($\hbar\to 1$), we have

$$Z^{(n)}_{\rm semi}(\zeta)=n[J_n((2\bar{q}^2\zeta/m\omega_c^2)^{1/2})]^2 \tag{4.39}$$

and

$$Z^{(n)}_{\rm dHvA}(\zeta)=\frac{\omega_c}{\pi\zeta}\left[\frac{\pi-2\pi\zeta/\omega_c}{2}\right]_{\rm per}\frac{n}{2}\left[\frac{2\bar{q}^2\zeta}{m\omega_c^2}\right]^{1/2}J_n((2\bar{q}^2\zeta/m\omega_c^2)^{1/2})[J_{n-1}((2\bar{q}^2\zeta/m\omega_c^2)^{1/2})-J_{n+1}((2\bar{q}^2\zeta/m\omega_c^2)^{1/2})] , \tag{4.40}$$

where the periodic linear "sawtooth" function $[(\pi-y)/2]_{\rm per}$ (Fig. 2) is defined as $(\pi-y)/2$ in the fundamental interval $0<y<2\pi$ and is periodically repeated outside this interval.

The low- and/or intermediate-field strength evaluation of

$$\mathfrak{z}^{(n)}(\zeta)=\mathfrak{z}^{(n)}_{\rm semi}(\zeta)+\mathfrak{z}^{(n)}_{\rm dHvA}(\zeta)$$

may also be carried out using the procedures of Ref. 4, but it is simpler to note that the only difference in the s integrands of $Z^{(n)}(\zeta)$ and $\mathfrak{z}^{(n)}(\zeta)$ is that $Z^{(n)}(\zeta)\sim\sinh(n\omega_c s/2)$, whereas $\mathfrak{z}^{(n)}(\zeta)\sim\cosh(n\omega_c s/2)$ with the other s-integrand factors being identical. For low fields this corresponds to $Z^{(n)}_{\rm semi}(\zeta)\sim n\omega_c s/2$, whereas $\mathfrak{z}^{(n)}_{\rm semi}(\zeta)\sim 1$, and since an s-integrand factor of s can be induced by differentiation with respect to ζ, we have the low-field relation ($\hbar\to 1$)

$$\frac{n\omega_c}{2}\frac{\partial\mathfrak{z}^{(n)}_{\rm semi}(\zeta)}{\partial\zeta}=Z^{(n)}_{\rm semi}(\zeta) , \tag{4.41}$$

whence

$$\mathfrak{z}^{(n)}_{\rm semi}(\zeta)=\frac{2}{\omega_c}\int_0^\zeta d\zeta'[J_n((2\bar{q}^2\zeta'/m\omega_c^2)^{1/2})]^2 \tag{4.42a}$$

$$=\frac{2m\omega_c}{\bar{q}^2}\int_0^{(2\bar{q}^2\zeta/m\omega_c^2)^{1/2}}dx\, xJ_n^2(x) , \tag{4.42b}$$

and performing the x integral [H. Margenau and G. Murphy, *Mathematics of Physics and Chemistry,* 2nd ed. (Van Nostrand, Princeton, NJ, 1967), p. 121], we obtain

$$\mathfrak{z}^{(n)}_{\rm semi}(\zeta)=\frac{2\zeta}{\omega_c}[J_n^2((2\bar{q}^2\zeta/m\omega_c^2)^{1/2})-J_{n-1}((2\bar{q}^2\zeta/m\omega_c^2)^{1/2})\times J_{n+1}((2\bar{q}^2\zeta/m\omega_c^2)^{1/2})] . \tag{4.43}$$

The evaluation of $\mathfrak{z}^{(n)}_{\rm dHvA}(\zeta)$ is most simply carried out by

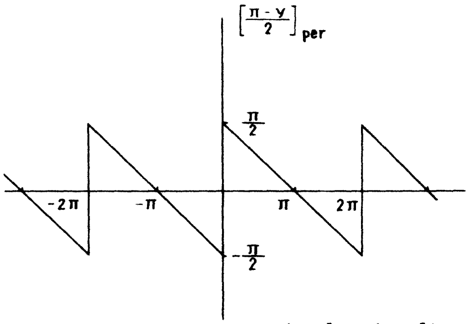

FIG. 2. Periodic linear "sawtooth" function $[(\pi-y)/2]_{\text{per}}=(\pi-y)/2$ for $0<y<2\pi$ in the fundamental interval, which is periodically repeated outside the fundamental interval.

using a slight variation of the procedures of Ref. 4, which is developed here in Appendix B. This yields ($\hbar\to 1$)

$$\mathfrak{z}^{(n)}_{\text{dHvA}}(\zeta)=\frac{\omega_c}{\pi}\left[\frac{\pi-(2\pi\zeta/\omega_c)}{2}\right]_{\text{per}}\frac{\partial}{\partial\zeta}\mathfrak{z}^{(n)}_{\text{semi}}(\zeta)\,, \tag{4.44}$$

and with Eq. (4.42a) we immediately obtain

$$\mathfrak{z}^{(n)}_{\text{dHvA}}(\zeta)=\frac{2}{\pi}\left[\frac{\pi-(2\pi\zeta/\omega_c)}{2}\right]_{\text{per}} \times[J_n((2\bar{q}^{\,2}\zeta/m\omega_c^2)^{1/2})]^2\,. \tag{4.45}$$

Finally, we present exact evaluations of $Z^{(n)}(\zeta)$ and $\mathfrak{z}^{(n)}(\zeta)$ which are most useful at higher-magnetic-field strengths (when few Landau levels are populated; $\hbar\omega_c\sim\zeta$ for the degenerate case and $\hbar\omega_c\sim kT=1/\beta$ for the nondegenerate case). These exact evaluations involve expansion in terms of a Landau series, which is the series form that would emerge in correspondence with a Landau eigenfunction expansion of the Green's function. The techniques involved in generating the Landau series are exhibited in Ref. 4, where it is shown that $Z^{(n)}(\zeta)$ is given by ($\hbar\to 1$)

$$\begin{aligned}Z^{(n)}(\zeta)=&\frac{1}{2}\left[\frac{\bar{q}^{\,2}}{2m\omega_c}\right]^n\exp\left[\frac{-\bar{q}^{\,2}}{2m\omega_c}\right]\\&\times\sum_{\pm}\sum_{r=0}^{\infty}\frac{r!}{(n+r)!}[L_r^n(\bar{q}^{\,2}/2m\omega_c)]^2\left[\eta_+(2r+1\mp1)f_0\left[\frac{\omega_c}{2}(2r+1\mp1)-\zeta\right]\right.\\&\left.-\eta_+(2n+2r+1\mp1)f_0\left[\frac{\omega_c}{2}(2n+2r+1\mp1)-\zeta\right]\right]\,.\end{aligned} \tag{4.46}$$

Recalling that the only difference in the s integrands of $Z^n(\zeta)$ and $\mathfrak{z}^{(n)}(\zeta)$ is that $Z^{(n)}(\zeta)\sim\sinh(n\omega_c s/2)$, whereas $\mathfrak{z}^{(n)}(\zeta)\sim\cosh(n\omega_c s/2)$ (a difference of exponentials versus a sum of exponentials), we find that the Landau-series development of $\mathfrak{z}^{(n)}(\zeta)$ differs from that above for $Z^{(n)}(\zeta)$ by changing the difference of the two terms in large square brackets in Eq. (4.46) into a sum of the same two terms (Ref. 3) ($\hbar\to 1$),

$$\begin{aligned}\mathfrak{z}^{(n)}(\zeta)=&\frac{1}{2}\left[\frac{\bar{q}^{\,2}}{2m\omega_c}\right]^n\exp\left[\frac{-\bar{q}^{\,2}}{2m\omega_c}\right]\\&\times\sum_{\pm}\sum_{r=0}^{\infty}\frac{r!}{(n+r)!}[L_r^n(\bar{q}^{\,2}/2m\omega_c)]^2\left[\eta_+(2r+1\mp1)f_0\left[\frac{\omega_c}{2}(2r+1\mp1)-\zeta\right]\right.\\&\left.+\eta_+(2n+2r+1\mp1)f_0\left[\frac{\omega_c}{2}(2n+2r+1\mp1)-\zeta\right]\right]\,.\end{aligned} \tag{4.47}$$

These results for $Z^{(n)}(\zeta)$ and $\mathfrak{z}^{(n)}(\zeta)$ are valid for all wave numbers, statistical regimes (temperatures), and magnetic field strengths, but they are most useful at high fields when only a few Landau levels are occupied, since the unoccupied levels do not contribute to Eqs. (4.46) and (4.47).

V. THE SLAB SURFACE-PLASMON DISPERSION RELATION IN QUANTIZING MAGNETIC FIELD

Our analysis of the magnetic field dependence of the dynamic nonlocal density perturbation response matrix $\mathscr{R}=\delta\rho/\delta V$ may be used to determine the effects of the magnetic field on the slab surface-plasmon dispersion relation as formulated by Newns in terms of $\mathscr{R}$. Newns's dispersion relation is given by[2]

$$-1=\epsilon_{\bar{q}}(\Omega)\,, \tag{5.1}$$

where $\epsilon_{\bar{q}}(\Omega)$ is defined as (put $(\tau/\pi)[\Omega+i\varepsilon]\to\Omega+i\varepsilon$)

$$\epsilon_{\bar{q}}(\Omega)=\left[\frac{4\bar{q}}{d}\sum_{q_z,q_z'}E^{-1}(\bar{q},q_z,q_z';\Omega+i\varepsilon)\right]^{-1}\,, \tag{5.2}$$

and E^{-1} is the matrix inverse of (labeling matrix rows and columns by q_z and q_z', respectively)

$$E(\bar{q},q_z,q_z';\Omega+i\varepsilon)=q^2\delta_{q_zq_z'}/\eta_{q_z}+4\pi\mathscr{R}(\bar{q},q_z,q_z';\Omega+i\varepsilon) . \tag{5.3}$$

[It should be noted that Newns's notation[2] will be employed throughout this section, and his $\mathscr{R}$ in Eq. (5.3) is the negative of ours.] Recognizing that

$$\mathscr{R}(\bar{q},q_z,q_z';\Omega+i\varepsilon)=D(\bar{q},q_z;\Omega+i\varepsilon)\delta_{q_zq_z'}/\eta_{q_z} - A(\bar{q},q_z,q_z';\Omega+i\varepsilon) \tag{5.4}$$

E may be rewritten as

$$E(\bar{q},q_z,q_z';\Omega+i\varepsilon)=4\pi[\Delta(\bar{q},q_z;\Omega+i\varepsilon)\delta_{q_zq_z'}/\eta_{q_z} - A(\bar{q},q_z,q_z';\Omega+i\varepsilon)] , \tag{5.5}$$

with the definition of the diagonal elements Δ,

$$\Delta(\bar{q},q_z;\Omega+i\varepsilon)=\frac{1}{4\pi}[\bar{q}^2+q_z^2+4\pi D(\bar{q},q_z;\Omega+i\varepsilon)] . \tag{5.6}$$

We have expressed this in a form which is valid for both antisymmetric modes $[f(\bar{r},z)=-f(\bar{r},d-z)]$ and symmetric modes $[f(\bar{r},z)=f(\bar{r},d-z)]$ as well. For antisymmetric modes, $q_z,q_z'=(2n+1)\pi/d$ $(n=0,1,2,\ldots,\infty)$ and $\eta_{q_z}\equiv 1$. For symmetric modes $q_z,q_z'=2n\pi/d$ $(n=0,1,\ldots,\infty)$ and $\eta_{q_z=0}=\frac{1}{2}$ and $\eta_{q_z>0}=1$. Supressing the explicit appearance of the matrix indices q_z and q_z' (as well as $\bar{q}$, Ω, etc.) and also η_{q_z}, we have symbolically[2]

$$\begin{aligned}E^{-1}&=\frac{1}{4\pi}(\Delta-A)^{-1}=\frac{1}{4\pi}\Delta^{-1}\sum_{n=0}^{\infty}(\Delta^{-1}A)^n\\&=\frac{1}{4\pi}[\Delta^{-1}+\Delta^{-1}(\Delta^{-1}A)+\Delta^{-1}(\Delta^{-1}A)^2+\cdots] .\end{aligned} \tag{5.7}$$

The leading term Δ^{-1} on the right-hand side yields the "diagonal" approximation, in which nondiagonal elements A are neglected, as

$$E^{-1}(\bar{q},q_z,q_z';\Omega+i\varepsilon)=\frac{\delta_{q_zq_z'}\eta_{q_z}}{\bar{q}^2+q_z^2+4\pi D(\bar{q},q_z;\Omega+i\varepsilon)} . \tag{5.8}$$

Forming $[\epsilon_{\bar{q}}(\Omega)]^{-1}$ in the diagonal approximation yields

$$[\epsilon_{\bar{q}}(\Omega)]^{-1}=\frac{4\bar{q}}{d}\sum_{q_z}\frac{\eta_{q_z}}{\bar{q}^2+q_z^2+4\pi D(\bar{q},q_z;\Omega+i\varepsilon)} . \tag{5.9}$$

We have already shown that, in the semi-infinite limit $d\rightarrow\infty$ [Eqs. (3.22)–(3.32)],

$$\begin{aligned}D(\bar{q},q_z;\nu)&=\frac{e^2}{4}\int\frac{d^2\bar{k}}{(2\pi)^2}\sum_{k_z=-\infty}^{+\infty}F(\bar{k},k_z,\bar{k}+\bar{q},k_z+q_z;\nu)\\&\Rightarrow\frac{q^2}{4\pi}[\epsilon^{\infty}(\mathbf{q};\nu)-1] .\end{aligned} \tag{5.10}$$

The essential feature of this consideration was the merging of the discrete states for electron motion across the slab (characterized by discrete $k_z=n\pi/d$ with d finite) into a continuum of states for $d\rightarrow\infty$, so that the replacement $\sum_{k_z}\rightarrow(d/\pi)\int dk_z$ could be made. It is worthwhile to observe that the very same consideration is valid for a *finite* slab provided that the electron dynamics are described *classically*: this is to say, that in the transition from a quantum description of electron dynamics to a classical description of electron dynamics for a finite slab, the discrete set of states for motion across the slab merges into a continuum (even for d finite) and the replacement $\sum_{k_z}\rightarrow(d/\pi)\int dk_z$ can be made. Hence, if we employ a semiclassical model which treats electron dynamics on a classical basis, but averages with respect to an initial Fermi distribution, Eq. (5.10) is valid for a finite slab on the understanding that $\epsilon^{\infty}(\mathbf{q},\nu)$ is to be taken as the semiclassical limit of the magnetic-field-dependent bulk infinite-space RPA dielectric function. On this basis we have used Eqs. (5.9) and (5.10) to determine the local limit of the magnetic-field-dependent slab surface-plasmon dispersion relation (further detail is supplied in Ref. 8) as follows:

(a) Antisymmetric modes, $\Omega\rightarrow\Omega_A$,

$$-1=\left[1-\frac{\omega_p^2}{\Omega^2}\right]^{-1/2}\left[1-\frac{\omega_p^2}{\Omega^2-\omega_c^2}\right]^{-1/2}\times\tanh\left[\frac{\bar{q}d}{2}\left[\frac{1-\omega_p^2/(\Omega^2-\omega_c^2)}{1-\omega_p^2/\Omega^2}\right]^{1/2}\right] . \tag{5.11a}$$

(b) Symmetric modes, $\Omega\rightarrow\Omega_S$,

$$-1=\left[1-\frac{\omega_p^2}{\Omega^2}\right]^{-1/2}\left[1-\frac{\omega_p^2}{\Omega^2-\omega_c^2}\right]^{-1/2}\times\coth\left[\frac{\bar{q}d}{2}\left[\frac{1-\omega_p^2/(\Omega^2-\omega_c^2)}{1-\omega_p^2/\Omega^2}\right]^{1/2}\right] . \tag{5.11b}$$

In the thick limit $d\rightarrow\infty$, Eq. (5.11) yields

$$\Omega_A^2=\Omega_S^2=(\omega_p^2+\omega_c^2)/2 , \tag{5.12}$$

and in the thin limit $d\rightarrow 0$, Eq. (5.11) yields

$$\Omega_A^2\cong\omega_p^2,\quad \Omega_S^2\cong\omega_c^2+2\pi e^2\rho^{2\mathrm{D}}\bar{q}/m , \tag{5.13}$$

where $\rho^{2\mathrm{D}}=\rho d$ is the two-dimensional density.

In the case of the semi-infinite limit $d\rightarrow\infty$, we have $\sum_{q_z}\rightarrow(d/2\pi)\int dq_z$, so that the sum in Eq. (5.9) also becomes an integral, and we obtain the semi-infinite surface-plasmon dispersion relation as

$$-1=\frac{2\bar{q}}{\pi}\int_0^{\infty}dq_z\frac{1}{(\bar{q}^2+q_z^2)\epsilon^{\infty}(\mathbf{q},\Omega)} . \tag{5.14}$$

The structure of this dispersion relation was studied in the absence of a magnetic field by Ritchie and Marusak[9] as well as by Newns,[2] and the magnetic field dependence of it was partially explored by Cheng and Harris.[10] We have employed Eq. (5.14) to carry out a low-wave-number analysis [Eq. (3.47)] of magnetic field effects in the nonlocal semi-infinite surface-plasmon spectrum and have found the following results[11,12] (further detail is supplied in Refs. 11 and 12): The principal semi-infinite surface-plasmon mode in magnetic field and its linear wave-vector shift are given for $\omega_p > \omega_c$ by

$$\Omega^2 = \frac{\omega_p^2+\omega_c^2}{2} + \frac{\bar{q}}{2}\frac{\omega_p^2+\omega_c^2}{\omega_p^2-\omega_c^2}\left[\frac{3}{ms_2}\frac{\omega_p^4-\omega_c^4}{\omega_p^2}\right]^{1/2}, \tag{5.15}$$

where $s_2 \equiv \rho^\infty/\alpha^\infty$ [Eqs. (3.48)]. This mode suffers heavy natural damping for $\omega_c > \omega_p$ (surface-plasmon damping[15] in a quantizing magnetic field is analyzed in Ref. 15). A similar analysis[14] of quantum magnetic field effects in the nonlocal shift of coupled semi-infinite surface-plasmon—surface-optical-phonon modes was presented in Ref. 14. Furthermore, there is a nonlocal semi-infinite surface Bernstein mode near $\Omega \sim 2\omega_c$ as given by

$$\Omega_{(2\omega_c)}^2 = (2\omega_c)^2 + \frac{\bar{q}^{\,2}\omega_p^2}{ms_1\omega_c^2}\frac{1}{c}, \tag{5.16a}$$

where $s_1 = \rho^\infty/\sigma^\infty$ [Eqs. (3.48)] and

$$c = (1-\omega_p^2/3\omega_c^2) - (1-\omega_p^2/4\omega_c^2)^{-1}. \tag{5.16b}$$

Such Bernstein modes for propagation perpendicular to the magnetic field are undamped in the bulk,[1(a)] but the *surface* Bernstein mode under consideration here does suffer natural damping.[13]

The few examples which we have just discussed are the simplest illustrations of the usefulness of the material developed here in analyzing quantum (and classical) magnetic field effects in the nonlocal, longitudinal, electrostatic slab surface-plasmon spectrum. We have extended these considerations to include the effects of retardation associated with the finite velocity of light for the thin-slab limit of a two-dimensional plasma in several studies relevant to inversion layers,[16–19] and much important related 2D work of other authors is cited in the comprehensive review article on the electronic properties of two-dimensional systems by Ando, Fowler, and Stern.[20]

Recently, several important papers by Gumbs and his collaborators have treated problems relating to the linear longitudinal dielectric response properties of a bounded solid-state slab plasma in a quantizing magnetic field from a different point of view. Their work includes studies of the nonlocal surface magnetoplasmon spectrum,[21–23] and static shielding by a magnetoplasma near a surface,[24–26] and it embodies an effort to account for the role of "quantum interference term" counterparts of our nondiagonal elements $-A(\bar{q},q_z,q_z';\nu)$. The fully detailed evaluation of the nondiagonal elements $-A(\bar{q},q_z,q_z';\nu)$ which we have presented here provides the basis for a more refined and accurate analysis of the roles of the quantizing magnetic field, nonlocality, and spatial inhomogeneity in the surface magnetoplasmon spectrum, natural damping, dynamic screening, and static shielding. In this connection, it should be noted that the neglect of nondiagonal elements in obtaining the semi-infinite surface-plasmon dispersion relation Eq. (5.14) yields a description of nonlocal surface plasmons in magnetic field which ignores surface-induced changes of the dielectric polarization properties of the medium: this deficiency can be remedied by the incorporation of the nondiagonal elements set forth in detail in this paper. Moreover, this detailed information specifying the longitudinal dielectric response properties of a bounded solid-state slab magnetoplasma also provides the means to analyze its dynamic, nonlocal inhomogeneous surface interactions,[27,28] and its exchange and correlation phenomena.[29] Such applications will be reported separately.

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APPENDIX A: TWO-DIMENSIONAL INTEGRALS $\rho^{2D}(\zeta)$, $\sigma^{2D}(\zeta)$, $\mu^{2D}(\zeta)$, and $\chi^{2D}(\zeta)$

Exact evaluations of $\rho^{2D}(\zeta)$ and $\sigma^{2D}(\zeta)$ have been developed in Ref. 4, with the results ($\hbar \to 1$)

$$\rho^{2D}(\zeta) = (m\omega_c/\pi)\sum_{r=0}^{\infty}\eta_+(r\omega_c)f_0(r\omega_c-\zeta), \tag{A1}$$

$$\sigma^{2D}(\zeta) = (m\omega_c/\pi)\sum_{r=0}^{\infty}\eta_+(r\omega_c)f_0(r\omega_c-\zeta)r\omega_c. \tag{A2}$$

Approximations appropriate to low- and intermediate-magnetic-field strengths for $\rho^{2D}(\zeta)$ and $\sigma^{2D}(\zeta)$ are also given in Ref. 4, Eqs. (43) and (44), respectively. Furthermore, nondegenerate evaluations of $\rho^{2D}(\zeta)$ and $\sigma^{2D}(\zeta)$ appear in the same reference, Eqs. (46) and (47), respectively.

Our treatment[3] of $\mu^{2D}(\zeta)$ and $\chi^{2D}(\zeta)$ will start with the verification of the identity Eq. (4.34). In terms of the s integrands involved, we have

$$\chi^{2D}(\zeta) \sim \frac{\cosh(\omega_c s/2)}{[\sinh(\omega_c s/2)]^3}, \quad \frac{\rho^{2D}(\zeta)}{2} \sim \frac{\cosh(\omega_c s/2)}{\sinh(\omega_c s/2)},$$

$$\mu^{2D}(\zeta) \sim \frac{\cosh^3(\omega_c s/2)}{\sinh^3(\omega_c s/2)},$$

whence

$$\chi^{2D}(\zeta) + \frac{\rho^{2D}(\zeta)}{2} \sim \frac{\cosh(\omega_c s/2)\{1+[\sinh(\omega_c s/2)]^2\}}{[\sinh(\omega_c s/2)]^3} \sim \frac{\cosh^3(\omega_c s/2)}{\sinh^3(\omega_c s/2)}$$

and thus we verify that

$$\chi^{2D}(\zeta) + \tfrac{1}{2}\rho^{2D}(\zeta) = \mu^{2D}(\zeta). \tag{A3}$$

This identity will be employed to evaluate $\chi^{2D}(\zeta)$ after we have evaluated $\mu^{2D}(\zeta)$. The evaluation of $\mu^{2D}(\zeta)$ is best done by employing Ref. 4, Eqs. (28) and (31), with $\mu^{2D}(\zeta)$ as given by Eq. (4.32), which yields ($\hbar \to 1$)

$$\mu^{2D}(\zeta)=\omega_c \sum_{r=0}^{\infty} \eta_+(r\omega_c) f_0(r\omega_c-\zeta) \times \oint_{c_0} \frac{dz}{2\pi i} e^{r\omega_c z} \frac{m\omega_c}{4\pi} \frac{\cosh^3(\omega_c z/2)}{\sinh^3(\omega_c z/2)} .$$

The integral c_0 about the origin is given by $(x=\omega_c z/2)$

$$\oint_{c_0} \frac{dz}{2\pi i}(\cdots)=\frac{m}{2\pi}\oint_{c_0}\frac{dx}{2\pi i}e^{2rx}\frac{1}{(\tanh x)^3} =\frac{m}{2\pi}\oint_{c_0}\frac{dx}{2\pi i}e^{2rx}\left[\frac{1}{x^3}+\frac{1}{x}+\text{analytic}\right] =\frac{m}{\pi}(r^2+\tfrac{1}{2}) ,$$

whence

$$\mu^{2D}(\zeta)=\frac{m\omega_c}{\pi}\sum_{r=0}^{\infty}\eta_+(r\omega_c)f_0(r\omega_c-\zeta)(r^2+\tfrac{1}{2}) . \tag{A4}$$

Since $\chi^{2D}(\zeta)=\mu^{2D}(\zeta)-\frac{1}{2}\rho^{2D}(\zeta)$ we have

$$\chi^{2D}(\zeta)=\frac{m\omega_c}{\pi}\sum_{r=0}^{\infty}\eta_+(r\omega_c)f_0(r\omega_c-\zeta)r^2 . \tag{A5}$$

Approximations of $\mu^{2D}(\zeta)$ and $\chi^{2D}(\zeta)$ appropriate to low- and intermediate-magnetic-field strengths and nondegenerate evaluations may be obtained using Ref. 4, Eqs. (42) and (45).

APPENDIX B: REMARKS ON THE EVALUATION OF THE dHvA OSCILLATORY PART OF TWO-DIMENSIONAL INTEGRALS FOR LOW- AND INTERMEDIATE-FIELD STRENGTH

We wish to point out that the techniques developed in Ref. 4, Eqs. (35)—(44), for evaluating two-dimensional integrals at low- and intermediate-field strength can be expressed in a more convenient form for obtaining the dHvA oscillatory part.[3] Using the notation of Ref. 4, we consider integrals of the general form J

$$J=\int_0^{\infty} d\omega f_0(\omega)\int_{-i\infty+\delta}^{+i\infty+\delta}\frac{ds}{2\pi i}e^{\omega s}j(s) , \tag{B1}$$

where $j(s)$ has isolated singularities at $s_n=\pm i2\pi n/\hbar\omega_c$ and is periodic with period s_1. Following Ref. 4, Eqs. (35)—(44), we have

$$J=\oint_{c_0(\text{origin})}\frac{ds}{2\pi i}\frac{e^{\zeta s}}{s}j(s)+\sum_{n\neq 0}e^{\zeta s_n}\oint_{c_0}\frac{dz}{2\pi i}\frac{e^{\zeta z}}{z+s_n}j(z) . \tag{B2}$$

The substance of the low- and intermediate-field approximation is that $z\sim 1/\zeta \ll s_n \sim 1/\hbar\omega_c$ so that $z+s_n\to s_n$ and then

$$\sum_{n\neq 0}\to\left[\sum_{n\neq 0}\frac{e^{\zeta s_n}}{s_n}\right]\oint_{c_0}\frac{dz}{2\pi i}e^{\zeta z}j(z) \to\frac{\hbar\omega_c}{\pi}\left[\frac{\pi-(2\pi\zeta/\hbar\omega_c)}{2}\right]_{\text{per}}\oint_{c_0}\frac{dz}{2\pi i}e^{\zeta z}j(z) . \tag{B3}$$

This is clearly the dHvA oscillatory term

$$J_{\text{dHvA}}=\frac{\hbar\omega_c}{\pi}\left[\frac{\pi-(2\pi\zeta/\hbar\omega_c)}{2}\right]_{\text{per}}\oint_{c_0}\frac{dz}{2\pi i}e^{\zeta z}j(z) , \tag{B4}$$

whereas the first term on the right-hand side of Eq. (B2) yields the semiclassical limit when evaluated for low field by using just the leading term of the Laurent expansion

$$J_{\text{semi}}=\oint_{c_0}\frac{ds}{2\pi i}\frac{e^{\zeta s}}{s}j(s) . \tag{B5}$$

Noting that

$$\oint_{c_0}\frac{dz}{2\pi i}e^{\zeta z}j(z)=\frac{\partial}{\partial\zeta}\oint_{c_0}\frac{ds}{2\pi i}\frac{e^{\zeta s}}{s}j(s)=\frac{\partial}{\partial\zeta}J_{\text{semi}} , \tag{B6}$$

we have

$$J_{\text{dHvA}}=\frac{\hbar\omega_c}{\pi}\left[\frac{\pi-(2\pi\zeta/\hbar\omega_c)}{2}\right]_{\text{per}}\frac{\partial}{\partial\zeta}J_{\text{semi}} \tag{B7}$$

and applying this to $\mathfrak{z}^{(n)}(\zeta)$ yields Eq. (4.44) $(\hbar\to 1)$. The full result for J at low and intermediate fields in the zero-temperature degenerate limit may be written as

$$J=J_{\text{semi}}+\frac{\hbar\omega_c}{\pi}\left[\frac{\pi-(2\pi\zeta/\hbar\omega_c)}{2}\right]_{\text{per}}\frac{\partial}{\partial\zeta}J_{\text{semi}} . \tag{B8}$$

APPENDIX C: EVEN AND ODD PROPERTIES OF THE REAL AND IMAGINARY PARTS OF $\mathscr{R}(\bar r,z,\bar r',z';\nu+i\epsilon)$

We shall prove here that the real and imaginary parts of $\mathscr{R}(\bar r,z,\bar r',z';\nu+i\epsilon)$ are even and odd functions of frequency ν. We write Eq. (3.14) in the form $(\nu\to-\nu)$

$$\mathscr{R}(\bar r,z,\bar r',z';-\nu+i\epsilon)=-\frac{i4}{d^2}\int\frac{d^2\bar k}{(2\pi)^2}\int\frac{d^2\bar k'}{(2\pi)^2}e^{i(\bar k-\bar k')\cdot(\bar r-\bar r')} \times\sum_{k_z}\sum_{k_{z'}}\Pi(k_z,k_z';z,z')\left[\left[-\int_0^{\infty}dt\,e^{-it(-\nu-i\epsilon)}\bar G_<(\bar k,k_z;-t)\bar G_>(\bar k',k_z';t)\right]^* +\left[\int_{-\infty}^{0}dt\,e^{-it(-\nu+i\epsilon)}\bar G_>(\bar k,k_z;-t)\bar G_<(\bar k',k_z';t)\right]\right] , \tag{C1}$$

with the definition

$$\Pi(k_z,k_z';z,z')\equiv\sin(k_z z)\sin(k_z z')\sin(k_z' z)\sin(k_z' z')=\Pi(k_z',k_z;z,z') \ . \tag{C2}$$

Noting that

$$\bar{G}_<(\bar{k},k_z;-t)=\bar{G}_<^*(\bar{k},k_z;t);\ \bar{G}_>(\bar{k},k_z;-t)=\bar{G}_>^*(\bar{k},k_z;t) \ , \tag{C3}$$

we have

$$\begin{aligned}\mathscr{R}^*(\bar{r},z,\bar{r}',z';-\nu+i\epsilon)&=+\frac{i4}{d^2}\int\frac{d^2\bar{k}}{(2\pi)^2}\int\frac{d^2\bar{k}'}{(2\pi)^2}e^{-i(\bar{k}-\bar{k}')\cdot(\bar{r}-\bar{r}')}\\&\quad\times\sum_{k_z}\sum_{k_z'}\Pi(k_z,k_z';z,z')\left[\left[-\int_0^\infty dt\,e^{it(\nu+i\epsilon)}\bar{G}_<(\bar{k},k_z;-t)\bar{G}_>(\bar{k}',k_z';t)\right]\right.\\&\quad\left.+\left[\int_{-\infty}^0 dt\,e^{-it(-\nu+i\epsilon)}\bar{G}_>(\bar{k},k_z;-t)\bar{G}_<(\bar{k}',k_z';t)\right]^*\right] \ .\end{aligned} \tag{C4}$$

If we now interchange $t\leftrightarrow -t$ and $\bar{k}\leftrightarrow\bar{k}'$ and $k_z\leftrightarrow k_z'$ and employ Eq. (C2), we find

$$\begin{aligned}\mathscr{R}^*(\bar{r},z,\bar{r}',z';-\nu+i\epsilon)&=\frac{i4}{d^2}\int\frac{d^2\bar{k}}{(2\pi)^2}\int\frac{d^2\bar{k}'}{(2\pi)^2}e^{i(\bar{k}-\bar{k}')\cdot(\bar{r}-\bar{r}')}\\&\quad\times\sum_{k_z}\sum_{k_z'}\Pi(k_z,k_z';z,z')\left[\left[-\int_{-\infty}^0 dt\,e^{-it(\nu+i\epsilon)}\bar{G}_>(\bar{k},k_z;-t)\bar{G}_<(\bar{k}',k_z',t)\right]\right.\\&\quad\left.+\left[\int_0^\infty dt e^{it(-\nu+i\epsilon)}\bar{G}_<(\bar{k},k_z;-t)\bar{G}_>(\bar{k}',k_z',t)\right]^*\right] \ ,\end{aligned} \tag{C5}$$

and comparing with Eq. (3.14) we have

$$\mathscr{R}^*(\bar{r},z,\bar{r}',z';-\nu+i\epsilon)=\mathscr{R}(\bar{r},z,\bar{r}',z';\nu+i\epsilon) \ , \tag{C6}$$

whence

$$\mathrm{Re}\mathscr{R}(\bar{r},z,\bar{r}',z';-\nu+i\epsilon)=\mathrm{Re}\mathscr{R}(\bar{r},z,\bar{r}',z';\nu+i\epsilon)=\text{even} \tag{C7}$$

and

$$\mathrm{Im}\mathscr{R}(\bar{r},z,\bar{r}',z';-\nu+i\epsilon)=-\mathrm{Im}\mathscr{R}(\bar{r},z,\bar{r}',z';\nu+i\epsilon)=\text{odd} \tag{C8}$$

so that $\mathrm{Re}\mathscr{R}$ is an even function of frequency ν whereas $\mathrm{Im}\mathscr{R}$ is an odd function of frequency ν. These even and odd properties are quite general for $\mathscr{R}$.

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