PHYSICAL REVIEW B VOLUME 33, NUMBER 6 15 MARCH 1986

Magnetoplasma modes of the two-dimensional electron gas at nonintegral filling factors

H. C. A. Oji and A. H. MacDonald
National Research Council, Ottawa, Canada K1A 0R6
(Received 14 May 1985; revised manuscript received 10 October 1985)

Two approaches are used to discuss the collective excitations of a two-dimensional electron gas in a perpendicular magnetic field B, which occur at energies near $\hbar\omega_c = \hbar eB/mc$. The first approach, a time-dependent Hartree-Fock approximation, becomes exact at integral Landau-level filling in the strong-field limit. The second approach, a generalized single-mode approximation, incorporates the strong correlations among electrons in a partially filled Landau level and is more reliable but can be conveniently implemented only in the strong-field limit. Both approaches predict the existence of poles in the density response function at finite wave vector for $\omega = \omega_c$, which may lead to anomalies in the cyclotron-resonance line shape.

I. INTRODUCTION

It is well known[1,2] that a two-dimensional (2D) electron gas can support charge-density-wave oscillations with the plasma frequency given, at long wavelength, by

$$\omega(\mathbf{q}) = \left[\frac{2\pi n e^2 q}{m}\right]^{1/2} . \tag{1}$$

These collective excitation energies can be determined by locating the poles of the density response function

$$\chi(\mathbf{q},\omega) = \frac{\Pi(\mathbf{q},\omega)}{1 - V(\mathbf{q})\Pi(\mathbf{q},\omega)} , \tag{2}$$

where $\Pi(\mathbf{q},\omega)$ is the proper polarizability and $V(\mathbf{q}) = 2\pi e^2/q$ is the 2D Fourier transform for the Coulomb interaction. Frequently, $\Pi(\mathbf{q},\omega)$ is approximated by the polarizability of a noninteracting system (the random-phase approximation,[3] or RPA), but higher-order corrections have been studied by some workers.[4-6] In this paper we study the long-wavelength density oscillations of a 2D electron gas in a perpendicular magnetic field (B) where, because of the Landau quantization of the energy levels, excitation modes occur close to integral multiples of ω_c. At long wavelength the lowest of these modes is plasmonlike and is frequently referred to as the magnetoplasmon mode. While modes appearing near other multiples of ω_c can be treated as easily, we explicitly discuss only this case. The magnetoplasmons have been studied previously in the RPA (Refs. 7 and 8) and, more recently, in the Hartree-Fock approximation (HFA) (Refs. 9–12). The previous studies have been restricted to integral filling factors and, for the most part, to the strong-magnetic-field limit. Here we consider the case of nonintegral filling factors and find that a pole occurs in the density response function for finite wave vector at $\omega = \omega_c$. The associated mode is primarily of spin-wave character but has a partial density-wave character. We suggest that the existence of these modes may be the reason for the as yet unexplained anomalies which sometimes occur in the cyclotron-resonance line shape.[13-15]

We will discuss two approaches to the case of fractional filling factor ν: $\nu = 2\pi a_L^2 n$ where n is the electron density and $a_L = (\hbar c/eB)^{1/2}$. In Sec. II we use finite temperatures to introduce the possibility of fractional filling factors in the HFA. (The RPA may be obtained as a simplification of the HFA.) Numerical results following from this approximation are reported on in Sec. III (the effect of finite temperature) and Sec. IV (the filling factor dependence of resonance frequency). Although fractional filling factors do not cause any formal difficulty in the finite-temperature HFA, the approximation does not describe the strong correlations between electrons associated with the macroscopic degeneracy which occurs when any Landau level is fractionally occupied. In Sec. V we employ the generalized single-mode approximation (GSMA),[16,17] which is readily implemented only in the strong-field limit, to include these correlations in the estimation of the resonant frequencies. The GSMA does not describe the broadening of the collective modes, but is reliable for wave vectors smaller than $\sim a_L^{-1}$, where these modes are expected to be well defined.[16,17] Both approaches show that the spin-wave-like mode which has an extremum for $\omega < \omega_c$ produces a pole in the *density* response function. We conclude in Sec. VI with a discussion of the possible connection between these modes and the cyclotron-resonance line shape.

II. FINITE-TEMPERATURE HARTREE-FOCK APPROXIMATION

We consider an interacting 2D electron gas which is perturbed by a spin-dependent external potential ($V_{\text{ext}}^{\sigma} e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$), where σ denotes the spin variable. The collective excitation modes are determined, as mentioned earlier, by the poles of the density response function $\chi(\mathbf{q},\omega)$, which in this case is related to the (linear) response $\delta\rho^{\sigma}(\mathbf{q},\omega)$ by

$$\delta\rho^{\sigma}(\mathbf{q},\omega) = \sum_{\sigma'} \chi_{\sigma,\sigma'}(\mathbf{q},\omega) V_{\text{ext}}^{\sigma'} . \tag{3}$$

One approach for calculating $\chi(\mathbf{q},\omega)$ is to apply the standard perturbation theory using the quite general manybody techniques in which one ends up summing a certain class of diagrams.[9] The class considered in Ref. 9 is equivalent to a self-consistent Hartree-Fock approximation so that one can alternately use a perturbation theory in terms of the single-particle wave functions,[11,18] which is the approach we follow here. The starting point is the equation of motion for the single-particle wave functions

$$i\hbar\frac{\partial\psi_\alpha(\mathbf{r},t)}{\partial t}=H(t)\psi_\alpha(\mathbf{r},t)\ , \tag{4}$$

where

$$H(t)\psi_\alpha(\mathbf{r},t)=\left[\frac{1}{2m}\left[\mathbf{p}+\frac{e}{c}\mathbf{A}\right]^2+H_{\text{ext}}(t)\right]\psi_\alpha(\mathbf{r},t)$$
$$+\sum_{\alpha'}n_F(\varepsilon_{\alpha'})\int d\mathbf{r}'\frac{e^2}{|\mathbf{r}-\mathbf{r}'|}[\psi^*_{\alpha'}(\mathbf{r}',t)\psi_{\alpha'}(\mathbf{r}',t)\psi_\alpha(\mathbf{r},t)-\psi^*_{\alpha'}(\mathbf{r}',t)\psi_\alpha(\mathbf{r}',t)\psi_{\alpha'}(\mathbf{r},t)]\ . \tag{5}$$

Once $\psi_\alpha(\mathbf{r},t)$ is known, the charge-density which is given by

$$\rho(\mathbf{r},t)=\delta\rho(\mathbf{r},t)+\rho_0(\mathbf{r},t)=-e\sum_\alpha\psi^*_\alpha(\mathbf{r},t)\psi_\alpha(\mathbf{r},t)n_F(\varepsilon_\alpha) \tag{6}$$

can be calculated. In Eq. (5),

$$n_F(\varepsilon_\alpha)=\{\exp[(\varepsilon_\alpha-\mu)/k_BT]+1\}^{-1}$$

is the equilibrium Fermi function and ε_α is the energy eigenvalue in the absence of the external perturbation.

Application of the standard time-dependent perturbation theory within the single-particle picture,[19] as describe in detail in Refs. 11 and 18, leads to the following results in the presence of a perpendicular magnetic field:

$$\delta\rho^\sigma(\mathbf{q},\omega)=(2\pi a_L^2)^{-1}\sum_{n',n}\delta\Delta^\sigma_{n',n}(\mathbf{q},\omega)F_{n',n}(-\mathbf{q})\ , \tag{7}$$

where $\delta\Delta^\sigma_{n',n}(\mathbf{q},\omega)$ satisfies the equation

$$\delta\Delta^\sigma_{n',n}(\mathbf{q},\omega)=\frac{\hbar^{-1}[n_F(\varepsilon_{n,\sigma})-n_F(\varepsilon_{n',\sigma})]}{\omega-\hbar^{-1}(\varepsilon_{n',\sigma}-\varepsilon_{n\sigma})+i\eta}\left[V^\sigma_{\text{ext}}F_{n,n'}(\mathbf{q})+\frac{e^2}{a_L}\sum_{m,m'}H(n,m;n',m';\mathbf{q})\sum_{\sigma'}\delta\Delta^{\sigma'}_{m',m}(\mathbf{q},\omega)\right.$$
$$\left.-\frac{e^2}{a_L}\sum_{m,m'}X(n,m;n',m';\mathbf{q})\delta\Delta^\sigma_{m',m}(\mathbf{q},\omega)\right]\ , \tag{8}$$

and the Hartree-Fock energy for the nth Landau level, $\varepsilon_{n,\sigma}$, is

$$\varepsilon_{n,\sigma}=\hbar\omega_c\left[n+\frac{1}{2}+\frac{1}{4}\sigma g\frac{m}{m_0}\right]-\frac{e^2}{a_L}\sum_m n_F(\varepsilon_{m\sigma})X(n,m;n,m;\mathbf{q}=0)\ . \tag{9}$$

In Eq. (9) $\sigma=\pm1$, for up and down spins, respectively, g is the effective g factor, and m/m_0 is the ratio of the effective electron mass to the bare mass. The terms which involve $H(n,m,n',m';\mathbf{q})$ and $X(n,m,n'm';\mathbf{q})$ in Eq. (8) originate from electron-electron interactions in the Hartree-Fock approximation where the Coulomb local field is determined by

$$H(n,m;n',m';\mathbf{q})=\frac{1}{qa_L}F_{n,n'}(\mathbf{q})F_{m',m}(-\mathbf{q})\ , \tag{10}$$

and the exchange local field by

$$X(n,m;n',m';\mathbf{q})=\int_0^\infty dq'a_L\int_0^{2\pi}d\theta\frac{1}{2\pi}\exp[ia_L^2(q_yq'_x-q_xq'_y)]F_{n,m}(\mathbf{q}')F_{m',n'}(-\mathbf{q}')\ . \tag{11}$$

The function $F_{n',n}(\mathbf{q})$ appearing here is given by

$$F_{n',n}(\mathbf{q})=\left[\frac{n!}{n'!}\right]^{1/2}\left[\frac{(iq_x-q_y)a_L}{\sqrt{2}}\right]^{n'-n}\times\exp\left[\frac{-q^2a_L^2}{4}\right]L_n^{n'-n}(q^2a_L^2/2)\ , \tag{12}$$

where $L_n^\alpha(x)$ is a generalized Laguerre polynomial.

Numerical results following from this approximation are reported in the following two sections. Examination of Eq. (8) makes its connection with other approximations apparent. One obtains the well-known lowest-order expression for $\chi(\mathbf{q},\omega)=\Pi^0(\mathbf{q},\omega)$ by substituting Eq. (8) into Eq. (7) with $e^2=0$. The finite-temperature RPA follows if the Coulomb local fields are retained but the exchange local fields are set to zero. The relationship between the present approach and the diagrammatic method has already been pointed out in Ref. 11, for the zero-temperature case where it was noted that in the limit $(e^2/\epsilon a_L)/\hbar\omega_c \ll 1$ the present approximation reduces to the same result obtained by Kallin and Halperin.[9] In the diagram language, the density response function in the Hartree-Fock approximation as determined by Eqs. (3), (7), and (8) would correspond to summing the set of diagrams shown in Fig. 1.

III. TEMPERATURE DEPENDENCE OF RESONANT FREQUENCIES

According to Eq. (7), the poles of the function $\delta\Delta^{\sigma}_{n,n'}(\mathbf{q},\omega)$ determine the magnetoplasmon dispersion relation. To derive these resonant frequencies, we rewrite Eq. (8) into the form

$$\sum_{m,m',\sigma'} [\hbar\omega\delta_{n',m'}\delta_{n,m}\delta_{\sigma',\sigma} - E(n',n,\sigma;m',m,\sigma';\mathbf{q})]\delta\Delta^{\sigma'}_{m',m}(\mathbf{q},\omega) = [n_F(\varepsilon_{n,\sigma}) - n_F(\varepsilon_{n'\sigma})]V^{\sigma}_{\text{ext}}F_{n,n'}(\mathbf{q}) , \tag{13}$$

where

$$E(n',n,\sigma;m',m,\sigma';\mathbf{q}) = (\varepsilon_{n',\sigma'} - \varepsilon_{n,\sigma})\delta_{n',m'}\delta_{n,m}\delta_{\sigma',\sigma} + [n_F(\varepsilon_{n,\sigma}) - n_F(\varepsilon_{n',\sigma})]\frac{e^2}{a_L}[H(n,m;n',m';\mathbf{q}) - \delta_{\sigma'\sigma}X(n,m;n',m';\mathbf{q})] . \tag{14}$$

It is clear from the above equations that $\delta\Delta^{\sigma}_{m',m}(\mathbf{q},\omega)$ vanishes for $\omega \neq 0$ if $n = n'$. Thus the interesting cases are restricted to Landau levels such that $n \neq n'$, in which case the poles of the density response function are obtained, as readily seen from Eq. (13), by diagonalizing the matrix given by Eq. (14). This matrix is already diagonal in the absence of interaction yielding the resonant frequencies $\omega = (n'-n)\omega_c$. We focus our attention on the dispersion relations of modes occurring close to the cyclotron frequency $\omega(\mathbf{q}) \sim \omega_c$ corresponding to $|n'-n| = 1$.

We have diagonalized the matrix, Eq. (14), numerically. The Hartree-Fock energies $\varepsilon_{n,\sigma}$ [see Eq. (9)] are evaluated by an iterative process at a given filling factor $\nu = (eB/hc)^{-1}n$, where n is the areal electron density, and

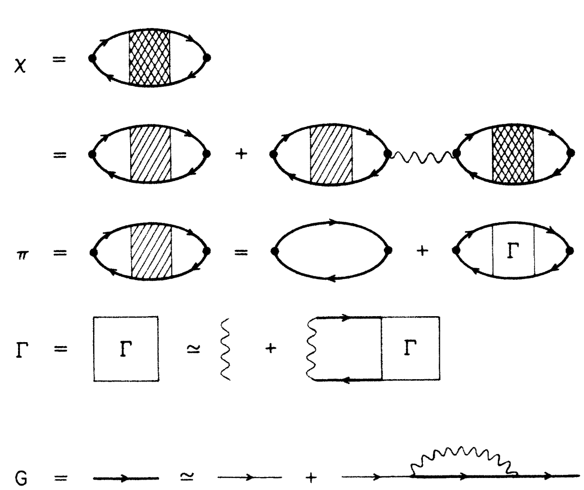

FIG. 1. General diagrammatic expressions for $\chi(\mathbf{q},\omega)$ and $\Pi(\mathbf{q},\omega)$ where the last two approximative diagrams, representing the vertex function Γ and Greens function G define the so-called Hartree-Fock approximation for $\chi(\mathbf{q},\omega)$. The wavy line is the bare interaction potential. The approximations for Γ and G lead to the same result for $\chi(\mathbf{q},\omega)$ obtained by our single-particle description.

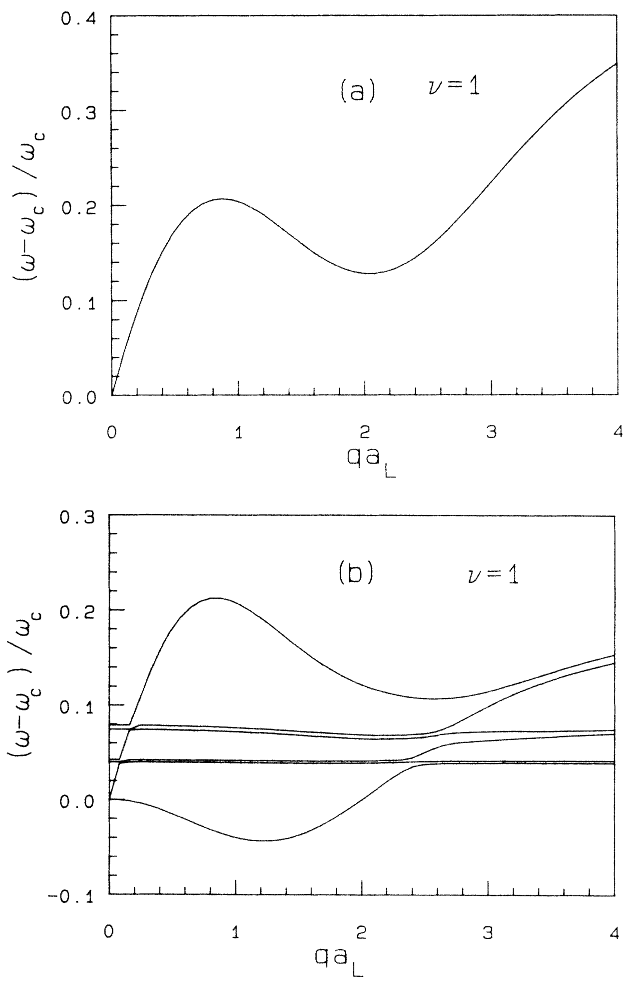

FIG. 2. Shift of the resonant frequencies from the cyclotron frequency as a function of wave vector for filling factor $\nu = 1$ and for various values of the (reduced) temperature. The Coulomb interaction parameter $(e^2/\epsilon a_L)/\hbar\omega_c = 1$ and in (a) $k_B T/\hbar\omega_c = 10^{-2}$ and 10^{-1}. The curves are indistinguishable on this scale. In (b) we illustrate the dispersion of the modes occurring near ω_c at the same value of ν for $k_B T = 0.5\hbar\omega_c$ (see text).

also for fixed values of the ratios $(e^2/\epsilon a_L)/\hbar\omega_c$ and $k_BT/\hbar\omega_c$. The numerical procedures followed to ensure convergence with respect to the order of the truncated matrix are described more fully in Ref. 11. The minimal effect of finite temperature on the dispersion relation is shown in Fig. 2(a), where we plot the magnetoplasmon dispersion for $\nu=1$ and $e^2/a_L=\hbar\omega_c$ at $k_BT/\hbar\omega_c=10^{-2}$, 10^{-1}. For GaAs parameters $\hbar\omega_c/k_BT \sim 20B[\mathrm{T}]/T[\mathrm{K}]$ so these values correspond to $T=2$ K and $T=20$ K, respectively, at a magnetic field of 10 T. (The effective g factor is taken to be 0.49 and the effective mass is $0.068m_0$ as appropriate for the 2D electrons in a GaAs/AlGaAs heterojunction.) A stronger temperature effect might have been expected to be present at this filling factor because of the small spin splitting compared to $\hbar\omega_c$ However, the exchange correlation affects the energy levels in such a manner that the minority-spin energy levels (spin-up levels) remain essentially unaltered while the majority-spin energy levels are significantly lowered. The lowering of majority-spin energy levels by the exchange interaction dramatically reduces the occupation of the minority-spin $n=0$ Landau level which thermal broadening would otherwise produce. It is not until k_BT is several times larger than $g_{\mathrm{eff}}\hbar\omega_c$ ($g_{\mathrm{eff}}=\frac{1}{4}g\,m/m_0$) that this gigantic exchange enhancement of the spin splittings is quenched. In Fig. 2(b) we show the dispersion of magnetoplasmon modes occurring for $\nu=1$ at $k_BT/\hbar\omega_c=0.5$, which corresponds to $T=100$ K at $B=10$ T for GaAs. Because both spins are nearly equally occupied two principle modes occur. As we discuss in more detail later the higher-energy mode is primarily of charge-density character while the lower-energy mode is primarily of spin-density character. We have also shown in Fig. 2(b) the anticrossings between these modes and the corresponding modes associated with transitions between $n=1$ and $n=2$ Landau levels and between $n=2$ and $n=3$ Landau levels. These modes are relevant here because of the significant thermal occupation of the $n=1$ and $n=2$ Landau levels.

The exchange enhancement of the spin polarization, mentioned above, is illustrated in Fig. 3 for $k_BT/\hbar\omega_c=0.05$, which corresponds to $T=10$ K at $B=10$ T in GaAs. At this temperature only the $n=0$ Landau levels have significant occupations. In the RPA, i.e., using bare energies, up- and down-spin occupancies are nearly equal while in the HFA, i.e., including the effect of exchange on the energy levels, the spin polarization is nearly complete for $\nu<1$.

IV. FILLING FACTOR DEPENDENCE OF RESONANT FREQUENCIES

We have performed a series of numerical calculations using Eq. (13) to determine the dependence of the resonant frequencies on the total filling factor. We consider the $k_BT \ll \hbar\omega_c$ limit which is easily reached in low-temperature experiments. In the HFA the resonant frequencies depend smoothly on the filling factor. One qualitatively new feature enters when fractional filling factors are considered and, in order to explain it clearly, we find it useful to consider the high magnetic field limit ($e^2/\epsilon a_L \ll \hbar\omega_c$) and to discuss in detail the case of filling factors between 0 and 2. This allows us to truncate the secular equation [Eq. (13)] to modes with $n=m=0$ and

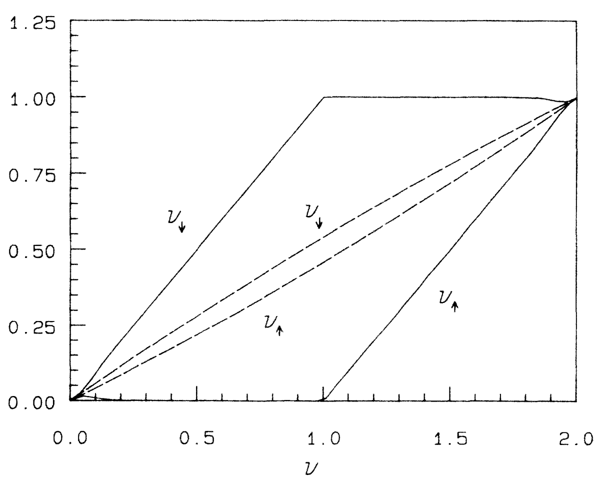

FIG. 3. Filling factor of the majority- and minority-spin Landau levels, $\nu_\downarrow$ and $\nu_\uparrow$, respectively, versus total filling factor $\nu=\nu_\downarrow+\nu_\uparrow$ in the HFA (solid line) and RPA (dashed line) at $k_BT/\hbar\omega_c=0.05$ and $(e^2/\epsilon a_L)/\hbar\omega_c=1$. In the RPA thermal smearing causes nearly equal occupancies for minority and majority spins. In the HFA exchange lowers the majority-spin energy compared to the minority-spin energy except as ν approaches 0 and 2 and dramatically reduces the effect of thermal smearing.

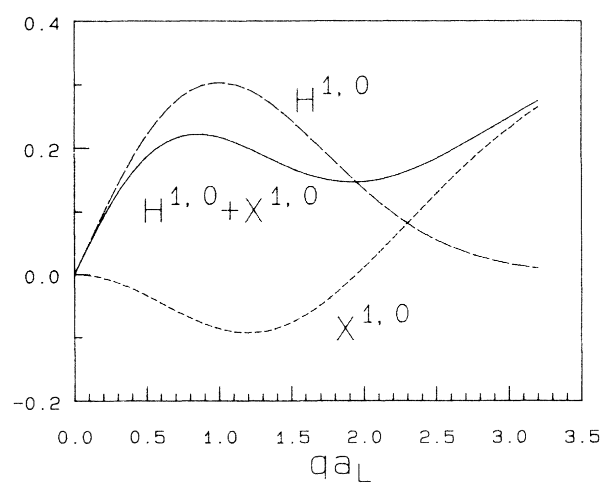

FIG. 4. The functions $H^{1,0}(\mathbf{q})$ (long-dashed) curve and $X^{1,0}(\mathbf{q})$ (short-dashed curve), defined by Eqs. (16), versus wave vector. The sum $H^{1,0}(\mathbf{q})+X^{1,0}(\mathbf{q})$ (solid curve) is the shift of the chargelike excitation energy from $\hbar\omega_c$ at $\nu_\downarrow=1$, $\nu_\uparrow=0$ in the strong-field limit. At $\nu_\uparrow=1$, $\nu_\downarrow=1$, in the strong-field limit, $\omega_+(q)=2H^{1,0}(\mathbf{q})+X^{1,0}(q)$ and $\omega_-(q)=X^{1,0}(q)$.

$n'=m'=1$. For filling factors between 0 and 1 only the majority spin need be considered and the resonant frequency is

$$\omega(\mathbf{q})=\omega_c+\hbar^{-1}\nu_\downarrow[H^{1,0}(\mathbf{q})+X^{1,0}(q)]\ , \tag{15}$$

where we have separated the Hartree contribution to the shift from ω_c,

$$H^{1,0}(\mathbf{q})=\left[\frac{e^2}{a_L}\right]H(0,0;1,1;q) = \left[\frac{e^2}{a_L}\right]\frac{qa_L}{2}\exp\left[\frac{-q^2a_L^2}{2}\right]\ , \tag{16a}$$

and the exchange contribution

$$X^{1,0}(\mathbf{q})=\frac{e^2}{a_L}[-X(0,0;1,1;\mathbf{q})-X(1,0;1,0;\mathbf{q}=\mathbf{0}) +X(0,0;0,0;\mathbf{q}=\mathbf{0})]\ . \tag{16b}$$

In Fig. 4 $H^{1,0}(\mathbf{q})$, $X^{1,0}(\mathbf{q})$, and $H^{1,0}(\mathbf{q})+X^{1,0}(\mathbf{q})$ are plotted versus q. The resonant frequency shift is simply proportional to $\nu_\downarrow$ and the previous results are recovered when $\nu_\downarrow=1$.

More interesting is the region between $\nu=1$ and $\nu=2$ where the minority-spin Landau level is being filled $(\nu_\uparrow=\nu-1)$. In this case both spins need to be considered and the secular equation produces two resonant frequencies:

$$\omega_\pm(\mathbf{q})=\omega_c+\left[\frac{H^{1,0}(\mathbf{q})+X^{1,0}(\mathbf{q})}{2}\right](1+\nu_\uparrow)\pm\left[\left[\frac{[H^{1,0}(\mathbf{q})+X^{1,0}(\mathbf{q})](1-\nu_\uparrow)}{2}\right]^2+\nu_\uparrow H^{1,0}(\mathbf{q})^2\right]^{1/2}\ . \tag{17}$$

It is instructive to solve Eq. (13) to determine the residues in the density response, $\delta\rho^\uparrow+\delta\rho^\downarrow=\delta\rho$, to a spin-independent $(V^\uparrow=V^\downarrow=V)$ external potential. The result is

$$\delta\rho(\mathbf{q},\omega_\pm)(\omega-\omega_\pm)\,|_{\omega=\omega_\pm}=\hbar^{-1}\alpha_\pm(\mathbf{q})\,|F_{1,0}(\mathbf{q})|^2V(2\pi a_L^2)^{-1}\ , \tag{18a}$$

where

$$\alpha_\pm(\mathbf{q})=\left[\frac{1+\nu_\uparrow}{2}\right]\pm\left[H^{1,0}(\mathbf{q})\left[\frac{1+\nu_\uparrow}{2}\right]^2+X^{1,0}(\mathbf{q})\left[\frac{1-\nu_\uparrow}{2}\right]^2\right]\left[\left[\frac{H^{1,0}(\mathbf{q})+X^{1,0}(\mathbf{q})(1-\nu_\uparrow)}{2}\right]^2+\nu_\uparrow[H^{1,0}(\mathbf{q})]^2\right]^{-1/2}\ . \tag{18b}$$

The nature of the two modes can be understood if we consider two limits. As $q\to 0$, $|X^{1,0}(\mathbf{q})|\ll|H^{1,0}(\mathbf{q})|$ and we recover the RPA. In this limit $\omega_+(q)=\omega_c+\nu H^{1,0}(q)$ and $\omega_-(q)=\omega_c$, i.e., $\omega_+(q)$ becomes a pure charge-density excitation whose frequency shift depends on the total filling factor and not on its spin decomposition while $\omega_-(q)$ is a pure spin-density excitation whose resonance frequency as $q\to 0$ is unshifted. On the other hand, as $q\to\infty$, $|X^{1,0}(\mathbf{q})|\gg|H^{1,0}(q)|$. In this limit we can ignore the direct Coulomb interactions and the excitations in the two spin channels become independent. [The higher-frequency mode becomes the majority spin with excitation $\omega_+(q)=\omega_c+X^{1,0}(q)$ and the lower-frequency mode becomes $\omega_-(q)=\omega_c+\nu_\uparrow X^{1,0}(q)$.] The transition between the two regimes is quite abrupt as we see in Fig. 5 where we have plotted $\alpha_\pm(q)$ versus q for $\nu_\uparrow=0.5$. [$\alpha_\pm(q)$ is a measure of the charge-density-wave component in the resonance.] Note that as $q\to 0$, $\alpha_+(q)\to(1+\nu_\uparrow)$ and $\alpha_-(q)\to 0$ while for $q\to\infty$, $\alpha_+(q)\to 1$ and $\alpha_-(q)\to\nu_\uparrow$.

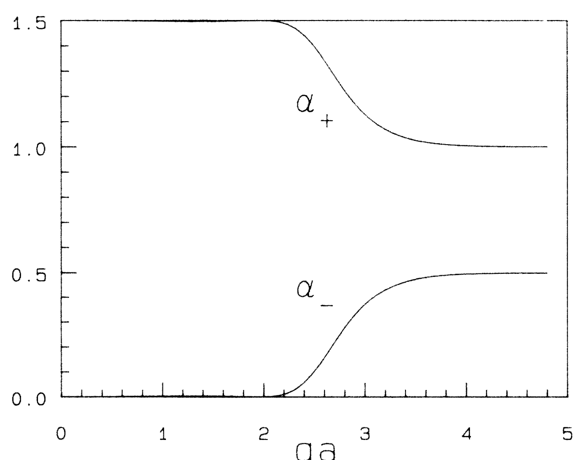

FIG. 5. $\alpha_\pm(q)$ versus wave vector where $\alpha_\pm(q)$, given by Eq. (18), is a measure of the CDW component in the resonance.

In Fig. 6 we have plotted the energies of the density-wave-like mode $[\omega_+(q)]$ and the spin-wave-like mode $[\omega_-(q)]$ calculated in the HFA for several values of ν between 1 and 2 and for $(e^2/a_L)/\hbar\omega_c=1$. With GaAs parameters this corresponds to a magnetic field of ~ 6.1 T and to a density of $\sim 1.5\nu\times 10^{11}$ cm^{-2}. From Eq. (17) it follows that in the strong-field limit $[(e^2/\epsilon a_L)/\hbar\omega_c\ll 1]$ $\omega_-(q)$ crosses ω_c when $X^{1,0}(\mathbf{q})=0$, which according to Fig. 4 occurs at $qa_L\sim 1.94$, for all values of $\nu_\uparrow$. Comparing Figs. 6 and 4 we see that the minimum in $\omega_-(q)$ tends to deepen and the wave vector at which $\omega_-(q)$ crosses ω_c increases with $(e^2/a_L)/\hbar\omega_c$, although the crossing wave vector is still nearly independent of $\nu_\uparrow$. Nevertheless, the results illustrated in Figs. 6 are qualitatively identical to

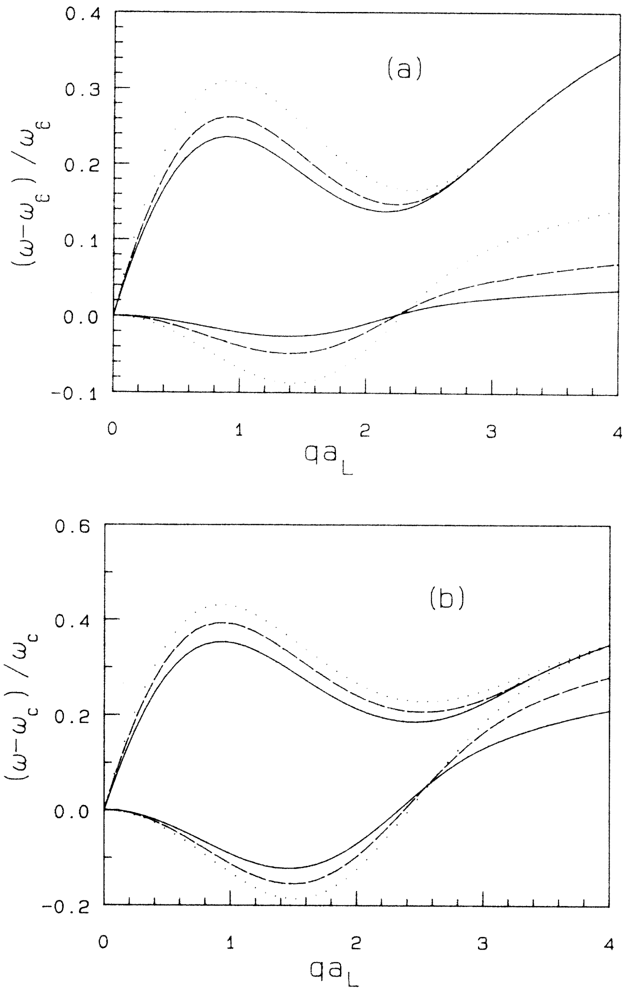

FIG. 6. Magnetoplasmon excitation curves (cf. Fig. 2) as affected by a variation of the filling factor, $1<\nu\leq 2$, at a constant temperature $k_BT/\hbar\omega_c=5\times 10^{-2}$. The ratio $(e^2/\epsilon a_L)/\hbar\omega_c=1$. As the filling factor is increased or reduced, the curves approach gradually the results one would obtain for $\nu=2$ or $\nu=1$, respectively. (a) $\nu=1.1$ (solid curve), $\nu=1.2$ (dashed curve), and $\nu=1.4$ (dotted curve); (b) $\nu=1.6$ (solid curve), $\nu=1.8$ (dashed curve); and $\nu=2$ (dotted curve).

those discussed above which hold for the strong-field limit of the HFA and indicate that, as far as these excitation modes are concerned, the strong-field approximation is reasonably reliable at realistic magnetic fields and 2D electron densities.

V. GENERALIZED SINGLE-MODE APPROXIMATION

For the 2D electron gas in a perpendicular magnetic field a single-mode approximation (SMA) was used by Girvin *et al.*[17] to evaluate the intra-Landau-level modes relevant to the fractional quantum Hall effect,[20] and has been generalized (GSMA) in Ref. 16 to treat other excitations. This approximation is expected to be applicable whenever the excitations are primarily collective in nature and is essentially based on the assumption that the excited state can be constructed by forming density waves in the ground state. Because of the Landau quantization of the energy levels, however, these excitations must be grouped according to the Landau levels involved, i.e., we assume the relevant excitation can be expanded in the form

$$|\psi_{\mathbf{k}}\rangle=\sum_{n',n,\sigma} C^{\sigma}_{n',n}(\mathbf{k})\rho^{n',n}_{\mathbf{k},\sigma}|\psi_0\rangle\ , \tag{19}$$

where $\rho^{n',n}_{\mathbf{k},\sigma}$ is the part of the density operator which transfers electrons of spin σ from the nth to the n'th Landau level (see Ref. 16). In the strong-field limit (to which we restrict ourselves in this section) and for $\nu<1$, $|\psi_0\rangle$ is formed entirely among the majority-spin electrons of spin σ. Then, as discussed by Girvin *et al.*[17,21] for intra-Landau-level excitations $|\psi_{\mathbf{k}}\rangle\sim\rho^{0,0}_{\mathbf{k}}|\psi_0\rangle$ and the excitation energy

$$\omega_0(k)\sim\langle\omega_0|\rho^{0,0}_{-\mathbf{k},\downarrow}[H,\rho^{0,0}_{\mathbf{k},\downarrow}]|\psi_0\rangle/\langle\psi_0|\rho^{0,0}_{-\mathbf{k}\downarrow}\rho^{0,0}_{\mathbf{k}\downarrow}|\psi_0\rangle$$

where H is the Hamiltonian for the interacting system. For $\nu<1$, a situation which has been treated in Ref. 16, the excitations which occur at energies near $\hbar\omega_c$ are approximately described by $|\psi_{\mathbf{k}}\rangle\sim\rho^{1,0}_{\mathbf{k},\downarrow}|\psi_0\rangle$. For $\nu>1$, however, both spins must be considered and the details of the calculation are summarized below.

We use the notation and definition of Ref. 16 where, for the case of interest here, the approximate excited states and their excitation energies are determined by solving the secular equation

$$\sum_{\sigma}[E^{\sigma',\sigma}_{pl}(1,0;1,0;\mathbf{k})-\omega_{\pm}(k)S^{\sigma',\sigma}_{pl}(1,0;1,0;\mathbf{k})]C^{\sigma'}(\mathbf{k})=0\ , \tag{20a}$$

where

$$E^{\sigma',\sigma}_{pl}(1,0;1,0;\mathbf{k})=\langle\psi_0|\rho^{0,1}_{-\mathbf{k},\sigma'}[H,\rho^{1,0}_{\mathbf{k},\sigma}]|\psi_0\rangle\ , \tag{20b}$$

$$S^{\sigma',\sigma}_{pl}(1,0;1,0;\mathbf{k})=\langle\psi_0|\rho^{0,1}_{-\mathbf{k}\sigma'}\rho^{1,0}_{\mathbf{k}\sigma}|\psi_0\rangle\ , \tag{20c}$$

and $|\psi_0\rangle$ is the ground state in which the majority (down) spin $n=0$ Landau level is completely full and the electrons in the partially occupied minority (up) spin $n=0$ Landau level are highly correlated. The matrix elements in Eqs. (20) can be expressed in terms of the partial correlation functions of the electrons. The calculations leading to these expressions are very similar to those detailed in Ref. 16, and we simply quote the result:

$$E_{pl}^{\sigma',\sigma}(1,0;1,0;\mathbf{k})=N_\sigma \exp(-k^2a_L^2/2)\frac{k^2a_L^2}{2}\{\delta_{\sigma',\sigma}(X[h_{\sigma,\sigma};\mathbf{k}]+\omega_c)+\nu_{\sigma'}H^{1,0}(\mathbf{k})\}\ , \tag{21a}$$

where

$$X[h_{\sigma\sigma};\mathbf{k}]=e^2\int\frac{d^2q}{(2\pi)}a_L q^{-1}\{h_{\sigma\sigma}(q)[(1-q^2/2)e^{i(\mathbf{k}\times\mathbf{q})_z}-1]q^2e^{k^2/2-\mathbf{k}\cdot\mathbf{q}}h_{\sigma\sigma}(k-q)/2\} \tag{21b}$$

and

$$h_{\sigma\sigma}(k)=\nu_0\int_0^\infty dr\, r a_L^{-2}[g_{\sigma\sigma}(r)-1]J_0(kr) \tag{21c}$$

is the Fourier transform of the partial pair-correlation function for electrons of spin σ and $J_0(x)$ is a Bessel function. Similarly,

$$S_{pl}^{\sigma',\sigma}(1,0;1,0;\mathbf{k})=\delta_{\sigma',\sigma}N_\sigma \exp(-k^2a_L^2/2)k^2a_L^2/2\ , \tag{22}$$

which when combined with Eqs. (21) and (20) gives the following expression for the magnetoplasmon-mode energies:

$$\omega_\pm(\mathbf{k})=\omega_c+\left[\frac{H^{1,0}(\mathbf{k})(1+\nu_\uparrow)+X[h_{\downarrow\downarrow};\mathbf{k}]+X[h_{\uparrow\uparrow};\mathbf{k}]}{2}\right]$$
$$\pm\left[\left[\frac{H^{1,0}(\mathbf{k})(1-\nu_\uparrow)+X[h_{\downarrow\downarrow};\mathbf{k}]-X[h_{\uparrow\uparrow};\mathbf{k}]}{2}\right]^2+\nu_\uparrow[H^{1,0}(\mathbf{k})]^2\right]^2\ , \tag{23}$$

which can be compared with the corresponding Hartree-Fock result. Since $h_{\downarrow\downarrow}(k)=-e^{-k^2/2}$, the result for a full $n=0$ Landau level, comparing Eq. (21b) with Eqs. (11), (12), and (16b) shows that $X[h_{\downarrow\downarrow};\mathbf{k}]=X^{1,0}(\mathbf{k})$. The HFA is recovered if $h_{\uparrow\uparrow}(\mathbf{k})$ is replaced by its uncorrelated values, $-\nu_\uparrow e^{-k^2/2}$, which would imply that $X[h_{\uparrow\uparrow};\mathbf{k}]=\nu_\uparrow X^{1,0}(\mathbf{k})$.

In Fig. 7 we have compared the GSMA magnetoplasmon energies with the HFA energies for the case of $\nu_\uparrow=\frac{1}{3}$, where the correlation function of the minority-spin electrons can be approximated by that of the state proposed by Laughlin[22,23] in connection with the quantum Hall effect.[20] As $k\to 0$ the Hartree local field in Eq. (21a) dominates and the two approximations yield identical results. As $k\to\infty$ the Hartree local field vanishes and the two-spin channels become independent. For the majority spin, which has a full Landau level, the two approximations agree, but for the minority spin the shift of the mode from ω_c is underestimated by the HFA. Physically, this is because the HFA does not reflect the strong correlations among the minority-spin electrons, and therefore underestimates the energy cost of promoting one of these electrons to the next Landau level. From Eq. (21b) we can show that

$$\lim_{k\to\infty}X[h_{\sigma\sigma},\mathbf{k}]=\frac{-2E_0[h_{\sigma\sigma}]}{N_\sigma}-\frac{\nu_\sigma e^2}{a_L}\left[\frac{\pi}{8}\right]^{1/2}\ , \tag{24}$$

where $E_0[h_{\sigma\sigma}]$ is the contribution to the total energy from correlations among electrons of spin σ. Since

$$E_0[h_{\downarrow\downarrow}]=-\left[\frac{\pi}{8}\right]^{1/2}\frac{e^2}{a_L}N_\downarrow$$

and for the case illustrated ($\nu_\uparrow=\frac{1}{3}$), $E_0[h_{\uparrow\uparrow}]\approx-0.4100(e^2/a_L)N_\uparrow$,[24,25]

$$\lim_{k\to\infty}X[h_{\downarrow\downarrow}(k)]=\left[\frac{\pi}{8}\right]^{1/2}\frac{e^2}{a_L}\simeq 0.627\frac{e^2}{a_L} \tag{25a}$$

as in the HFA, while

$$\lim_{k\to\infty}X[h_{\uparrow\uparrow}(k)]\approx 0.611\frac{e^2}{a_L}\ , \tag{25b}$$

compared to $0.209e^2/a_L$ in the HFA. In Fig. 8 we show the finite residue in the density response function for the spin-wave-like mode is not an artifact of the HFA but rather that the magnitude of the residue is actually increased in the GSMA. In fact, this result should not be

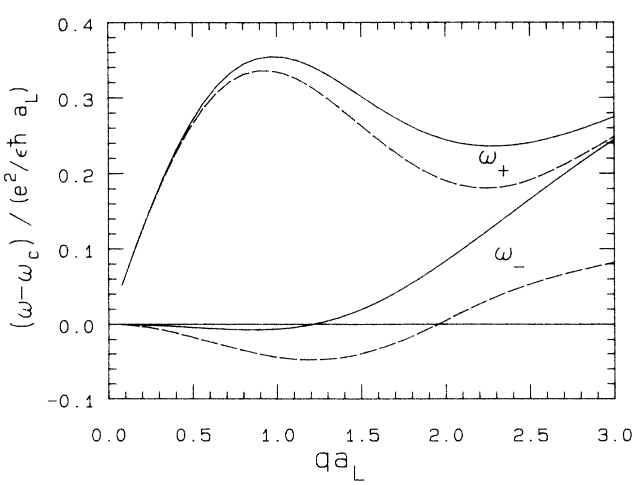

FIG. 7. Shift of n excitation energies from $\hbar\omega_c$ in units of $e^2/\epsilon a_L$ for chargelike and spinlike magnetoplasmon modes. The solid lines are for the GSMA and the dashed lines are for the HFA. The curves are for the case $\nu_\downarrow=1$ and $\nu_\uparrow=\frac{1}{3}$.

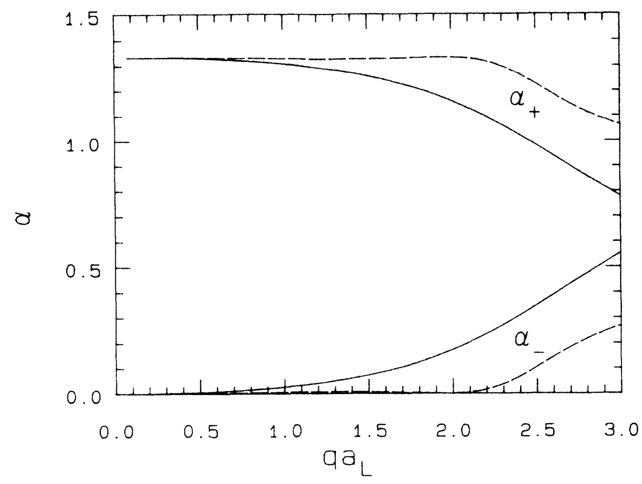

FIG. 8. Chargelike component in the nominally chargelike and nominally spinlike magnetoplasmon modes. The solid lines are for the GSMA and the dashed lines for the HFA. $\nu_\downarrow = 1$ and $\nu_\uparrow = \frac{1}{3}$ as in Fig. 7.

surprising since spin and charge degrees of freedom are not decoupled whenever the system is spin polarized.

VI. DISCUSSION

We conclude this paper with a discussion of the possibility that the partial density-wave character in the spin-wave-like magnetoplasmon mode may be responsible for the anomalies which have been observed in cyclotron-resonance line shape,[13–15] at least when the 2D electron gas is formed at a GaAs/AlGaAs interface. According to the data of Schlesinger *et al.*,[13] the anomaly reflects a crossover from a regime at lower magnetic fields where the cyclotron resonance is shifted from ω_c toward lower frequencies to a regime at higher fields where the resonance is shifted toward higher frequencies. Our argument is based on the theory of Kallin and Halperin,[26] who treat the mixing of magnetoplasmon modes at different wave vectors by a disorder potential. In an ideal system the cyclotron-resonance mode is the $\mathbf{q} \rightarrow 0$ limit of the chargelike magnetoplasmon mode. In the presence of disorder the cyclotron-resonance line shape reflects the broadened spectral weight of this mode.

For qualitative purposes it is sufficient to focus on the extrema of the magnetoplasmon dispersions since the density of modes is infinite at these energies; referring to Figs. 4–7, the chargelike mode has a maxima of $qa_L = X_+^{(1)}$ and a minima at $qa_L = X_+^{(2)} > X_+^{(1)}$, while the spinlike mode has a minima at $X_-^{(1)} \approx X_+^{(1)}$. If only the chargelike modes are considered, repulsion between $q=0$ and finite q modes tends to shift the cyclotron resonance toward lower frequencies. This is the result obtained by Kallin and Halperin[26] for $\nu=1$ and $\nu=2$. If, on the other hand, the spinlike mode dominates, the shift will be toward higher frequencies. Note that the nominally spinlike mode might be expected to be more weakly coupled to the $\mathbf{q} \rightarrow 0$ density mode because of its smaller densitylike component, but this may be more than compensated for by the proximity of its energy minima to the bare cyclotron-resonance energy. A crossover from a negative shift to a positive shift with increasing field can occur because the coupling to the densitylike modes near $q = X_+^{(2)} a_L^{-1}$ drops exponentially as q becomes larger than α^{-1}, where α is the set-back distance of the remote ionized donors expected to dominate disorder scattering in these systems $[a_L^{-1} \propto (B)^{1/2}]$. In fact, for $\nu=3$, the lowest integer value for which a spin-wave-like mode with some density-wave character occurs, Kallin and Halperin find a much broader cyclotron-resonance mode which may be indicative of a crossover behavior. If this mechanism is correct the anomaly should be, to a good approximation, associated with a critical value for α/a_L. Qualitatively, this is consistent with the observed dependence of the magnetic field at which the anomaly occurs on the 2D electron density. Higher electron densities would tend to be associated with shorter set-back distances so that the magnetic field at which a given value of α/a_L occurs would increase. We believe it would be worthwhile to undertake a systematic experimental test of this picture.

ACKNOWLEDGMENTS

We have benefited from discussions with many colleagues and would particularly like to thank A. J. Berlinsky, S. M. Girvin, C. Kallin, and Z. Schlesinger.

[1]F. Stern, Phys. Rev. Lett. **8**, 546 (1967).

[2]First observed experimentally by C. C. Grimes and G. Adams, Phys. Rev. Lett. **36**, 146 (1976).

[3]See, e.g., A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971), Chap. 5.

[4]M. Jonson, J. Phys. C **9**, 3055 (1976).

[5]P. F. Maldague, Solid State Commun. **26**, 133 (1978).

[6]A. Czachor, A. Holas, S. R. Sharma, and K. S. Singwi, Phys. Rev. B **25**, 2144 (1982).

[7]H. Chiu and J. J. Quinn, Phys. Rev. B **9**, 4724 (1974).

[8]N. Horing and M. Yildiz, Ann. Phys. (N.Y.) **97**, 216 (1976).

[9]C. Kallin and B. I. Halperin, Phys. Rev. B **30**, 5655 (1984).

[10]Yu. A. Bychkov, S. V. Iordanskii, and G. M. Eliashberg, Zh. Eksp. Teor. Fiz. Pis'ma Red. 33, 152 (1981) [JETP Lett. **33**, 143 (1981)].

[11]A. H. MacDonald, J. Phys. C **18**, 1003 (1985).

[12]The case of a ferromagnetic 2D gas has been treated by J.-W. Wu, Parwel Hawrylak, and J. J. Quinn, Phys. Rev. B **31**, 6592 (1985).

[13]Z. Schlesinger, S. J. Allen, J. C. M. Hwang, P. M. Platzman, and N. Tzoar, Phys. Rev. B **30**, 435 (1984).

[14]B. A. Wilson, S. J. Allen, and D. C. Tsui, Phys. Rev. B **24**, 5887 (1981).

[15]Th. Englert, J. C. Maan, Ch. Uihlein, D. C. Tsui, and A. C. Gossard, Solid State Commun. **46**, 545 (1983).

[16]A. H. MacDonald, H. C. A. Oji, and S. M. Girvin, Phys. Rev. Lett. **55**, 2208 (1985).

[17]S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. Lett. **54**, 581 (1985).

[18]A. H. MacDonald, Phys. Rev. B **30**, 4392 (1984).

[19]An example of a similar procedure, employed to calculate the density response function (for a 3D electron gas in absence of a magnetic field) in the RPA may be found in, e.g., N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt, Rinehart, and Winston, New York, 1976), Chap. 17.

[20]For a recent review, see D. Yoshioka, Prog. Theor. Phys. (to be published).

[21]S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. B **33**, 2481 (1986).

[22]R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[23]A. H. MacDonald, G. C. Aers, and M. W. C. Dharma-wardana, Phys. Rev. B **31**, 5529 (1985). We use for $h_{\uparrow\uparrow}(k)$ the modified-hyprenetted-chain result from this paper.

[24]D. Levesque, J. J. Weis, and A. H. MacDonald, Phys. Rev. B **30**, 1056 (1984).

[25]R. Morf and B. I. Halperin, Phys. Rev. B **33**, 2221 (1986).

[26]C. Kallin and B. I. Halperin, Phys. Rev. B **31**, 3635 (1985).