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Critical behavior of Mott variable-range hopping in Si:As near the metal-insulator transition

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dc conductivity measurements of barely insulating uncompensated Si:As samples yield results showing Mott variable-range-hopping behavior [$\ln\sigma \propto (T_0/T)^{1/4}$] in the temperature range $0.15 < T < 10$ K. The exponent $\frac{1}{4}$ has been confirmed using the logarithmic-derivative technique. The characteristic temperature T_0 shows scaling behavior of the form $T_0 \propto (1-N/N_c)^p$ with $2.3 < p < 2.9$ for $1-N/N_c < 0.07$ and $p \simeq 8.4$ for $0.1 < 1-N/N_c < 0.16$. Both the exponent $\frac{1}{4}$ in the Mott law and the localization length exponent $\nu = p/3$ derived from the scaling of T_0 suggest a filling-in of the Coulomb gap and a reduced role of electron-electron interactions in the temperature range of these measurements.

Considerable understanding of the scaling of electronic behavior near the metal-insulator (MI) transition has been gained from recent experimental studies of the dielectric susceptibility as the transition is approached from the insulating side[1,2] and of the dc conductivity, extrapolated to $T=0$, on the metallic side.[3,4] Both quantities show scaling behavior dependent on a localization (or correlation) length $\xi = \xi_0 \epsilon^{-\nu}$, where $\epsilon = |1-N/N_c|$, with N the donor concentration and N_c the critical concentration for the MI transition. In uncompensated Si:P and Si:As the critical exponent $\nu \sim \frac{1}{2}$ is observed on both sides of the transition in contrast to the value $\nu = 1$ predicted by the scaling theory of localization[5] without electron-electron (e-e) interactions.

We have made measurements of the temperature-dependent dc conductivity $\sigma(T,N)$, for $N < N_c$, which also exhibits scaling behavior. Mott[6] predicted that the conductivity would proceed by variable-range hopping (VRH) with a temperature dependence given by

$$\sigma(T) = \sigma_0 \exp[-(T_0/T)^m] \quad , \tag{1}$$

with $m = \frac{1}{4}$ in three dimensions. His model assumed a relatively constant density of states near the Fermi level and gave $T_0 = \beta/k_B N(E_F)\xi^3$ with k_B the Boltzmann constant, $N(E_F)$ the density of states at the Fermi level, and $\beta \simeq 18$ a numerical constant. Efros and Shklovskii[7] considered the role of e-e interactions neglected in Mott's model. Their calculation yields a result, $\sigma \propto \exp[-(T_0'/T)^{1/2}]$, based on a soft Coulomb gap in the $T=0$ density of states.

In compensated semiconductors there have been conflicting claims as to the value of the exponent m in the VRH law. Benzaquen and Walsh[8] have considered the situation and emphasized the need for accurate measurements to determine the exponent. Their data on n-type GaAs give $m \simeq \frac{1}{4}$ as does the earlier work of Allen and Adkins[9] on Ge:Sb. On the other hand, evidence that $m \simeq \frac{1}{2}$ in n-type GaAs was reported by Redfield[10] and in Ge:As by Ionov, Shlimak, and Matveev.[11] In uncompensated Si:P, Kobayashi and co-workers[12] found $\sigma(T)$ to fit Eq. (1) with $m = \frac{1}{4}$ of $2 \leq T \leq 10$ K, although they did not attempt to determine the exponent accurately. None of the above work quantitatively discussed scaling behavior near the MI transition. Hess, De Conde, Rosenbaum, and Thomas,[1] however, reported VRH and the scaling of T_0 as inferred from ac conductivity measurements at 400 MHz for several Si:P samples. The more extensive Si:As results presented here show a much stronger scaling of T_0.

We have made four-terminal measurements on uncompensated Si:As in the concentration range 7.30×10^{18} cm^{-3} $< N < 8.50\times10^{18}$ cm^{-3}. We used a series of bar-shaped samples cut from a single ingot. The samples were etched in a CP4 solution and $Au_{0.98}Sn_{0.02}$ wire-welded contacts were attached, as described previously.[13] The most accurate measurements were made in the temperature range $4.2\ \mathrm{K} > T > 0.5$ K in a ^{3}He refrigerator. Additional measurements have been made to lower temperatures in a dilution refrigerator and up to 77 K with a heater and calibrated Ge resistor. Currents were kept low so that the electric field in the samples was always small enough ($E/T < 3\times10^{-3}$ V/cm K) to assure there was no sample heating or non-Ohmic conductance.[14]

Donor concentrations were initially determined from four-point room-temperature resistivity measurements using the calibration based on neutron activation analysis by Newman and Holcomb[15] (NH). The ratio $r = \rho(4.2\ \mathrm{K})/\rho(298\ \mathrm{K})$ was then used to establish more accurately relative concentrations and to compare our samples with those of NH which are directly tied to the neutron activation analysis. Fixing our values for r vs N to the NH results in an overlapping concentration range resulted in a small systematic displacement. Consequently, our previously reported concentrations[13] and our earlier value of N_c are about 3.5% smaller than the corrected values in this work. If we use the r vs N curve (which neglects possible systematic shifts in the absolute concentration scale) sample concentrations are accurate to better than 0.5%.

Our results for $\sigma(T)$ are shown in Fig. 1 as a function of $T^{-1/4}$ for seven samples and show VRH conductivity below $T \simeq 10$ K. In addition, two of these, with $N = 7.57\times10^{18}$ and 8.48×10^{18} cm^{-3}, were measured to lower temperatures and, although there was less accuracy in the determination of the sample temperature, showed the VRH behavior continuing down to $T \sim 0.15$ K. Although Fig. 1 assumes the Mott form for $\sigma(T)$, a plot of $\ln\sigma(T)$ vs $T^{-1/2}$ does not fit the data nearly as well. We were able to establish the exponent directly by taking the data in groups of three points equally spaced in $1/T$, enabling experimental determination

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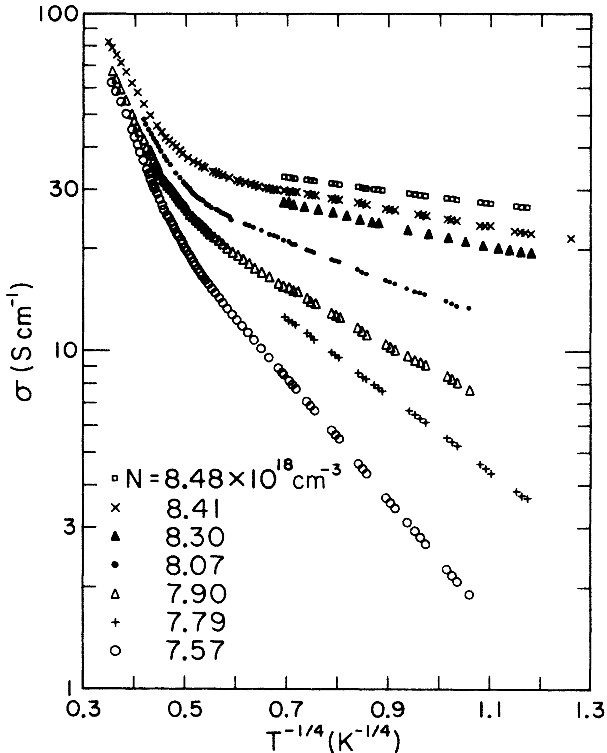

FIG. 1. The dc conductivity of Si:As vs $T^{-1/4}$ for six concentrations showing variable-range hopping for $0.37 < T < 10$ K.

of the quantity $\delta = -d\ln\sigma/d\ln(1/T)$. For

$$\sigma \propto (1/T)^s \exp[-(T_0/T)^m],\ \delta = m(T_0/T)^m - s\ .$$

The slope of δ vs $1/T$ on a log-log scale yields the exponent m. The simplest explanation of the data yields $m \sim \frac{1}{4}$ and s close to zero. The data for the most concentrated sample are fit equally well with $m = 0.18$ and $s = 0$, or with $m = 0.25$ and $s = -0.03$. An attempt to fit $\ln\delta$ vs $\ln(1/T)$ with $m = \frac{1}{2}$ requires a density-dependent $s(N)$ increasing from -0.42 at $N = 7.57\times10^{18}$ cm^{-3} to 0 as $N \rightarrow N_c$. Furthermore, the T_0 values obtained with $m = \frac{1}{2}$ (Coulomb-gap model) disagree by orders of magnitude with the theoretical expression[7] $T_0' = 2.8e^2/\kappa(N)\xi$ as $N \rightarrow N_c$ employing $\nu = \frac{1}{2}$.

Since Eq. (1) gives the best fit to the data, T_0 is determined from the slope of the linear region in Fig. 1 and is shown as a function of As concentration in the inset of Fig. 3. Within the narrow 15% change in concentration, T_0 decreases by almost five orders of magnitude. Two of the T_0 values shown are smaller than the temperature range of the measurements. It is important to note, however, that the Mott VRH distance is given by

$$R_{\rm hop} \sim 0.4\xi(T_0/T)^{1/4}\ .$$

For $\epsilon << 1$ one has $\xi >> d = n^{-1/3} \sim 4a_D^*$, where a_D^* is the donor Bohr radius. Thus, in the critical regime with $T_0/T << 1$, one has $R_{\rm hop} > d$ even though $R_{\rm hop} < \xi$. The critical regime is therefore different than the dilute regime, with $\xi < d$ where one requires $T_0/T >> 1$ to observe VRH conduction. Although the change in $\sigma(T)$ is small for these two samples with $T_0/T << 1$, the derivative plot in Fig. 2 confirms that the Mott law is the best fit. The magnitudes of $\sigma(T)$ reported here are consistent with those in Si:P in Kobayashi *et al.*[12] in the range $2\ {\rm K} < T < 10$ K. All of the samples shown in Fig. 1, however, are closer (smaller ϵ) to the MI transition than the closest insulating sample in Ref. 12, and therefore, the values of $\sigma(T)$ in Fig. 1 are larger. These data are also qualitatively similar to that of Hess *et al.*,[1] although those authors report a much weaker scaling of T_0 with ϵ.

Mott's expression for T_0 with the scaling form of the localization length gives

$$T_0 = [\beta/k_B N(E_F)\xi_0^3](1 - N/N_c)^{3\nu}\ , \tag{2}$$

which vanishes as $N \rightarrow N_c$. While this can be used to determine N_c, we obtained N_c independently from the scaling of $\sigma(N > N_c, T = 0)$. We used the data of Newman and Holcomb[4] along with additional values we obtained from the $T = 0$ intercepts of $\ln\sigma$ vs $T^{1/2}$ for samples measured to lower temperatures and closer to N_c than in Ref. 4. These results give

$$N_c = (8.55 \pm 0.10)\times10^{18}\ {\rm cm}^{-3}\ .$$

The scaling behavior of T_0 is indicated in Fig. 3, where T_0 is shown on a log-log scale versus ϵ. Two sets of points are shown with $N_c = 8.55\times10^{18}$ cm^{-3} and with $N_c = 8.60\times10^{18}$ cm^{-3}, with the latter giving the best fit for the samples closest to N_c. In each case, $T_0(\epsilon)$ shows an abrupt change to a steeper slope for $\epsilon > 0.07$. For $\epsilon < 0.07$, T_0 exhibits scaling behavior, but the slope, and therefore the exponent ν, depends strongly on the value of N_c. From the two fits shown, we find ν in the range 0.77–0.97. This exponent is larger than previous results[1-4] in Si:P and Si:As and is closer to the localization prediction $\nu = 1$. We speculate that this may be due to the higher-temperature range of these experiments at which electrons will be excited to states above the Fermi level, but below the mobility edge. This thermal ex-

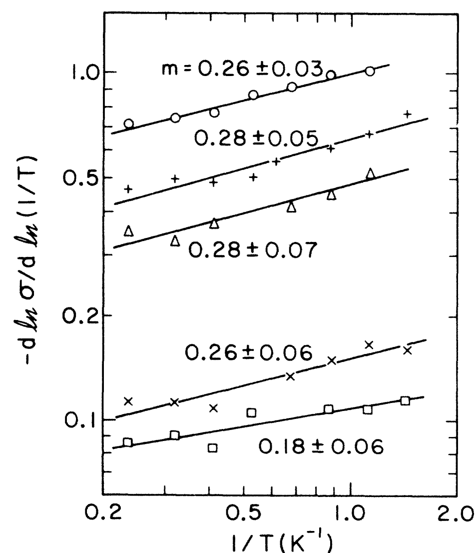

FIG. 2. The derivative $-d\ln\sigma/d\ln(1/T)$ with the slope giving a direct measure of the exponent m in the variable-range-hopping law. The lines shown are the best fit to the data with $s = 0$ and the errors quoted are two standard deviations. The symbols correspond to the same samples as in Fig. 1.

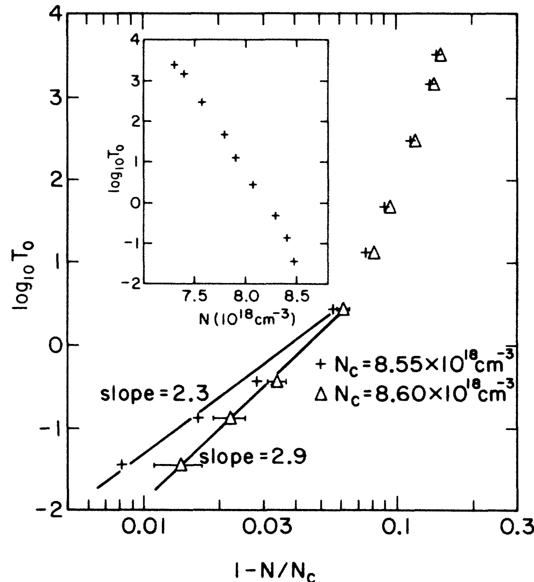

FIG. 3. $\log_{10} T_0$ vs $\log_{10}(1-N/N_c)$ for two values of N_c (T_0 in K). T_0 shows scaling behavior within 7% of the MI transition and changes abruptly to a much steeper dependence above 7%. The inset shows T_0 decreasing by five orders of magnitude with a 15% change in N.

citation produces empty sites in the immediate vicinity of the Fermi level. The situation is then analogous to that in a compensated system as studied by Thomas *et al.*[16] They found that compensation in Ge:Sb increased ν, and that 20% compensation was sufficient to change ν from $\frac{1}{2}$ to ~ 1. They suggested that this increase might be due to the increased number of random scattering centers in the system, reducing the relative role of e-e interactions. This suggestion is supported in our higher-temperature case by Davies, Lee, and Rice.[17] Their numerical studies showed the Coulomb-gap filling in at finite temperatures, indicating a reduced role of Coloumb interactions.

To check quantitative agreement of our results with Eq. (2), we use it to derive a value of $N(E_F)$. Using $\xi_0 = 15.45$ Å, the donor Bohr radius for As, $\beta = 18$ and $N_c = 8.60\times10^{18}$ cm^{-3}, yields $N(E_F) = 6.6\times10^{21}$ $\mathrm{eV}^{-1}\mathrm{cm}^{-3}$, a factor of 9 larger than the free-electron value of $N(E_F)$ at N_c. On the other hand, Thomas, Ootuka, Kobayashi, and Sasaki[18] employed specific-heat measurements to determine $N(E_F)$ for Si:P and obtained a value very close to the free-electron value at N_c. We offer no explanation for this difference, although a smaller value of $N(E_F)$ would be obtained with either a smaller β or a larger prefactor ξ_0.

For $\epsilon > 0.07$, T_0 changes abruptly to a much stronger dependence on ϵ. The slope in Fig. 3 is ~ 8.4 for the most dilute samples. A possible explanation for this is a rapid decrease in $N(E_F)$, although it would have to be reduced by a surprisingly large factor of about 70 to explain the increase in T_0 when $\epsilon = 0.15$.

In conclusion, we have observed Mott VRH conductivity in Si:As with an exponent $m \simeq \frac{1}{4}$. This indicates a lack of importance of e-e interactions in agreement with the Coulomb-gap filling in at finite temperatures. The parameter T_0 decreases rapidly as $N \rightarrow N_c$ and exhibits scaling behavior. The exponent ν of the localization length within 7% of N_c is closer to 1, as predicted by the localization theory with no interactions, than $T=0$ scaling results in Si:As (Refs. 2 and 4) and Si:P.[1,3] Further from the transition T_0 changes abruptly to a much stronger dependence on $1-N/N_c$ which may result from a sharply decreasing density of states.

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