Penetration depth of V_3Si

J. W. Blezius and J. P. Carbotte
Physics Department, McMaster University, Hamilton, Ontario, Canada L8S 4M1
(Received 23 October 1985)

The London-limit penetration depth, $\lambda_L(T)$, has now been calculated as a function of temperature, T, for V_3Si using two recent tunneling-derived spectra of the electron-phonon interaction. Significantly different predictions for $\lambda_L(T)/\lambda_L(0)$ resulted from the two spectra. Comparison with recent experimental data revealed good agreement in one case, but not in the other.

The penetration depth of V_3Si has recently been measured.[1] The London limit applies to the sample considered, and the temperature dependence of the penetration depth was found to deviate significantly from Bardeen-Cooper-Schrieffer (BCS) behavior. This is revealed in a plot of $\lambda_L^2(0)/\lambda_L^2(T)-(1-t^4)$ vs T. In this expression, $\lambda_L(T)$ is the London-limit penetration depth, $1-t^4$ is the prediction of the two-fluid model for $\lambda_L^2(0)/\lambda_L^2(T)$, and t is the reduced temperature, T/T_c, where T_c is the critical temperature. A good fit to the data can be achieved by introducing a phenomenological correction for strong coupling based on a value of 3.88 for the ratio $2\Delta_0/k_BT_c$, where Δ_0 is the gap energy.

Two recent tunneling studies[2,3] on V_3Si have been analyzed to obtain the electron-phonon spectral density, $\alpha^2F(\omega)$. Two very different results are obtained. In one case,[3] the electron-phonon mass renormalization parameter, λ, derived from $\alpha^2F(\omega)$ is found to be about 1 with a corresponding conventional value for the Coulomb repulsion parameter, μ^*; in the other case,[2] λ is greater than 2, and μ^* is anomalously large with a value much greater than 1 which does not seem to be physically reasonable.

In this Brief Report, the results are reported for detailed strong-coupling calculations of the London-limit penetration depth of V_3Si, based on the two tunneling-derived $\alpha^2F(\omega)$ spectra, and are compared with the results of the experiment. Convenient theoretical expressions for the penetration depth in the London limit, as well as for the coherence distance, are found in the paper of Lemberger, Ginsberg, and Rickayzen.[4] While these authors work in the BCS limit, the same prescriptions also apply in strong-coupling theory, as can be seen from the paper by Nam.[5] The only modification is that the full Eliashberg strong-coupling equations are used, rather than their weak-coupling limits,[5-7] to obtain the values of the Matsubara gaps. According to Lemberger *et al.*,[4] the normalized, local-limit penetration depth, λ_l, is given by

$$\frac{\lambda_l(T)}{\lambda_l(0)}=\lim_{T\to 0}\left[T\sum_{n\geq 1}\frac{\tilde{\Delta}_n^2}{\tilde{\omega}_n^2+\tilde{\Delta}_n^2}\right]\Bigg/T\sum_{n\geq 1}\frac{\tilde{\Delta}_n^2}{\tilde{\omega}_n^2+\tilde{\Delta}_n^2}\ . \quad (1)$$

In Eq. (1), the $\tilde{\Delta}_n$ are the Matsubara gaps and the $\tilde{\omega}_n$ are the renormalized frequencies satisfying the Eliashberg equations:[6,8,9]

$$\tilde{\Delta}_n=\pi T\sum_m[\lambda(m-n)-\mu^*]\frac{\tilde{\Delta}_m}{(\tilde{\omega}_m^2+\tilde{\Delta}_m^2)^{1/2}}+\frac{1}{2\tau}\frac{\tilde{\Delta}_n}{(\tilde{\omega}_n^2+\tilde{\Delta}_n^2)^{1/2}}, \quad (2)$$

$$\tilde{\omega}_n=\omega_n+\pi T\sum_m\lambda(m-n)\frac{\tilde{\omega}_m}{(\tilde{\omega}_m^2+\tilde{\Delta}_m^2)^{1/2}}+\frac{1}{2\tau}\frac{\tilde{\omega}_n}{(\tilde{\omega}_n^2+\tilde{\Delta}_n^2)^{1/2}}, \quad (3)$$

where

$$\omega_n=\pi T(2n-1);\ n\in I\ , \quad (4)$$

$$\lambda(m-n)=2\int\frac{\Omega\alpha^2F(\Omega)d\Omega}{\Omega^2+(\omega_n-\omega_m)^2}\ . \quad (5)$$

τ is the impurity scattering time, μ^* is the Coulomb repulsion parameter, and $\alpha^2F(\Omega)$ is the electron-phonon spectral density as a function of phonon energy, Ω. $\alpha^2F(\Omega)$ is taken directly from the experiment. Since the present calculation is done on the imaginary-frequency axis and since tunneling inversion is done on the real axis, μ^* is treated as an adjustable parameter which is chosen so as to give the exact T_c as described by Mitrović and Carbotte.[10] It should be noted that the gap edge, Δ_0, is then completely determined from these calculations.

The local-limit penetration depth applies when the mean free path, l, is much smaller than the zero-temperature coherence length, $\xi(0)$. According to Lemberger *et al.*,[4] the London-limit penetration depth, which applies for $\lambda >> \xi$, is related to λ_l by

$$\frac{\lambda_L^2(T)}{\lambda_L^2(0)}=\frac{\xi(0)}{\xi(T)}\frac{\lambda_l^2(T)}{\lambda_l^2(0)}\ , \quad (6)$$

where the coherence distance is

$$\xi=\frac{\hbar v_F}{2}\left[\sum_{n\geq 1}\frac{\tilde{\Delta}_n^2}{(\tilde{\omega}_n^2+\tilde{\Delta}_n^2)^{3/2}}\Bigg/\sum_{n\geq 1}\frac{\tilde{\Delta}_n^2}{\tilde{\omega}_n^2+\tilde{\Delta}_n^2}\right]\ , \quad (7)$$

v_F is the Fermi velocity, and $\hbar$ is Planck's constant over 2π.

Equations (2) and (3) have been solved for the $\alpha^2F(\Omega)$, which was determined by Kihlstrom[3] and also for that of Bangert *et al.*[2] From the solutions, the coherence length [Eq. (7)] and the local-limit penetration depth [Eq. (1)] have been computed. In each case, T_c has been taken from the tunneling experiment. This is somewhat different from the value of Christen *et al.*[1] Some of the characteristic properties of these solutions can be found in a paper by Mitrović and Carbotte[10] and are not repeated here.

Figure 1 shows the temperature variation of the normalized coherence length, $\xi(T)/\xi(0)$, based on the spectrum of Kihlstrom.[3] The solid curve is for infinite mean free path, l, while the dotted curve is for $l=28$ nm, a value that will be introduced later as being realistic for the sample which was analyzed by Christen *et al.*[1] It should be noted that as the temperature reaches T_c in the pure case, $\xi(T)/\xi(0)$ drops to a value of about 0.80, which is larger than the BCS value of 0.76. Of course, one should not compare directly with BCS theory, since the calculation at hand gives $2\Delta_0/k_BT_c=3.88$. For this value of the ratio, the

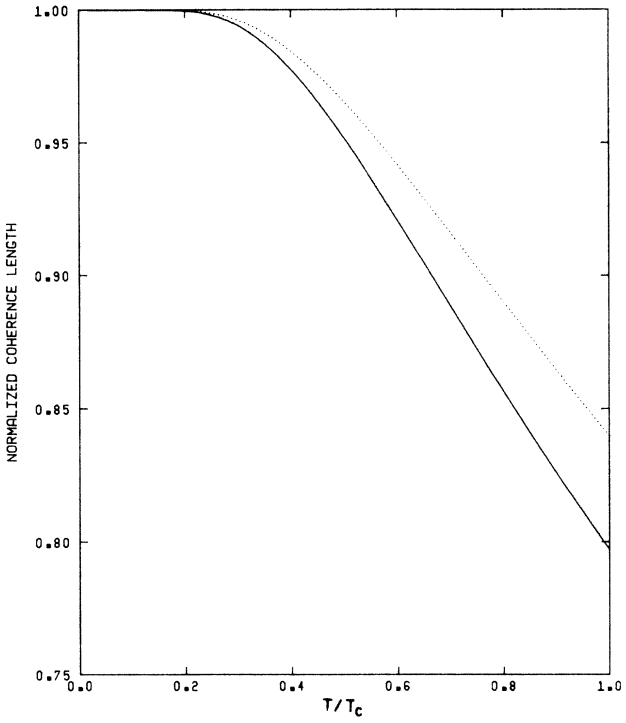

FIG. 1. The coherence length normalized with respect to its zero-temperature value as a function of temperature normalized with respect to the superconducting critical temperature. The material is V_3Si. The solid curve is the pure case ($l=\infty$); the dotted curve is the slightly dirty case ($l=28.0$ nm).

semiempirical (scaling) calculation of Ginsberg[11] predicts a drop in the value of $\xi(T)/\xi(0)$ at T_c to 0.84. The present strong-coupling value is between this value of Ginsberg[11] and the BCS value. That the strong-coupling result for $\xi(T)/\xi(0)$ falls with temperature faster than the scaling result has already been noted by Kerchner and Ginsberg[7] for other materials.

To obtain the absolute magnitude of the zero-temperature coherence distance, $\xi(0)$, a value for the bare Fermi velocity is needed. The value of 1.31×10^7 cm/sec which was quoted in the report of Orlando, McNiff, Foner, and Beasley[12] was used in the present calculations. These authors[12] take care to treat fully the mass renormalization factors of $(1+\lambda)$, which are ignored by many other authors. This value of v_F is in agreement with that quoted by Muto, Toyota, Noto, Kutsu, Isino, and Fukase,[13] once care is taken to extract the renormalization, $(1+\lambda)$, from their quoted value. This value of v_F gives $\xi(0)=6.40$ nm as compared with the value of 5.6 nm quoted by Orlando *et al.*,[12] and with 4.9 nm by Muto *et al.*[13] The agreement is even more reasonable when one considers that a correction for impurity content should be introduced before a comparison is made. A mean free path of 28.0 nm is assumed and the present calculation yields a value of 5.11 nm for the impure $\xi(0)$. This value, which is obtained by direct evaluation of Eq. (7), obeys, to within 2%, the simple rule

$$\frac{1}{\xi(0,l)}=\frac{1}{\xi(0,\infty)}+\frac{1}{l}\ . \tag{8}$$

As a test of the calculated value of $\xi(0)$, a v_F-independent ratio can be calculated, following Kerchner and Ginsberg:[7]

$$\xi(0,\infty)\Big/\frac{\hbar v_F}{\pi\Delta_0(1+\lambda)}\ ,$$

where the denominator is an approximate expression for the coherence length, which includes some electron-phonon renormalization through the $1+\lambda$ factor. The resulting value is 1.14, which is intermediate between the BCS value of 1 and the value of 1.28 for Pb quoted by Kerchner and Ginsberg.[7] This is reasonable since $2\Delta_0/k_BT_c$ is 4.55 for Pb, much larger than the value for V_3Si.

The results for the temperature variation of the normalized London-limit penetration depth, $\lambda_L(T)/\lambda_L(0)$, will now be described. In addition to $\xi(T)/\xi(0)$, a knowledge is required of the local-limit penetration depth as given by Eq. (1). This factor is independent of the impurity concentration. As is done by Christen *et al.*, $[\lambda_L(0)/\lambda_L(T)]^2-(1-t^4)$ is plotted in Fig. 2. The dot-dashed curve is based on the spectrum of Kihlstrom.[3] For comparison, the experimental points of Christen *et al.*[1] are reproduced in the figure. The agreement observed is reasonable, particularly when it is remembered that a correction to the theoretical curve is needed for impurity content. This correction will push the curve up somewhat, as will be described presently. It is noted that the present calculations are for samples with T_c values which are slightly different from the experimental results of Christen *et al.*[1] This is expected not to make a significant difference, however.

The solid curve will be briefly considered since it serves as a test of the numerical work. It was obtained from a full strong-coupling calculation based on the spectrum of Al.[14] It falls almost exactly on the weak-coupling curve given in

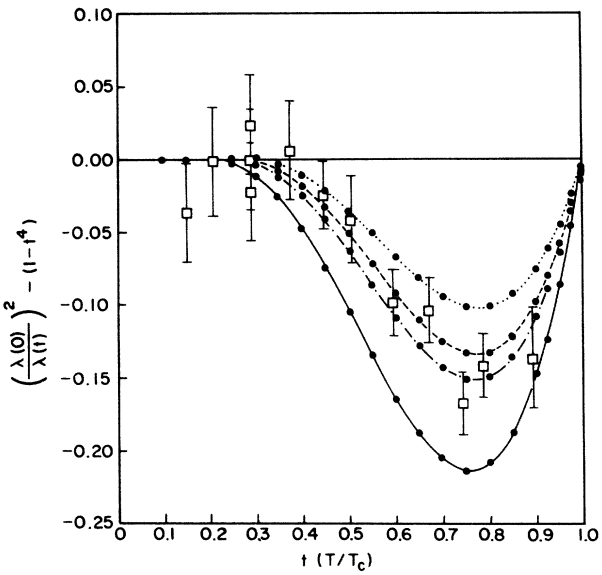

FIG. 2. The strong-coupling difference from the two-fluid prediction of the inverse square of the London-limit penetration depth normalized to its zero-temperature value as a function of temperature normalized with respect to the superconducting critical temperature. The solid curve is both Al and the BCS prediction; the dot-dashed curve is pure Kihlstrom *et al.*[3] V_3Si; the short-dashed curve is Kihlstrom *et al.*[3] V_3Si with impurities in the amount prescribed by Christen *et al.*;[1] the dotted curve is pure Bangert *et al.*[2] V_3Si. The data with error bars are those of Christen *et al.*[1]

Fig. 2 of Christen *et al.*[1] This is considered to be an excellent check on the numerical accuracy.

The dotted line in Fig. 2 is the results based on the spectrum of Bangert *et al.*[2] with the unusually large μ^*. The value of μ^* was 1.9 in the current calculation. The poor agreement with the experimental data should be noted. If a correction were applied for impurity content, the agreement would become even worse.

The effects of impurities on the $[\lambda_L(0)/\lambda_L(T)]^2 - (1-t^4)$ curve will now be discussed. A mean free path of 28.0 nm has been estimated from data which were provided by Christen *et al.*:[1] They quote $l=(0.882/0.2)\xi(0)$. If the numerically obtained value of $\xi(0)$ is used, this formula of Christen *et al.*[1] yields $l=28.0$ nm, and hence an inverse scattering time, $1/\tau=0.485$ meV. This value is needed in Eqs. (2) and (3). With this correction, the dashed curve of Fig. 2 shows satisfactory agreement with the experiment. A correction of the same magnitude to the Bangert *et al.*[2] curve (dotted line) would yield a curve which deviates greatly from the experimental data.

Finally, the strong-coupling corrections to the BCS value of the zero-temperature, London penetration depth are compared. In absolute terms

$$\lambda_L(0)=\left[\frac{mc^2}{4\pi ne^2}\right]^{1/2}\left[2\pi T\sum_{n\geq 1}\frac{\tilde{\Delta}_n^2}{(\tilde{\omega}_n^2+\tilde{\Delta}_n^2)^{3/2}}\right]^{1/2}, \tag{9}$$

where m is the electron mass, c the speed of light, e the electron charge, and n the free-electron density. The value of the second quantity is 1 in BCS theory and the prefactors are the same in strong-coupling and BCS theories. When the electron-phonon renormalization is taken into account in the $\tilde{\omega}_n$ channel, a factor of $(1+\lambda)$ naturally arises. Thus, it is best to quote

$$\left[(1+\lambda)2\pi T\sum_{n\geq 1}\frac{\tilde{\Delta}_n^2}{(\tilde{\omega}_n^2+\tilde{\Delta}_n^2)^{3/2}}\right]^{1/2} \tag{10}$$

as the strong-coupling correction and it should be near 1. For Al it was found to be 1.006, and for Kihlstrom[3] V_3Si it was 0.998, with $l=\infty$ in both cases. The effect, therefore, is small. This is expected, since Kerchner and Ginsberg[7] found 2% effects in the case of Pb which is coupled more strongly than V_3Si. When the mean free path of $l=28.0$ nm is accounted for, expression (10) gives a value of 0.892 for Kihlstrom[3] V_3Si, reflecting the change in coherence length at zero temperature.

In summary, the penetration depth indicates that the Kihlstrom[3] spectrum for the electron-phonon interaction gives good agreement with the experiment, whereas the Bangert *et al.*[2] spectrum does not. This is consistent with the anomalous values of λ and μ^* for the latter.

The authors wish to thank J. M. Coombes for solving the Eliashberg equations for the spectrum of Bangert *et al.* This research was supported by the Natural Sciences and Engineering Research Council of Canada.

[1]D. K. Christen, H. R. Kerchner, S. T. Seluka, and Y. K. Chang, in *Proceedings of the Seventeenth International Conference on Low Temperature Physics,* edited by U. Eckern, A. Schmid, W. Weber, and H. Wühl (North-Holland, Amsterdam, 1984).

[2]W. Bangert, J. Geerk, and P. Schweiss, Phys. Rev. B **31**, 6066 (1985).

[3]K. E. Kihlstrom, Phys. Rev. B **32**, 2891 (1985).

[4]T. R. Lemberger, D. M. Ginsberg, and G. Rickayzen, Phys. Rev. B **18**, 6057 (1978).

[5]S. B. Nam, Phys. Rev. **156**, 470 (1967); **156**, 487 (1967).

[6]P. D. Scholten, J. D. Lejeune, W. M. Saslow, and D. G. Naugle, Phys. Rev. B **16**, 1068 (1977).

[7]H. R. Kerchner and D. M. Ginsberg, Phys. Rev. B **8**, 3190 (1973).

[8]D. Rainer and G. Bergmann, J. Low Temp. Phys. **14**, 501 (1974).

[9]J. M. Daams and J. P. Carbotte, J. Low Temp. Phys. **43**, 263 (1981).

[10]B. Mitrović and J. P. Carbotte, Phys. Rev. B **33**, 591 (1986).

[11]D. M. Ginsberg, Phys. Rev. B **7**, 146 (1973).

[12]T. P. Orlando, E. J. McNiff, Jr., S. Foner, and M. R. Beasley, Phys. Rev. B **19**, 4545 (1979).

[13]Y. Muto, N. Toyota, K. Noto, K. A. Kutsu, M. Isino, and T. Fukase, J. Low Temp. Phys. **34**, 617 (1979).

[14]H. K. Leung, J. P. Carbotte, and C. R. Leavens, J. Low Temp. Phys. **24**, 25 (1976).