Critical behavior of disordered degenerate semiconductors. I. Models, symmetries, and formalism

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A model that describes the qualitative properties of the electronic states of a disordered degenerate semiconductor with a finite number of degeneracy points is proposed. I introduce an effective Hamiltonian of the form of a Dirac operator coupled to randomly distributed fields. It is shown that there is a phase transition between the semimetal and metallic phases followed by a localization transition. The symmetry breaking associated with this transition is related to the nonsymmorphic character of the space group. The density of states plays the role of the order parameter and the elastic mean free path is the correlation length. A path-integral representation is introduced and used to characterize the universality class of this transition. The lower critical dimension is 2. A mapping of the two-dimensional case to one-dimensional self-interacting Fermi systems is presented. Applications to zero-gap semiconductors and other systems are discussed.

I. INTRODUCTION

This is the first of a series of papers in which I present a study of the effects of disorder on the spectral and transport properties of degenerate semiconductors. A brief account of ideas and results was presented elsewhere.[1,2] Degenerate (or zero-gap) semiconductors are systems whose valence and conduction bands are degenerate at a finite number of points.[3] Examples are HgTe (and other III-V systems) and SnTe (and other IV-VI systems) alloys, and thus systems with significant randomness, have also been studied, in particular $Cd_xHg_{1-x}Te$ and $Pb_xSn_{1-x}Te$. The semimetal-semiconductor transition observed in these alloys is related to the problems studied in this paper. Still other more "academic" examples exist like "two-dimensional graphite,"[4] and two-dimensional tight-binding systems in a magnetic field with one-half of quantum of flux per plaquette (square lattice).[5] A possible physical realization of the latter could be an array of tunneling junctions in a magnetic field with one-half of the quantum of flux per plaquette (for regular lattices such magnetic fields are prohibitively high).

Disordered degenerate semiconductors have a number of properties that make them different from other disordered electronic systems. In a dirty metal, the average density of states (DOS) is nonzero and is a smooth function of the impurity density. The elastic mean free path is also a finite slowly varying function of the impurity density and typically very small, of the order of atomic spacing. By contrast degenerate semiconductors are semimetals in their pure state. Since the bands are degenerate at points, the energy-momentum curves are linear near degeneracy points.[6] As a result the DOS varies like ϵ^{d-1} ($\epsilon = E - E_{\rm deg}$ and d is the dimension) and thus it is very small. The elastic mean free path can be very large. In typical systems the Fermi energy is exactly at the degeneracy energy $E_{\rm deg}$ which I will set to zero hereafter. From localization theory one expects localized states to occur at band edges or generally where the DOS is small.[7,8] A naive argument would suggest that disorder might continuously draw states from energies where the DOS is large to where it is small. The previous argument might suggest that they are localized states. In this paper I give arguments that indicate that in three dimensions these systems are semimetals up to a critical impurity density at which the DOS increases as a power law above the transition. The elastic mean free path is found to diverge at the transition.

The origin of the semimetal-metal transition is a spontaneous breakdown of the (discrete) sublattice symmetries these systems possess. Typically[3] they contain more than one atom in their unit cell and their associated space groups are nonsymmorphic, i.e., the point group is not a subgroup of the space group. As a result a translation connecting two atoms inside the unit cell becomes a symmetry operation if combined with either a reflection or rotation (i.e., screw axes and gliding planes). Such crystalline symmetries cannot survive in a dirty system. However, I show that if the probability distribution of the impurities has a certain symmetry, ensemble-averaged quantities may not break the sublattice symmetries unless the impurity concentration exceeds a critical value (in three dimensions).

It is also possible to present an alternative view of the physics of these systems by regarding one of the directions as imaginary time and performing analytic continuation to "real time." The result is a $(d-1)$-dimensional self-interacting Fermi-field theory in Minkowski space. Much is known about such theories in $1+1$ dimensions, particularly about their spectrum. This suggests that a version of the models presented in this paper should be exactly solvable.

In this paper I present the models and their symmetries. A detailed analysis of their critical properties is presented in the following paper.[9] In Sec. II the models are presented. It is shown that this problem is equivalent to the study of the spectral and transport properties of the Dirac

operator.[10] In Sec. III symmetries and symmetry breaking is discussed. In Sec. IV a path-integral representation of the Green's functions is presented. I show that temperature plays the role of a symmetry-breaking field. I present arguments that show that two is the lower critical dimension. In Sec. V the mapping to $(d-1)$-dimensional self-interacting Fermi fields is presented. Conclusions and possible experimental realizations are discussed in Sec. VI.

II. THE MODELS

We are interested in developing an effective Hamiltonian for states near the degeneracy points. I will assume there are N_c such points. Typical values are $N_c=1$ for HgTe (degenerate at the Γ point)[11] if spin-dependent scattering is neglected and $N_c=2$ if spin scattering is involved. For SnTe we have $N_c=4$ (degenerate at the L points like other IV-VI systems)[12] and $N_c=8$ if spin-dependent scattering is included. I will not consider effects of spin-orbit scattering (although it may be important). Electron-electron and electron-phonon interactions are not included; their effect may not be important if the coupling constants are weak. Of course they could (and in many cases they do) trigger instabilities which would alter the conclusions of this theory. A detailed model of the pure systems was presented by Kane several years ago.[6] In this paper I keep only those states with momenta close to the degeneracy points and energies close to the Fermi energy which happens to lie exactly at the degeneracy energy $E_{\rm deg}$ (which is set to zero). Two quantum numbers are needed to label the states $|\alpha a\rangle$, where $\alpha=1,2$ labels conduction- and valence-band states (i.e., $\epsilon>0$ and $\epsilon<0$) and $a=1,\ldots,N_c$ labels the various degeneracy points.[13] Momentum is measured relative to the respective degeneracy points. After appropriately shifting the momenta and setting the energy scale so that the Fermi velocity equals one, the pure-system effective Hamiltonian takes the Dirac form

$$H_0=i\boldsymbol{\alpha}\cdot\nabla\ , \tag{2.1}$$

where the 2×2 Dirac matrices are just the three Pauli matrices (in three dimensions). In two dimensions (i.e., superlattices) I take α_x and α_y to equal σ_x and σ_y. To make the notation simpler the coordinates x_i are labeled from 1 to d (the dimension). Thus we have $\alpha_i=\sigma_i$.

The effects of disorder are included simply by considering all possible scattering processes that may mix both conduction and valence states and different degeneracy points. A simple symmetry argument indicates that the most general form the scattering potentials may take yields a Hamiltonian of the form

$$H=i\boldsymbol{\alpha}\cdot\nabla+a(\mathbf{x})+\boldsymbol{\alpha}\cdot\mathbf{a}_0(\mathbf{x})+a_{0a}(\mathbf{x})T_a+\boldsymbol{\alpha}\cdot\mathbf{a}_a(\mathbf{x})T_a \tag{2.2a}$$

in three dimensions, and

$$H=i\boldsymbol{\alpha}\cdot\nabla+a(\mathbf{x})+\boldsymbol{\alpha}\cdot\mathbf{a}_0+a_{0a}(\mathbf{x})T_a+\mathbf{a}_a(\mathbf{x})\cdot\boldsymbol{\alpha}T_a+b(\mathbf{x})\beta+b_a(\mathbf{x})\beta T_a \tag{2.2b}$$

in two dimensions. Here $\beta=\sigma_3$ (this choice is arbitrary). In Eqs. (2.2a) and (2.2b) I introduced the set of N_c^2-1 $N_c\times N_c$ matrices T_a, which are the generators of the group SU(N_c). The interpretation of the various terms is very simple. Consider, for instance, Eq. (2.2a). Scattering processes in which states are not mixed are represented by the (random) amplitude $a(\mathbf{x})$. All terms that contain α matrices indicate that conduction- and valence-band states may be mixed while terms with T matrices may mix states of different degeneracy points. A model of this sort can be derived from a tight-binding model as in the example of Fisher and Fradkin.[5]

The Hamiltonians of Eqs. (2.2a) and (2.2b) were derived under the assumption that the lattice disorder has a white-noise character for the entire range of Fourier components. However, if the disorder is only appreciably random on scales larger than typical separations between atoms of two different sublattices, it will not induce scattering matrix elements connecting different degeneracy points. In that case, terms including T matrices should not be included. I show in the following paper that this affects significantly the nature of the phase transition in two dimensions. Thus as the coherence length of the random potential is varied, a crossover between $N_c=1$ and $N_c\neq1$ behavior should be observed.

The problem thus reduces to finding the spectral properties of the Dirac operator in the presence of random fields. The random fields represent the influence of the randomly distributed impurities or, in tight-binding picture, random site and bond potentials. They are assumed to be independently distributed unbiased (i.e., zero mean) Gaussian variables (i.e., white noise). For the sake of simplicity I will initially assume that they all have the same variance. However, this is by no means necessary. As a matter of fact this choice is not invariant under the renormalization group (RG) and some of the widths (or variances) grow (or decrease) under the RG flow. I argue below that non-Gaussian probability distributions, which differ in the higher moments, do not affect the critical properties (at least near two dimensions) since they contribute only to irrelevant operators. Thus non-Gaussian pieces of the distribution will affect the exact value of the critical impurity density but not the exponents. It should be stressed that corrections to the linear terms in the dispersion curves will produce a similar effect. This is why a precise prediction of the critical impurity density requires a careful numerical calculation that would be strongly affected by the microscopic details of the band structure. The Hamiltonians of Eqs. (2.2a) and (2.2b) are Hermitian but generally cannot be made real symmetric. The underlying lattice Hamiltonians thus contain scattering amplitudes that are complex (or involve spin) and hence violate (locally) time-reversal invariance. The result is that these Hamiltonians are invariant under unitary SU(N_c) transformations. If only real scattering amplitudes are allowed the resulting Hamiltonian can be made real symmetric and the symmetries are orthogonal [O($2N_c$) on this case]. If spin-orbit scattering is included the symmetry is expected to be simplectic.[14]

III. SYMMETRIES AND SYMMETRY BREAKING

The Hamiltonians, Eqs. (2.2a) and (2.2b) have a number of symmetries which can be regarded as coarse-grained

versions of the underlying lattice symmetries. It was mentioned before that it is characteristic of these systems that their space groups are nonsymmorphic. Thus a translation connecting two sites of the same unit cell becomes a symmetry operation if combined with a rotation or a reflection depending on the case. The point here is that the pure-system Hamiltonian is *odd* under such symmetry operations if it is symmetric under particle-hole symmetry. If it is not exactly invariant under a particle-hole transformation, this is only an approximate symmetry of the vicinity of the degeneracy points. The operators that break symmetry can be shown to be irrelevant in the RG sense since they contain higher powers of the gradient operator.[15] Such irrelevant operators can affect the physics away from the transition but their effect switches off very rapidly as the transition is approached.

Consider first the two-dimensional case. It can be shown by carefully deriving the Hamiltonian from a tight-binding model that the relevant symmetry operation involves multiplying the states by one of the Pauli matrices, say β, so that states associated with different sublattice sites (or bands) transform differently.[15] For example let

$$|\alpha'a'\rangle=\beta|\alpha a\rangle \tag{3.1}$$

at all points of space. The Hamiltonian in the new basis is

$$H'[a,\mathbf{a}_0,a_{0a},\mathbf{a}_a,b,b_a] =\beta H\beta=-H[-a,a_0,-a_{0a},\mathbf{a}_a,-b,-b_a]\ . \tag{3.2}$$

The trace of the one-particle Green's function is

$$\mathrm{tr}G_E(\mathbf{x},\mathbf{x}')_{f,g}=\sum_{\alpha,a}\left\langle \alpha a\mathbf{x}\left|\frac{1}{\hat{H}_{f,g}-E}\right|\alpha a\mathbf{x}'\right\rangle , \tag{3.3}$$

where f denotes the random fields associated with operators even under β and g with operators odd under β, transforms like

$$\mathrm{tr}G_E(\mathbf{x},\mathbf{x}')_{f,g}=-\mathrm{tr}G_{-E}(\mathbf{x},\mathbf{x}')_{-f,g}\ . \tag{3.4}$$

If I now assume that the probability distribution of the random fields f is even under $f\to -f$ (i.e., has zero mean) then the ensemble-averaged Green's function obeys

$$\langle \mathrm{tr}G_E(\mathbf{x},\mathbf{x})\rangle=-\langle \mathrm{tr}G_{-E}(\mathbf{x},\mathbf{x})\rangle\ , \tag{3.5}$$

unless the symmetry is spontaneously broken. In particular, Eq. (3.5) implies that the DOS at the Fermi energy

$$N_0(0)=\frac{1}{\pi}\mathrm{Im}\langle \mathrm{tr}G_{E=0}(\mathbf{x},\mathbf{x})\rangle \tag{3.6}$$

must be zero if the symmetry is not broken. It is also absolutely clear that if the system is sufficiently disordered it cannot keep track of the different sublattices and thus the symmetry cannot be respected.

From this argument we see that the DOS at zero energy plays the role of the order parameter of this transition. The correlation length is the distance over which fluctuations of the density of states are correlated. In the sequel I will show that the distance is proportional to the phase-coherence length [i.e., the distance over which $\langle G_E(\mathbf{x},\mathbf{x}')\rangle$ decays], which is just the elastic mean free path.

In three dimensions and analogous argument can be made. The only difference is that there is no analog of the matrix β. Thus one uses any of the α matrices (say σ_3) and repeats the same argument. The caveat here is that this is not a symmetry of H unless it is followed by a reflection in the same coordinate $x_3\to -x_3$). Hence in three dimensions parity operations are intrinsically necessary.

A similar phenomenon is known to exist in quantum-field theory. In that framework the symmetry operations described above are related to chiral symmetry and its breakdown. Recently this has been the subject of intensive investigations.[10] This analogy will be used in Sec. IV.

IV. PATH-INTEGRAL AND LOWER CRITICAL DIMENSION

We are interested in calculating the ensemble-averaged Green's functions of this problem. The (unaveraged) one-particle Green's function (or resolvent) at energy E is

$$G_E(\mathbf{x}a\alpha\,|\,\mathbf{x}'a'\alpha')=\left\langle \mathbf{x}a\alpha\left|\frac{1}{\hat{H}-E+i\epsilon}\right|\mathbf{x}'a'\alpha'\right\rangle , \tag{4.1}$$

where $\epsilon>0$ (and small). It turns out to be more convenient to work with the anti-Hermitian operator $\hat{\mathcal{D}}=i\hat{H}$, so that

$$G_E(\mathbf{x}a\alpha\,|\,x'a'\alpha')=i\left\langle \mathbf{x}a\alpha\left|\frac{1}{\hat{\mathcal{D}}-iE-\epsilon}\right|\mathbf{x}'a'\alpha'\right\rangle . \tag{4.2}$$

Thus

$$N_0(E,\mathbf{x})=+\frac{1}{\pi}\mathrm{Im}\,\mathrm{tr}G_E(\mathbf{x},\mathbf{x}) =\frac{1}{\pi}\mathrm{Re}\sum_{\alpha,a}\left\langle \mathbf{x}a\alpha\left|\frac{1}{\hat{\mathcal{D}}-iE-\epsilon}\right|\mathbf{x}'a\alpha\right\rangle . \tag{4.3}$$

Since $\hat{\mathcal{D}}$ is a Dirac operator it is natural to represent the Green's function [Eq. (4.3)] in terms of fermionic (i.e., Grassmann) variables. Let us introduce a set of Grassman (anticommuting) variables[16] $\psi_{aa}(\mathbf{x})$, $\bar{\psi}_{aa}(\mathbf{x})$ at every point of space. The path-integral representation of the unaveraged Green's function is

$$G_E(\mathbf{x}a\alpha\,|\,\mathbf{x}'a'\alpha')=i\langle\bar{\psi}_{aa}(\mathbf{x})\psi_{a'a'}(\mathbf{x}')\rangle =i\frac{\int\mathcal{D}\bar{\psi}\mathcal{D}\psi\,\bar{\psi}_{aa}(\mathbf{x})\psi_{a'a'}(\mathbf{x}')e^{-S}}{\int\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-S}}\ , \tag{4.4}$$

with the action $S=\int d^dx\,\mathcal{L}$ and the Lagrangian $\mathcal{L}$ in two dimensions is

$$\mathcal{L}=\bar{\psi}\nabla\psi-\epsilon\bar{\psi}\psi-ia\bar{\psi}\psi-i\bar{\psi}\mathbf{a}\cdot\boldsymbol{\alpha}\psi-i\bar{\psi}a_{0a}T_a\psi -i\bar{\psi}a_aT_a\psi-i\bar{\psi}b\gamma_5\psi-i\bar{\psi}_bT_a\gamma_5\psi\ . \tag{4.5}$$

In Eq. (4.5) I have introduced the two-dimensional Euclidean γ matrices with the properties[17]

$$\gamma_i=\alpha_i \quad i=1,2\ , \tag{4.5a}$$

$$\gamma_5=\beta\ , \tag{4.5b}$$

$$\{\gamma_i,\gamma_j\}=2\delta_{ij}\ , \tag{4.5c}$$

$$\{\gamma_i,\gamma_5\}=0\ , \tag{4.5d}$$

and

$$\not{\nabla}=\gamma_i\nabla_i\ , \tag{4.5e}$$

$$\not{a}=\gamma_i a_i\ . \tag{4.5f}$$

Repeated indices are summed and all indices in the ψ fields and the space dependence of the random fields has been omitted to simplify the notation. In three dimensions there is an analogous Lagrangian except that i runs from one to three and there is no γ_5 [since there is no β in Eq. (2.2a)]. Thus the zero-temperature Green's functions have a simple representation in terms of averages of Grassman fields.

At a finite temperature T (measured in energy units) the Green's functions must be calculated using the Matsubara formalism.[18] The net result is that *at finite temperature in a Fermionic system* the ϵ term in Eq. (4.1) must be replaced by the minimum Matsubara frequency $\omega_n=\pi T(k_B=1)$. The same modification carries through to the Lagrangian Eq. (4.5) as well as to the one to be derived below. Thus temperature breaks the symmetry explicitly and one expects that, at finite temperature, the transition will be smeared and that the mean free path and the conductivity will be finite. In particular, the unaveraged DOS is

$$N_0(E,x)=\frac{1}{\pi}\mathrm{Im}\,\mathrm{tr}G_E(\mathbf{x},\mathbf{x})=\frac{1}{\pi}\mathrm{Re}\langle\bar{\psi}(\mathbf{x})\psi(\mathbf{x})\rangle \equiv\frac{1}{\pi}\langle\bar{\psi}(\mathbf{x})\psi(\mathbf{x})\rangle\ . \tag{4.6}$$

It will also be of interest to calculate current-current correlation functions to study transport properties. Since the scattering is elastic (the random fields are time independent) the energy of each individual particle is separately conserved. Hence the retarded current-current correlation functions

$$G_{\mu\nu}(x,y)=-i\theta(x_0,y_0)\langle 0\,|\,[J_\mu(x),J_\nu(y)]\,|\,0\rangle \tag{4.7}$$

can be obtained from the retarded two-particle kernel with fixed energies. In two dimension the components of this kernel are

$$K_{00}(x,y)=-\mathrm{tr}\langle x\left|\frac{1}{\hat{\not{\mathcal{D}}}-\epsilon}\right|y\right\rangle\left\langle y\left|\frac{1}{\hat{\not{\mathcal{D}}}+\epsilon}\right|x\right\rangle\ , \tag{4.8a}$$

$$K_{ij}(x,y)=-\mathrm{tr}\left\langle x\left|\frac{1}{\hat{\not{\mathcal{D}}}-\epsilon}\gamma_i\right|y\right\rangle\left\langle y\left|\frac{1}{\hat{\not{\mathcal{D}}}+\epsilon}\gamma_j\right|x\right\rangle\ , \tag{4.8b}$$

$$K_{i0}(x,y)=-\mathrm{tr}\left\langle x\left|2\frac{1}{\hat{\not{\mathcal{D}}}-\epsilon}\gamma_i\right|y\right\rangle\left\langle y\left|\frac{1}{\hat{\not{\mathcal{D}}}+\epsilon}\gamma_j\right|x\right\rangle\ , \tag{4.8c}$$

and similar formulas in three dimensions. To generate these Green's functions it is generally necessary to introduce two sets of Grassmann variables, one associated with $+\epsilon$, the other with $-\epsilon$. However if $\hat{\not{\mathcal{D}}}$ happens to be odd under the γ_5 symmetry *without* having to flip the signs of some of the potentials, a single set of Grassmann variables suffices. In this case the γ_5 symmetry is continuous. This is generally not the case in all the problems that are considered in this paper.

To compute ensemble-averaged quantities it is necessary to use the replica trick. This requires us to introduce n sets of Grassman variables and to set $n\to 0$ at the end of the calculation. Thus the Fermi fields carry a number of indices. $\psi_{\alpha a r}(\mathbf{x})$, where α and a were explained before, $r=1,\ldots,2n$ and the first n indices are denoted with $+$ and the last n indices with $-$, depending on whether one is considering advanced or retarded functions. Hence the averaged quantities are (setting $E=0$)

$$N_0(0,\mathbf{x})=\frac{1}{\pi n}\langle\bar{\psi}_+(\mathbf{x})\psi_+(\mathbf{x})\rangle=-\frac{1}{\pi n}\langle\bar{\psi}_-(\mathbf{x})\psi_-(\mathbf{x})\rangle\ , \tag{4.9a}$$

$$K_{00}(\mathbf{x},\mathbf{y})=\langle\bar{\psi}_+(\mathbf{x})\psi_-(\mathbf{x})\bar{\psi}_-(\mathbf{y})\psi_+(\mathbf{y})\rangle\ , \tag{4.9b}$$

$$K_{ij}(\mathbf{x},\mathbf{y})=+\langle\bar{\psi}_+(\mathbf{x})\gamma_i\psi_-(\mathbf{x})\bar{\psi}_-(\mathbf{y})\gamma_j\psi_+(\mathbf{y})\rangle\ , \tag{4.9c}$$

and analogous formulas.

The effective Lagrangian used to compute these expectation values after averaging over equally distributed quenched random fields, is

$$\mathcal{L}=\bar{\psi}\not{\nabla}\psi-\epsilon\bar{\psi}\Lambda\psi +\frac{g}{2}[(\bar{\psi}\psi)^2+(\bar{\psi}\gamma_i\psi)^2+(\bar{\psi}\gamma_5\psi)^2 +(\bar{\psi}T_a\psi)^2+(\bar{\psi}\gamma_5T_a\psi)^2+(\bar{\psi}\gamma_iT_a\psi)^2] \tag{4.10a}$$

in two dimensions, and

$$\mathcal{L}=\bar{\psi}\not{\nabla}\psi-\epsilon\bar{\psi}\Lambda\psi+\frac{g}{2}[(\bar{\psi}\psi)^2+(\bar{\psi}\gamma_i\psi)^2 +(\bar{\psi}T_a\psi)^2+(\bar{\psi}\gamma_iT_a\psi)^2] \tag{4.10b}$$

in three dimensions. In Eqs. (4.10a) and (4.10b) I have used the matrix

$$\Lambda^{rr'}=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}_{rr'}\ , \tag{4.11}$$

which distinguishes advanced and retarded fields. A simple algebraic manipulation (Fierz transformation) reduces both (4.10a) to (4.10b) to the final form

$$\mathcal{L}=\bar{\psi}\nabla\psi-\epsilon\bar{\psi}\Lambda\psi-N_cg\bar{\psi}^a_r\psi^a_{r'}\bar{\psi}^{a'}_{r'}\psi^{a'}_r \tag{4.12}$$

after repeated use of the identity

$$\sum_{i=1}^{N_c^2-1}T_i^{ab}T_i^{cd}=N_c\delta_{ad}\delta_{bc}-\delta_{ab}\delta_{cd} \tag{4.13}$$

and a similar identity for Pauli matrices.

The upshot of this long discussion is that Lagrangian (4.12) will generate the average Green's functions of dirty degenerate semiconductors. It is natural to ask how general it is and if there are any corrections to it. Firstly, this Lagrangian follows from Eqs. (4.10a) and (4.10b) because I assumed that all scattering amplitudes have the *same*

distribution. Although this is natural, it is not general. The most general Lagrangian thus looks like Eq. (4.10a) or (4.10b) but with all quartic coefficients being different. In a separate publication (in preparation), I will show that Eq. (4.12) is *not* invariant under renormalization-group transformations and that, to leading order, an asymmetry term proportional to $(\bar{\psi}\psi)^2$ is generated. Secondly, there is the issue of non-Gaussian distributions. Such non-Gaussian terms differ from a white-noise distribution only in their higher moments. Hence they generate operators with more than four fermion operators. I will show now that, near two dimensions, they are irrelevant operators (in an RG sense) and thus do not change the nature of the singularities. Let us first show that two is the lower critical dimension for this transition. By dimensional analysis we know that $[\psi]=L^{-(d-1)/2}$, where L is a length. Thus ϵ scales like L^{-1} and g scales like L^{d-2}. A higher-order moment λ_n, say the coefficient of $2n$-fermion operator, scales like $\lambda_n \sim L^{(n-1)d-n}$. In two dimensions $\lambda_2 \equiv g$ is dimensionless and all operators of dimensions greater than two scale to zero at long distances. Thus all operators with $n>2$ are irrelevant. Therefore, up to anisotropies, Lagrangians (4.10a), (4.10b), or (4.12) are the most general renormalizable-field theory that one can write compatible with the symmetries of the problem.

Above two dimensions, g is also irrelevant. This means that the pure-system fixed point must be stable and so must be the semimetallic phase. Below two dimensions g is strongly relevant and hence the semimetallic phase is always unstable: disorder always dominates. Two dimensions is thus the marginal case. Therefore, due to quantum fluctuations, disorder always dominates even in two dimensions but this requires more than dimensional analysis. This will be discussed in the following paper.

V. RELATION TO (1 + 1)-DIMENSIONAL NONLINEAR FERMI-FIELD THEORIES

The Lagrangians (4.10a) and (4.10b) are the Euclidean version of self-interacting Fermi-field theories in d-dimensional Minkowski space. In two dimensions an (anti-) Wick rotation

$$x_2 \to i x_0 , \qquad x_1 \to x_1 , \tag{5.1}$$

yields the (1 + 1)-dimensional (Minkowski space) Lagrangian

$$\mathcal{L}=i\bar{\psi}\partial\psi-\epsilon\bar{\psi}\Lambda\psi+gN_c\bar{\psi}_r^a\psi_r^a\bar{\psi}_{r'}^{a'}\psi_a^{a'} , \tag{5.2}$$

with

$$\gamma_2 \to \gamma_0 , \qquad \gamma_1 \to +i\gamma_1 , \qquad \{\gamma_i,\gamma_j\}=2\delta_{ij} \to \{\gamma_\mu,\gamma_\nu\}=2g_{\mu\nu} , \tag{5.3}$$

in the Bjorken-Drell convention for $g_{\mu\nu}$.

The Lagrangian of Eq. (5.2) has the "wrong" sign for the coupling constant g.[19] Thus, for replica number n bigger than some n_c one expects the effective coupling to flow to *zero* at long distances. However in the replica limit $n \to 0$, the theory recovers its asymptotic freedom and the effective coupling grows in the infrared. Hence the replica limit (i.e., quenching) and the "wrong" sign of the coupling g conspire to produce nontrivial behavior at low energies. There are many exactly solvable self-interacting Fermi theories in $1+1$ dimensions.[20] This theory appears to be close to the non-Abelian chiral Fermi theories studied by Polyakov and Wiegmann,[21] except that the chiral symmetry here is discrete. It is instructive to transcribe the operators of Sec. IV into this language. The DOS becomes, in terms of time-ordered vacuum expectation values of the (1 + 1)-dimensional theory,

$$N(0,\mathbf{x}) \to \frac{1}{\pi}\langle 0 \mid \bar{\psi}(x)\psi(x) \mid 0\rangle \tag{5.4}$$

for ϵ fixed ($\epsilon \to 0$). In addition, the mean free path becomes the inverse of the mass gap in the Fermi-field theory. Expressions for the linear-response theory kernels Eqs. (4.9b) and (4.9c) are easily translated into two-particle Green's functions. Thus we learn that the localization length, which controls the decay of (4.9b) at long distances, is the inverse of a two-particle gap in the $+-$ sector. For this to be true it is necessary that the replica symmetry be spontaneously broken at $n=0$, otherwise there would not even be diffusion as stressed by Wegner[14] and McKane and Stone.[22]

VI. CONCLUSIONS

In this paper I have presented a qualitative model of disordered degenerate semiconductors. I have shown that both spectral and transport properties of the system can be studied in terms of an equivalent problem involving a Euclidean theory of self-interacting Dirac fermions in the replica limit. The relationship between the symmetries of both theories is established. I showed that the semimetal-metal transition can be viewed as a spontaneous breakdown of the underlying lattice symmetries and it is related to the nonsymmorphic character of their space groups. The average density of states at the Fermi level is shown to play the role of the order parameter of this transition and temperature is the symmetry breaking field. In the following publication I present a detailed analysis of the transition. I also discuss the relation with localization theory important in the regime in which the DOS is nonzero.

Throughout this paper I have ignored the effects of electronic correlations and electron-phonon interactions. They indeed may be important. In fact they are likely to dominate the semimetal-semiconductor transitions observed in the alloys $Pb_xSn_{1-x}Te$ and $Cd_xHg_{1-x}Te$. However if the electrons happens to interact weakly, usual instabilities cannot occur. This will be discussed elsewhere. Nevertheless, electron-electron interactions may still have an important effect even far from instabilities as it is known from localization theory.[23] This deserves a

separate study. From the experimental point of view there are a good number of systems that should be described by the model presented in this paper. Some degenerate semiconductors like HgTe exist naturally at high pressure. $Cd_{1-x}Hg_xTe$ and $Pb_xSn_{1-x}Te$ at the critical density are also good candidates. Still superlattices could be good candidates of two-dimensional systems. It would be highly desirable to have materials of this type which are superlattices of a monolayer in thickness.

Note added in proof. Recently B. Volkov and O. Pankratov [JETP Lett. **42**, 178 (1985)] have proposed a contact junction of PbTe-SnTe as an example of a two-dimensional zero-gap system.

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