Phenomenological dynamics of *XY* spin glasses

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We extend our previous (microscopic) study of the macroscopic dynamics of planar spin glasses to the case where the system has a remanent magnetization. In addition to employing, as macroscopic variables, the magnetization normal to the easy plane and the rotation angle about the axis normal to the easy plane, we utilize the result that random microscopic anisotropy which is bilinear in the spins leads to an in-plane macroscopic anisotropy with uniaxial symmetry. The present work is more phenomenological than the previous study, but as a consequence it is simpler to make predictions which are directly applicable to experiment. The zero-wave-vector mode, which involves the macroscopic variables identified above, is found to be field dependent when the static field is in the easy plane, so that it should be amenable to ESR measurements. (This mode can be described as a "longitudinal" mode, in the language of Heisenberg spin glasses.) As the measuring field is rotated in the plane, the resonance develops a complex angular dependence. This work also considers relaxation effects associated with the above macroscopic variables. Furthermore, since the in-plane macroscopic anisotropy is not fixed to the lattice, it is susceptible to change; we have, in addition, studied this slow anisotropy motion.

I. INTRODUCTION

The present paper is an extension of the work of Ref. 1, which considers the question of how one identifies the appropriate macroscopic variables for planar spin glasses and establishes the form of the macroscopic in-plane anisotropy which can be expected to occur. Having established the fundamental variables and anisotropy, in the absence of a remanent magnetization, the normal modes were studied in the long-wavelength limit. To go beyond this work and include a remanence, it is simplest to go to a more phenomenological approach. That has been done in the present paper, where we consider the effect of a remanence and an external field on the normal modes. In addition, we consider the effects of relaxation associated both with the macroscopic variables and with the anisotropy. For a brief review of the experimental and theoretical work which has already been done on *XY* spin glasses, the reader is referred to the Introduction of Ref. 1.

In Sec. II we study the normal modes for a remanent magnetization $\mathbf{m}_0$ and a field $\mathbf{H}$ in the plane (not necessarily parallel to one another). For the parallel case, we find a resonance involving (m_z, m_y, θ_z) for which

$$\omega^2=\gamma^2(K+m_0H)/\chi_{zz} \quad (\mathbf{H}\,||\,\mathbf{m}_0,\ \text{in plane}) \ , \qquad (1.1)$$

where χ_{zz} is the susceptibility normal to the plane. As a consequence, this resonance is tunable by varying H, and therefore it should be suitable for study with (fixed-frequency) electron-spin resonance (ESR) spectrometers. Moreover, by varying the orientation of the applied field, one can produce a more complex dependence of the resonance on field orientation; some examples of the possibilities are given.

In Sec. III we consider relaxation effects associated with the macroscopic variables and how this gives a finite linewidth to the normal modes. Section IV discusses what happens on a time scale long enough that the macroscopic anisotropy can reorient in the plane: this leads to the introduction of a memory angle Θ_z, and its dynamics is treated both in terms of time-dependent torque measurements and transverse susceptibilities. Finally, Sec. V provides a summary and discussion.

II. DYNAMICS FOR $\mathbf{H}\neq 0$, $\mathbf{m}_0\neq 0$

Equation (4.13) of Ref. 1 yields the result that, for an external field normal to the plane, the resonance frequency cannot be adjusted. This does not appear promising for one of the more useful probes of the spin-glass state: electron-spin resonance—because ESR spectrometers have only a limited amount of tunability. In order to give the resonance some tunability, we will permit the system to have a remanence $\mathbf{m}_0$, and we will consider that there is an applied field $\mathbf{H}$. Moreover, we will consider only the case $k=0$ (which is relevant to ESR); although from (4.13) of Ref. 1 it is clear that by replacing K by $(K+\rho_s k^2)$, one may incorporate the effect of finite k.

In Ref. 1 we have discussed why the transverse variables $\mathbf{m}_\perp$ and $\boldsymbol{\theta}_\perp$ are not good macroscopic variables. Nevertheless, in order to obtain an approximate macroscopic description of how the system behaves in the presence of a magnetic field $\mathbf{H}$, we will include them, use them, and then discard them. Note that we will only consider the case $\gamma\hbar H \ll J,D$, so that the magnetic field can be considered to be a perturbation. This is a very reasonable assumption, on comparison with the material parameters given in Table II of Ref. 2; for $\gamma\approx 2.8\times 10^7$ cgs, J typically corresponds to 10^4-10^5 G. It will be useful to introduce two sets of orthonormal triads, as in the case of the isotropic spin glass.[3,4] The first, $(\hat{\mathbf{n}},\hat{\mathbf{p}},\hat{\mathbf{q}})$, will be associated with the planar spin-glass state, with $\hat{\mathbf{q}}$ normal to the plane of the system and $\hat{\mathbf{n}}$ associated with the direction of the remanence, if any. The second, $(\hat{\mathbf{N}},\hat{\mathbf{P}},\hat{\mathbf{Q}})$, will be associated with the anisotropy energy, with $\hat{\mathbf{Q}}$ fixed along the normal to the plane (taken to be $\hat{\mathbf{z}}$) and $\hat{\mathbf{N}}$ being associated with the (history-dependent) preferred direction for $\hat{\mathbf{n}}$, as is taken to be the case for the isotropic spin glass.[5] We will thus replace the energy density (4.3)–(4.5) of Ref. 1 (which predicts no unidirectional anisotropy) by

$$\epsilon=\frac{m_z^2}{2\chi_{zz}}+\frac{\mathbf{m}_\perp^2}{2\chi_\perp}-\tfrac{1}{4}K[(\hat{\mathbf{n}}\cdot\hat{\mathbf{N}})^2+(\hat{\mathbf{p}}\cdot\hat{\mathbf{P}})^2] -\tfrac{1}{2}K_\perp(\hat{\mathbf{q}}\cdot\hat{\mathbf{Q}})^2-\mathbf{m}\cdot\left[\mathbf{H}+\frac{m_0}{\chi_\perp}\hat{\mathbf{n}}\right] . \tag{2.1}$$

In this expression, the first two terms give the energy cost due to magnetizing the system, where we have introduced the $\chi_\perp^{-1}\sim J$ term to permit the system to magnetize in the plane. The second two terms give the anisotropy energy, where the term in K $(\sim D_r^2/J)$ is due to random microscopic anisotropy D_r and serves to align $\hat{\mathbf{n}}$ with $\hat{\mathbf{N}}$ and $\hat{\mathbf{p}}$ with $\hat{\mathbf{P}}$, and we have introduced the $K_\perp$ $(\sim D)$ term to prevent $\hat{\mathbf{q}}$ and $\hat{\mathbf{Q}}$ from becoming significantly misaligned. The last term gives the interaction of the magnetization with the external field $\mathbf{H}$ and an internal field whose magnitude, being proportional to the remanence, is history dependent. For our purposes, we will treat the remanence as fixed and we will consider only the case where $\mathbf{H}$ is in the plane. The equilibrium conditions are that

$$0=\gamma\frac{\partial\epsilon}{\partial\phi_\alpha} , \tag{2.2}$$

$$0=\gamma\frac{\partial\epsilon}{\partial m_\alpha} , \tag{2.3}$$

where in (2.2), $(\mathbf{m},\hat{\mathbf{n}},\hat{\mathbf{p}},\hat{\mathbf{q}})$ change by $\delta\mathbf{m}=\delta\boldsymbol{\phi}\times\mathbf{m}$, etc., under the rotation $\delta\boldsymbol{\phi}$. In the limit of large D, the strongest constraints lead to $\hat{\mathbf{q}}\,||\,\hat{\mathbf{Q}}$ and $m_z=0$. Moreover, within the plane we have

$$\mathbf{m}_{\rm eq}=m_0\hat{\mathbf{n}}+\chi_\perp\mathbf{H}\quad (\mathbf{H}\text{ in plane}) . \tag{2.4}$$

Use of $\hat{\mathbf{q}}\,||\,\hat{\mathbf{Q}}$ enables us to reduce the anisotropy term in K to $-\frac{1}{2}K\cos^2(\theta_z-\Theta_z)$, where $\theta_z-\Theta_z$ gives the orientation within the plane of $\hat{\mathbf{n}}$ with respect to $\hat{\mathbf{N}}$. As a consequence, if we consider Θ_z fixed at the value 0, and take H to make an angle θ_H with respect to $\hat{\mathbf{N}}$, the energy may be rewritten as

$$\epsilon=\frac{m_z^2}{2\chi_{zz}}-\tfrac{1}{2}K\cos^2\theta_z-m_0H\cos(\theta_z-\theta_H) , \tag{2.5}$$

where we have dropped terms which do not depend upon θ_z or m_z and we take H to lie in the plane. Minimizing this, we deduce that

$$0=\frac{\partial\epsilon}{\partial\theta_z}=\tfrac{1}{2}K\sin(2\theta_z)+m_0H\sin(\theta_z-\theta_H) . \tag{2.6}$$

From (2.6) we find that the equilibrium angle θ_0 satisfies

$$\theta_H=\theta_0+\sin^{-1}\left[\left[\frac{K}{2m_0H}\right]\sin(2\theta_0)\right] \tag{2.7a}$$

or

$$\theta_H=\theta_0+\pi-\sin^{-1}\left[\left[\frac{K}{2m_0H}\right]\sin(2\theta_0)\right] , \tag{2.7b}$$

as appropriate. Note that by inverting (2.7) one can obtain θ_0 as a function of θ_H and thus the equilibrium anisotropy torque

$$\Gamma_0^{\rm an}=-\tfrac{1}{2}K\sin(2\theta_0) . \tag{2.8}$$

Let us now consider the problem of how the normal mode associated with m_z and θ_z is modified by the remanence and the applied field. Because of the large anisotropy, rotations about any axis in the plane will be suppressed. Physically, this means that if we formally consider a rotation $\boldsymbol{\theta}_\perp$ about an axis in the plane, then this variable can quickly adjust to its local equilibrium while the slower variables are changing in time. This means that the in-plane component of (2.3) continues to hold and therefore (2.4) is valid during the course of the low-frequency motion. As a consequence, (2.5) serves as the effective energy which drives θ_z. Thus, the equation of motion for m_z is given by (4.6) of Ref. 1 with ϵ given by (2.5):

$$\dot{m}_\alpha=-\gamma\frac{\partial\epsilon}{\partial\phi_\alpha} , \tag{2.9}$$

where $\mathbf{m}\rightarrow\mathbf{m}+\delta\boldsymbol{\phi}\times\mathbf{m}$ and $\boldsymbol{\theta}\rightarrow\boldsymbol{\theta}+\delta\boldsymbol{\phi}$ under the rotation $\delta\boldsymbol{\phi}$. Explicitly, on linearizing θ_z about θ_0, so $\theta_z=\theta_0+\delta\theta_z$, we have

$$\dot{m}_z=-[K\cos(2\theta_0)+m_0H\cos(\theta_0-\theta_H)]\delta\theta_z , \tag{2.10}$$

where θ_0 is given by (2.7). The equation of motion for $\delta\theta_z$ is given by (4.11) of Ref. 1:

$$\delta\dot{\theta}_z=\gamma\frac{\partial\epsilon}{\partial m_z}=\gamma\frac{m_z}{\chi_{zz}} . \tag{2.11}$$

Combining (2.10) and (2.11), we find that the oscillation frequency is given by

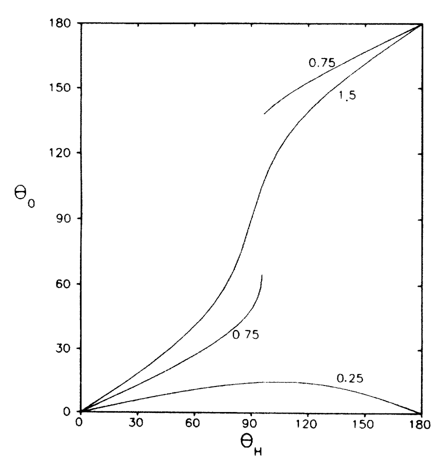

FIG. 1. θ_0 as a function of θ_H, for $\tilde{H}\equiv H/(K/m_0)$ representative of the three regimes $\tilde{H}>1$, $\frac{1}{2}<\tilde{H}<1$, and $0<\tilde{H}<\frac{1}{2}$. See discussion in Sec. II for details.

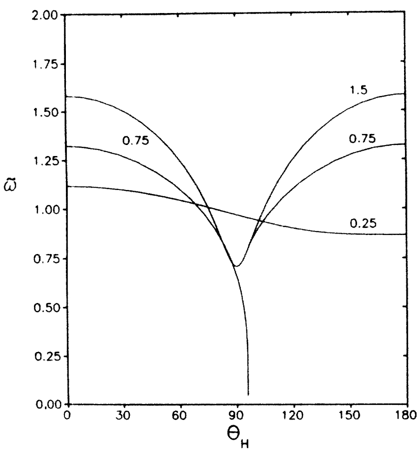

FIG. 2. $\tilde{\omega} \equiv \omega/\omega_0$ as a function of θ_H, for $\tilde{H}$ representative of the three regimes $\tilde{H} > 1$, $\frac{1}{2} < \tilde{H} < 1$, and $0 < \tilde{H} < \frac{1}{2}$. See discussion in Sec. II for details.

$$\omega^2 = \frac{\gamma^2[K \cos(2\theta_0) + m_0 H \cos(\theta_0 - \theta_H)]}{\chi_{zz}} . \quad (2.12)$$

In the simplest case, where $\mathbf{H}$ and $\hat{\mathbf{n}}$ are along $\hat{\mathbf{N}}$ (so that θ_0 and θ_H are zero), this yields the simple relationship

$$\omega^2 = \frac{\gamma^2(K + m_0 H)}{\chi_{zz}} \quad (\mathbf{H} \| \hat{\mathbf{n}} \| \hat{\mathbf{N}}) . \quad (2.13)$$

Thus, with a remanence and a static field in the plane, when one performs a transverse ESR experiment it should be possible to "tune" the resonance to the microwave cavity by varying the strength of the static field. Note that for H along $\hat{\mathbf{z}}$, ω is given by

$$\omega_0^2 = \gamma^2 \frac{K}{\chi_{zz}} \quad (\mathbf{H} \| \hat{\mathbf{z}}) . \quad (2.14)$$

In Figs. 1–5 we have plotted a number of relevant curves. To do so, it was useful to define

$$\tilde{H} \equiv H/(K/m_0), \quad \tilde{\omega} \equiv \omega/\omega_0 . \quad (2.15)$$

Figures 1–3 give θ_0, ω/ω_0, and Γ_0^{an}/K as a function of

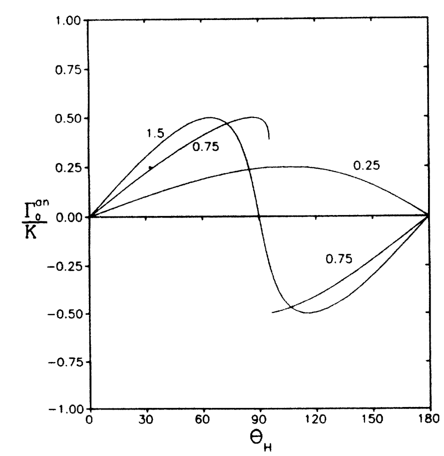

FIG. 3. Γ_0^{an}/K as a function of θ_H, for $\tilde{H}$ representative of the three regimes $\tilde{H} > 1$, $\frac{1}{2} < \tilde{H} < 1$, and $0 < \tilde{H} < \frac{1}{2}$. See discussion in Sec. II for details.

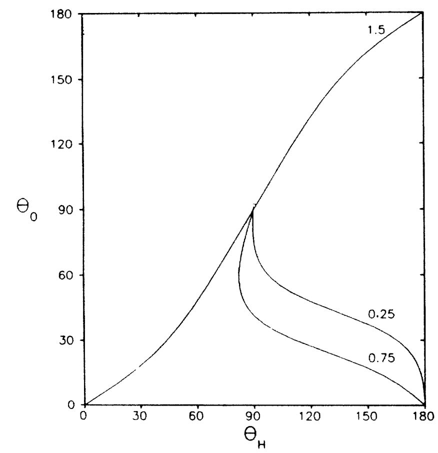

FIG. 4. θ_0 as a function of θ_H, for $\tilde{\omega}$ representative of the two regimes $\tilde{\omega} > 1$ and $0 < \tilde{\omega} < 1$. See discussion in Sec. II for details.

θ_H, for various values of $\tilde{H}$. For $\tilde{H} > 1$, (2.7a) applies for all θ_0 which was increased from 0 to π to produce the corresponding values of θ_H. For $\frac{1}{2} < \tilde{H} < 1$, (2.7a) applies for θ_0 increased from 0 to θ_0^{crit} in the first quadrant, where

$$\cos(2\theta_0^{crit}) \equiv - \left[\frac{4\tilde{H}^2 - 1}{3} \right]^{1/2} \quad (\tfrac{1}{2} < \tilde{H} < 1) , \quad (2.16)$$

and then (2.7b) applies beginning with the value of θ_0 which makes θ_H continuous; θ_0 is then increased to π. (This leads to a discontinuity in θ_0 because the system develops a dynamical instability, signified by $\tilde{\omega} = 0$.) For $0 < \tilde{H} < \frac{1}{2}$, (2.7a) applies for θ_0 increased from 0 to θ_0^{max} in the first quadrant, where

$$\sin(2\theta_0^{max}) \equiv 2\tilde{H} \quad (0 < \tilde{H} < \tfrac{1}{2}) , \quad (2.17)$$

and then (2.7b) applies as θ_0 is stepped down to 0.

In Figs. 4 and 5 we plot θ_0 and $\tilde{H}$ as a function of θ_H, for various values of $\tilde{\omega}$. This was done by noting that (2.6) and (2.12) may be combined to yield

$$\tan(\theta_H - \theta_0) = \frac{1}{2} \frac{\sin(2\theta_0)}{\tilde{\omega}^2 - \cos(2\theta_0)} , \quad (2.18)$$

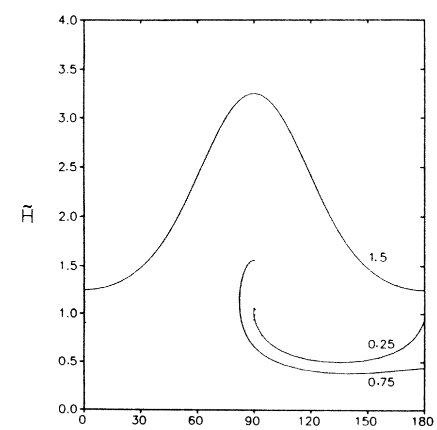

FIG. 5. $\tilde{H}$ as a function of θ_H, for $\tilde{\omega}$ representative of the two regimes $\tilde{\omega} > 1$ and $0 < \tilde{\omega} < 1$. See discussion in Sec. II for details.

which then leads to the following cubic equation for μ^2, where $\mu \equiv \cos 2\theta_0$:

$$\mu^6 - \mu^4(2\tilde{\omega}^2 + 3\cos^2\theta_H) + \mu^2[(\tilde{\omega}^2+1)(\tilde{\omega}^2+3)\cos^2\theta_H + \tilde{\omega}^4\sin^2\theta_H] - (\tilde{\omega}^2+1)^2\cos^2\theta_H = 0 \quad . \tag{2.19}$$

Since the quadrant is chosen to be consistent with (2.18), there are at most three acceptable real roots to (2.19). For $\tilde{\omega} > 1$ there is only one real solution. For $0 < \tilde{\omega} < 1$ there are either one or three real solutions, but we only choose those which can be reached by the following experimental procedure: starting with $\theta_0 = 0$ and $\tilde{H} = 0$, $\tilde{H}$ is applied at the orientation θ_H and increased until resonance occurs. If all real solutions had been accepted, the curves in Figs. 4 and 5 would be extended so as to be symmetrical about 90°. These solutions are represented by dashed lines.

III. PHENOMENOLOGICAL RELAXATION EFFECTS

Let us now consider the effect of relaxation on the variables m_z and θ_z, assumed to be not far from local equilibrium. We expect that we must augment the nondissipative terms by

$$\left[\frac{dm_z}{dt}\right]_d = -\frac{1}{T_z} m_z \quad , \tag{3.1}$$

$$\left[\frac{d\theta_z}{dt}\right]_d = -\frac{1}{U_z} \theta_z \quad , \tag{3.2}$$

where T_z and U_z are phenomenological relaxation times. These must be added to the right-hand sides of the appropriate parts of (2.9) and (2.11). For $\mathbf{H} || \hat{\mathbf{N}}$, (2.9) and (2.11) become

$$\dot{m}_z = -\gamma(K + m_0 H)\theta_z - \frac{1}{T_z} m_z \quad , \tag{3.3}$$

$$\dot{\theta}_z = \gamma \frac{m_z}{\chi_{zz}} - \frac{1}{U_z}\theta_z \quad . \tag{3.4}$$

The normal mode of (2.13) thus develops an imaginary part

$$\mathrm{Im}(\omega) = -\frac{1}{2}\left[\frac{1}{T_z} + \frac{1}{U_z}\right] \quad , \tag{3.5}$$

so that its damping is due both of m_z and θ_z relaxation processes. (Although this mode has a magnetization $m_y \approx m_0\theta_z$, we assume that its relaxation to this value is not relevant to the present discussion.)

IV. ANISOTROPY MOTION

To date, no experiments have been done on the in-plane anisotropy. However, on the basis of what we know of isotropic spin-glasses, it should be safe to assume that in-plane anisotropy is not much different for planar spin glasses than three-dimensional anisotropy is for isotropic spin glasses, except that now only one axis of rotation need be considered. Therefore, just as the anisotropy triad $(\hat{\mathbf{N}}, \hat{\mathbf{P}}, \hat{\mathbf{Q}})$ can rotate in the isotropic case, here $\hat{\mathbf{N}}$ and $\hat{\mathbf{P}}$ can rotate about $\hat{\mathbf{Q}}$. This means that Θ_z [introduced after (2.4)], which determines the orientation of $\hat{\mathbf{N}}$, develops a dynamics. Since it is possible to have $\hat{\mathbf{N}}$ far from its equilibrium orientation $\hat{\mathbf{n}}$, we will not assume that $\theta_z - \Theta_z$ is small. From Ref. 3 for the isotropic case, we can expect that $\dot{\Theta}_z$ is driven dominantly by the anisotropy torque, so we take

$$\dot{\Theta} = -\gamma F \Gamma_z^{(\mathrm{an})} = -\gamma \frac{FK}{2} \sin[2(\theta_z - \Theta_z)] \quad , \tag{4.1}$$

where F is a phenomenological constant with the dimension of inverse magnetization. This simple equation is surely an oversimplification, since in the isotropic case the anisotropy relaxation displays multiple relaxation times.[6] Nevertheless, it should provide some guidance. Setting $\gamma FK = \tau^{-1}$ and considering the case where θ_z and Θ_z are driven as $e^{-i\omega t}$, (4.1) yields

$$\Theta_z \approx \theta_z (1 - i\omega\tau)^{-1} \quad , \tag{4.2}$$

$$\Gamma_z^{\mathrm{an}} \approx -\tilde{K}\theta_z, \quad \tilde{K} \equiv K(1 + i/\omega\tau)^{-1} \quad . \tag{4.3}$$

Thus the anisotropy torque should be frequency dependent, with the important property that $\tilde{K} \to 0$ for $\omega \to 0$, since then the orientation Θ_z has enough time to completely relax.

A related phenomenon can be seen if one studies the in-plane transverse susceptibility. We will continue to assume that we are operating at sufficiently low frequencies that $\theta_\perp \approx 0$, so that (2.10) holds: for $H_y \neq 0$, this means that

$$m_y = m_0\theta_z + \chi_\perp H_y \quad . \tag{4.4}$$

We will assume that $\theta_z = \Theta_z = \theta_H = 0$ in equilibrium and study the response to small transverse fields. In that case we may employ (4.3) for the anisotropy torque. We then have, with the addition of an H_z and H_y oscillating as $e^{-i\omega t}$, that (2.10) and (2.11) are modified to read

$$-i\omega m_z = -\gamma(\tilde{K} + m_0 H)\theta_z + \gamma(m_0 - \chi_\perp H)H_y \quad , \tag{4.5}$$

$$-i\omega\theta_z = \frac{\gamma}{\chi_{zz}}(m_z - \chi_{zz}H_z) \quad . \tag{4.6}$$

For $H_y = 0$, the solution for the out-of-plane susceptibility is

$$\chi_{zz}(\omega) \equiv \frac{m_z}{H_z} = \left[\frac{\tilde{\omega}_0^2}{\tilde{\omega}_0^2 - \omega^2}\right]\chi_{zz} \quad , \tag{4.7}$$

where

$$\tilde{\omega}_0^2 \equiv \gamma^2 \frac{(\tilde{K} + m_0 H)}{\chi_{zz}} \quad . \tag{4.8}$$

Note that $\chi_{zz}(\omega) \to \chi_{zz}$ for $\omega \to 0$, as expected.

For $H_z = 0$, the in-plane transverse susceptibility is

$$\chi_\perp^{\mathrm{eff}}(\omega) \equiv \frac{m_y}{H_y} = \chi_\perp + \frac{\gamma^2 m_0^2}{(\tilde{\omega}_0^2 - \omega^2)\chi_{zz}} \quad . \tag{4.9}$$

Note that $\chi_\perp^{\mathrm{eff}}(\omega) \to m_x/H_x = \chi_\perp + m_0/H$ for $\omega \to 0$, as expected, where we use the fact that $\tilde{\omega}_0^2 \to \gamma^2 m_0 H/\chi_{zz}$ for

$\omega \to 0$, since $\tilde{K} \to 0$ in that limit. On the other hand, for $\tilde{\omega}_0^2 \approx \omega_0^2 >> \omega^2$, so that $\Theta_z \approx 0$ (frozen anisotropy), (4.9) yields

$$\chi_\perp^{\text{eff}} \approx \chi_\perp + \frac{m_0^2}{K + m_0 H} . \quad (4.10)$$

V. SUMMARY AND DISCUSSION

We have considered a number of the dynamical properties expected for planar spin glasses, emphasizing what can be expected when one includes anisotropy, remanence, and a magnetic field. Although they are qualitatively similar in behavior to isotropic (Heisenberg) spin glasses, they differ significantly in their detailed properties.

First, they have only one (rather than three) macroscopic normal mode, and this mode has a very different field dependence than any of the modes for the isotropic case.[7] Second, their macroscopic in-plane anisotropy has both a different microscopic source (only dipolar-like can produce an effect, whereas DM-like dominates in the isotropic case) and a different macroscopic symmetry (favoring 0° and 180° equally, rather than only 0°).[1] Experimental studies of ESR, transverse susceptibility, remanence properties (such as the isothermal remanence, the thermoremanence, and the remanence decay), and anisotropy decay would be quite useful.

Moreover, studies of the hysteresis loop should be quite interesting, since the predicted anisotropy energy is of the classic $\cos^2\theta$ form, but the magnetization is forced to lie in the easy plane, even when it is forced to "flip" (due to an instability). Hence, dynamic studies of the magnetization flip will not be hindered by any uncertainty in the path of the magnetization **m** (in **m** space) as it undergoes a flip. This same uncertainty will also be eliminated from studies of the anisotropy motion, since here it is sufficient to describe the movable anisotropy by a direction rather than by a triad (as in the isotropic case). Studies of the anisotropy decay to determine the dependence of τ (or of the multiple τ's[6]) on H and T would provide a revealing probe of the dynamical processes by which the system "learns" a preferred direction. This is a very general and outstanding problem whose answer is hardly more than qualitatively understood.[3,4,6,8] The most significant aspect of the anisotropy decay is that, because of the distinct separation of the τ's, it appears to be a collective phenomenon, unlike what happens in remanence decay. At present, we have no notion of what are the collective degrees of freedom which are relaxing, in contrast to the case of remanence decay, where individual spin flips or small cluster spin flips are most likely the dominant decay mechanism.

ACKNOWLEDGMENT

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