Microscopic approach to macroscopic dynamics in XY spin glasses

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Recently it has become possible to perform experimental studies on spin glasses with strong anisotropy forcing the spins into a plane. We have undertaken a theoretical investigation of the macroscopic dynamical behavior of such systems, beginning with a determination (at the microscopic level) of the appropriate macroscopic variables. Classical spins are considered at $T=0$. In the absence of weak random anisotropy in the plane, we predict spin waves with a linear dispersion and derive an expression for their velocity. When weak random anisotropy in the plane is included, a microscopic calculation shows a gap at zero wave vector. This uniform mode has no field dependence for a static field normal to the plane, and therefore would be difficult to observe using electron-spin resonance; however, the following paper shows that, in the presence of a remanence induced by a cooling field, the resonance frequency becomes field dependent for a static field in the plane. A formal expression for the in-plane macroscopic anisotropy constant is derived, employing both microscopic and macroscopic considerations. Assuming bilinear microscopic anisotropy, there is only uniaxial macroscopic in-plane anisotropy (to which Dzyaloshinsky-Moriya anisotropy does not contribute). A macroscopic study is made of the normal modes including both exchange and random anisotropy.

I. INTRODUCTION

The properties of spin glasses with single-ion uniaxial anisotropy have attracted considerable interest recently, both experimentally and theoretically. On the experimental side, Albrecht *et al.*[1] have been able to employ the same magnetic impurity (Mn) in three different hosts (Zn, Cd, Mg) with the same hexagonal crystal structure. The single-ion uniaxial anisotropy is found to yield Ising-like (ZnMn), XY-like (CdMn), and Heisenberg-like (MgMn) behavior. More recently, Baberschke *et al.*[2] have been able to employ the same hexagonal hosts (Y,Se) with four different impurities (Er, Dy, Tb, Gd), finding the same three categories of behavior. The positions in temperature of the susceptibility cusps were found to be at least in qualitative agreement with the mean-field theories.[3–6]

The above work concerned itself solely with the static behavior of these systems. It is also of interest to consider the dynamic behavior and, in particular, the normal modes (which are treated in the present paper). Of the three categories mentioned above, the Heisenberg case (negligible uniaxial anisotropy) has already been studied extensively, both experimentally[7–9] and theoretically.[9–13] A number of questions remain in that case. Of these, perhaps the most important are the difficulty in observing the second transverse mode,[9] the nature of the anisotropy relaxation,[12–18] and the remanence decay.[19] The normal modes are reasonably well understood, both with exchange only and with an additional weak random anisotropy, although it is still unclear why the $S(k,\omega)$ studies of Huber and co-workers[20,21] (not including anisotropy) do not yield peaks corresponding to spin waves with $\omega=ck$.

The microscopic Hamiltonian we shall consider has the form

$$H=H_{\rm ex}+H_D+H_{\rm an}\ , \tag{1.1}$$

where $H_{\rm ex}=-\frac{1}{2}\sum_{ij}J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j$ is the exchange term, $H_D=\frac{1}{2}D\sum_i S_{iz}^2$ is a strong uniaxial anisotropy term, and $H_{\rm an}$ includes the possibility of weak in-plane anisotropies (to be considered in Sec. III). The effects of H_D depend on the sign of D and its magnitude compared with J (which we take to be a value characteristic of the exchange interaction). For $D=0$, the spin directions (in an equilibrium configuration, with no external field) are random to the eye, i.e., they are distributed equally over directions in spin space. For small $|D|$, the spin components continue to span all directions in spin space, with some skewing towards ($D>0$) or away from ($D<0$) the XY plane. On the other hand, there are finite critical values of $|D/J|$, beyond which the spin components essentially collapse into the $\pm\hat{\mathbf{z}}$ direction (for $D<0$) or into the XY plane (for $D>0$). Then the equilibrium configurations will be exactly those of the Ising of XY spin glass with Hamiltonian H, and the magnitude of D will be manifested only through the dynamics. (The reason for this collapse in the XY case can be seen most easily by observing that the stability of the ground state is determined by studying the eigenvalues of a quadratic form: starting with a large D/J ratio, the planar solution is

stable, and only at a finite critical value of D/J does an instability occur, producing z components in the spins. At that point, the spins can undergo a continuous transition to a three-dimensional configuration, with some skewing towards the XY plane. Similar reasoning holds for large negative D/J.)

If the strong uniaxial anisotropy forces the spins normal to the plane ($D<0$), the dynamics is expected to be dominated by the strong anisotropy energy driving the individual spins back to equilibrium. Thus, if $\hat{\mathbf{z}}$ is the preferred direction, the system will absorb energy only when driven with a rf field along $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$. If the uniaxial anisotropy is the same for each spin, then the resonance frequency will vary as γD if the anisotropy constant D greatly exceeds J. (Here, γ is the gyromagnetic ratio.) Including exchange will lead to some broadening, which can be thought of as follows. Without accounting for J in the dynamics, the system is completely degenerate. (The up spins resonate at $\omega=\gamma D$, whereas the down spins resonate at $\omega=-\gamma D$.) When J is included as a weak perturbation, degenerate perturbation theory will lead to a splitting of these degenerate modes, with nearly all of the new modes possessing a net magnetization. The spread in the mode frequencies is expected to be on the order of J.

If the strong uniaxial anisotropy forces the spins into the plane ($D>0$), then one has the XY case. The dynamics has been studied numerically by a number of workers,[20–24] all in agreement that there is a normal mode with $\omega=ck$, as $k\to 0$. This corresponds to a nearly uniform rotation about the z axis producing a local magnetization m_z along the z axis. In this case, $c\propto(JD)^{1/2}$; in fact, all modes have frequencies which go as $(JD)^{1/2}$. It is the purpose of the present paper to provide an explicit expression for c, to discuss whether or not it makes sense to consider angle variables θ_x and θ_y to be conjugate to m_x and m_y (relevant to the macroscopic dynamics to be discussed in Sec. IV), and to include the effects of a weak, random anisotropy $D_r \ll J$. The latter introduces a macroscopic anisotropy $K\sim D_r^2/J$, so that $\omega\neq 0$ even as $k\to 0$: a gap develops and thus the uniform mode is no longer "hydrodynamic" (i.e., no longer a "Goldstone mode"). We find an anisotropy energy proportional to $\cos^2\theta$, whose coefficient is nonzero only for microscopic anisotropy with a symmetry of dipolar rather than Dzyaloshinsky-Moriya (DM) nature. (See the latter part of Sec. III for a more precise definition of our terminology.) [Note that, in a companion paper,[25] for $D\gg J$ we study the effect of a remanent magnetization $\mathbf{m}_0$ and a field $\mathbf{H}$ in the plane, not necessarily parallel to one another. For $\mathbf{m}_0||\mathbf{H}$, we find a $k=0$ resonance involving (m_z,m_y,θ_z) for which

$$\omega^2=\gamma^2(K+m_0H)/\chi_{zz} \quad (\mathbf{m}_0||\mathbf{H}, \text{ in plane}) ,$$

where χ_{zz} is the susceptibility normal to the plane. As a consequence, this resonance is tunable by varying H, and therefore it should be suitable for study with (fixed-frequency) electron-spin resonance (ESR) spectrometers. Moreover, if one rotates the orientation of the field in the plane, a more complex dependence of the resonance should be observed, analogous to what is found in the Heisenberg case.[7,9,11]]

In Sec. II we provide a discussion of the circumstances under which one can define a macroscopic rotation angle θ. (This is relevant to the issue of θ_z vs θ_x and θ_y.) In Sec. III we consider the computation of the spin-wave velocity and, with weak random anisotropy, the macroscopic anisotropy constant. Section IV considers the macroscopic dynamics with both spin stiffness and anisotropy and provides physical reasons for the invariance of the macroscopic anisotropy under rotations by π and why the DM interaction does not contribute to the macroscopic anisotropy. Finally, Sec. V provides a summary.

II. DEFINING A MACROSCOPIC ANGLE

In two previous papers Halperin and Saslow[10] and Saslow[13] have employed a macroscopic rotation angle $\delta\boldsymbol{\theta}$, assumed conjugate to the magnetization $\delta\mathbf{m}$, together with the corresponding commutation relations from which the macroscopic dynamics was derived. However, that derivation was carried out only in the context of isotropic Heisenberg spin glasses. Since the present problem includes strong planar anisotropy, the previous derivation must be generalized. We do not attempt a precise definition of $\delta\boldsymbol{\theta}$ for large rotations.

We first summarize the formalism of Ref. 13, which is based on that of Ginzburg.[26] (Our choice of units is such that $\hbar=1$.) We consider N classical spins of unit length, in a volume V, at temperature $T=0$, and cast the equations of motion in terms of the deviation angle $\delta\boldsymbol{\theta}_i$ of the spin $\mathbf{S}_i$ at site i. This is done in two ways. First, the linearized equation of motion for the deviations $\delta\mathbf{S}_i$ from the equilibrium orientation $\mathbf{S}_i^{(0)}$ is written in the form

$$\delta\dot{S}_{i\alpha}=-U_{ij\alpha\beta}\delta\theta_{j\beta} , \tag{2.1}$$

where $\delta S_{i\alpha}=\epsilon_{\alpha\beta\gamma}\delta\theta_{i\beta}S_{i\gamma}^{(0)}$ and $U_{ij\alpha\beta}$ will be determined. By replacing $\delta S_{i\alpha}$ by $\delta\dot{\theta}_{i\alpha}$ and $\delta\theta_{j\beta}$ by $\delta S_{j\beta}$, this can be rewritten in the form

$$\delta\dot{\theta}_{i\alpha}=W_{ij\alpha\beta}\delta S_{j\beta} \tag{2.2}$$

(where $W_{ij\alpha\beta}$ also will be determined). Then, (2.2) is inverted to yield

$$\delta S_{i\alpha}=\chi_{ij\alpha\beta}\delta\dot{\theta}_{j\beta} . \tag{2.3}$$

The time derivative of (2.3), combined with (2.1), then yields

$$\chi_{ij\alpha\beta}\delta\ddot{\theta}_{j\beta}=-U_{ij\alpha\beta}\delta\theta_{j\beta} . \tag{2.4}$$

The normal modes are found by letting $\delta\theta_{j\beta}\sim e^{-i\omega t}$, so that

$$U_{ij\alpha\beta}\delta\theta_{j\beta}=\omega^2\chi_{ij\alpha\beta}\delta\theta_{j\beta} . \tag{2.5}$$

The significance of (2.5) lies in the fact that both $\chi_{ij\alpha\beta}$ and $U_{ij\alpha\beta}$ are Hermitian, thus permitting conventional orthogonality theorems to be employed. The eigenmode $\delta\theta_{j\beta}^{(n)}$ has eigenfrequency ω_n and eigenvalue ω_n^2 (by time reversal, $\delta\theta_{j\beta}^{(n)*}$ has the eigenfrequency $-\omega_n$). Moreover, orthogonality of nondegenerate modes is expressed as

$$0=\delta\theta_{i\alpha}^{(m)*}\chi_{ij\alpha\beta}\delta\theta_{j\beta}^{(n)} \quad (\omega_n^2\neq\omega_m^2) . \tag{2.6}$$

For $\omega_n\neq 0$, one can multiply (2.6) by $-i\omega_n$ and then employ (2.3) to obtain

$$0=\delta\theta_{i\alpha}^{(m)*}\delta S_{i\alpha}^{(n)} \quad (\omega_n\neq 0) . \tag{2.7}$$

If we consider the case of uniaxial anisotropy, a uniform rotation about the z axis corresponds to $\omega_m=0$, so that with $\delta\theta_{i\alpha}^{(m)}=\delta\theta_z\neq 0$, (2.7) and (2.3) imply that

$$0=\sum_i \delta S_{iz}^{(n)}=\sum_{ij}\chi_{ijz\beta}\delta\theta_{j\beta}^{(n)} \quad (\omega_n\neq 0) . \tag{2.8}$$

This permits us to define the macroscopic rotation angle $\delta\theta_z$ by writing an arbitrary $\delta\theta_{jz}$ as a sum over $\omega_n\neq 0$ normal modes and over the uniform rotation mode:

$$\delta\theta_{j\beta}=\sum_n \delta\theta_{j\beta}^{(n)}+\delta\theta_z\delta_{\beta z} . \tag{2.9}$$

Then,

$$\sum_{ij}\chi_{ijz\beta}\delta\theta_{j\beta}=\sum_n\sum_{ij}\chi_{ijz\beta}\delta\theta_{j\beta}^{(n)}+\sum_{ij}\chi_{ijzz}\delta\theta_z = \left[\sum_{ij}\chi_{ijzz}\right]\delta\theta_z , \tag{2.10}$$

where we have employed (2.8). Hence, with

$$\tilde{\chi}_{zz}\equiv N^{-1}\sum_{ij}\chi_{ijzz} , \tag{2.11}$$

(2.10) becomes

$$\delta\theta_z=\tilde{\chi}_{zz}^{-1}\sum_{ij}\chi_{ijz\beta}\delta\theta_{j\beta} . \tag{2.12}$$

This will be correct, to $O(k)$, as a definition for the macroscopic angle associated with long-wavelength excitations.

Because a uniform rotation about another axis (e.g., $\hat{\mathbf{x}}$) is not associated with a zero-frequency mode, it is not possible to define $\delta\theta_x$ or $\delta\theta_y$ in the same rigorous fashion. Indeed, it may not make much sense to define such variables because they are not associated with a collective mode. Actually, the $\delta\theta_{jz}$ are a complete set of variables, since there are, for N spins, $2N$ transverse degrees of freedom satisfying a first-order differential equation, which can be rewritten in terms of $N\delta\theta_{jz}$'s satisfying a second-order differential equation. Indeed, this has been used as a strategy in explicit numerical studies.[20] Thus, some linear combinations of $\delta\theta_{jz}$'s, not corresponding to a hydrodynamic mode, must be made equivalent to a macroscopic $\delta\theta_x$ and $\delta\theta_y$. However, $\delta\theta_z$ has exhausted all of the true hydrodynamic modes, so that $\delta\theta_x$ and $\delta\theta_y$ would only be artificial constructs. The physics of this system is simply that, if a spatially uniform, oscillating magnetic field were applied in the x direction, as its frequency would be varied, different modes would cause absorption. Indeed, Grassl and Huber[24] find a broad distribution of eigenfrequencies in their study of the XY spin glass on a three-dimensional lattice. As a consequence, the picture presented in Ref. 27, in terms of m_x and θ_x, is correct in the order of magnitude of the eigenfrequency [$\sim(JD)^{1/2}$], but incorrectly gives the impression that there is a single such mode.

To derive the dynamics, we begin with the equation of motion

$$\dot{\mathbf{S}}_i=\gamma\mathbf{S}_i\times\mathbf{H}_i , \tag{2.13}$$

where

$$\mathbf{H}_i\equiv\gamma^{-1}\partial H/\partial\mathbf{S}_i \tag{2.14}$$

is the local field at site i. In the absence of an external field, (1.1) gives

$$\gamma\mathbf{H}_i=\sum_j J_{ij}\mathbf{S}_j-DS_{iz}\hat{\mathbf{z}} , \tag{2.15}$$

leading to the equilibrium condition

$$\sum_j J_{ij}\mathbf{S}_j^{(0)}=\lambda_i\mathbf{S}_i^{(0)} , \tag{2.16}$$

where λ_i is the local field in equilibrium. Linearizing about equilibrium, (2.15) leads to

$$\delta\dot{\mathbf{S}}_i=\delta\mathbf{S}_i\times\lambda_i\mathbf{S}_i^{(0)}+\mathbf{S}_i^{(0)}\times\left[\sum_j J_{ij}\delta\mathbf{S}_j-D\delta S_{iz}\hat{\mathbf{z}}\right] =\mathbf{S}_i^{(0)}\times\left[\sum_j J_{ij}\delta\mathbf{S}_j-\lambda_i\delta\mathbf{S}_i-D\delta S_{iz}\hat{\mathbf{z}}\right] . \tag{2.17}$$

We can now use (2.17) to obtain $U_{ij\alpha\beta}$, $W_{ij\alpha\beta}$, and $\chi_{ij\alpha\beta}$ more explicitly. Writing $\delta\mathbf{S}_i=\delta\boldsymbol{\theta}\times\mathbf{S}_i^{(0)}$, (2.17) yields

$$\delta\dot{S}_{i\alpha}=\epsilon_{\alpha\beta\gamma}S_{i\beta}^{(0)}\left[\sum_j J_{ij}\epsilon_{\gamma\mu\nu}\delta\theta_{j\mu}S_{j\nu}^{(0)}-\lambda_i\epsilon_{\gamma\mu\nu}\delta\theta_{i\mu}S_{i\nu}^{(0)} -D\delta_{\gamma z}\epsilon_{z\mu\nu}\delta\theta_{i\mu}S_{i\nu}^{(0)}\right] ; \tag{2.18}$$

comparison with (2.1) gives

$$U_{ij\alpha\nu}=[\delta_{\alpha\nu}(\mathbf{S}_i^{(0)}\cdot\mathbf{S}_j^{(0)})-S_{i\nu}^{(0)}S_{j\alpha}^{(0)}]\lambda_{ij} -D\delta_{ij}(\delta_{\alpha\nu}-\delta_{\alpha z}\delta_{\nu z}-S_{i\nu}^{(0)}S_{i\alpha}^{(0)}) , \tag{2.19}$$

where $\lambda_{ij}=\lambda_{ij}\equiv\lambda_i\delta_{ij}-J_{ij}$. Furthermore, with $\delta\dot{\mathbf{S}}_i=\delta\dot{\boldsymbol{\theta}}_i\times\mathbf{S}_i^{(0)}$, (2.17) and (2.2) yield

$$W_{ij\alpha\beta}=\lambda_{ij}\delta_{\alpha\gamma}^{Ti}\delta_{\gamma\beta}^{Tj}+D\delta_{ij}\delta_{\alpha z}\delta_{\beta z} , \tag{2.20}$$

where $\delta_{\alpha\gamma}^{Ti}\equiv\delta_{\alpha\gamma}-S_{i\alpha}^{(0)}S_{i\gamma}^{(0)}$. Note that the projectors in (2.20) properly have no inverse, but if we invert within the subspace of transverse spin components there are no difficulties, leading to

$$\chi_{ij\alpha\beta}=(W^{-1})_{ij\alpha\beta} . \tag{2.21}$$

An alternative and more transparent approach is to analyze (2.17) by components, employing δS_{iz} and $\delta\theta_{iz}$ as the variables for each site. [With these variables it will be straightforward to show that all eigenfrequencies vary as $(JD)^{1/2}$.] The z component yields

$$\delta\dot{S}_{iz}=-\lambda_{ij}\mathbf{S}_i^{(0)}\cdot\mathbf{S}_j^{(0)}\delta\theta_{jz} , \tag{2.22}$$

and the z component of the cross product of $\mathbf{S}_i^{(0)}$ and (2.17) yields

$$\delta\dot{\theta}_{iz}=\lambda_{ij}\delta S_{jz}+D\delta S_{iz}\approx D\delta S_{iz} \quad (D\gg J) . \tag{2.23}$$

Combining (2.22) and the time derivative of (2.23) then yields the N second-order differential equations

$$\delta\ddot{\theta}_{iz}=-D\lambda_{ij}\mathbf{S}_i^{(0)}\cdot\mathbf{S}_j^{(0)}\delta\theta_{jz} \quad (D\gg J) . \tag{2.24}$$

With $\delta\theta_{iz}\sim e^{-i\omega t}$, the eigenmodes satisfy

$$D\lambda_{ij}\mathbf{S}_i^{(0)}\cdot\mathbf{S}_j^{(0)}\delta\theta_{jz}^{(n)}=\omega_n^2\delta\theta_{iz}^{(n)} \quad (D\gg J) . \tag{2.25}$$

Since $\lambda_{ij} \sim |J|$, we see that $\omega \sim (DJ)^{1/2}$, as indicated earlier. Physically, this follows since δS_{iz} is driven by exchange torques (J) associated with $\delta\theta_{iz}$, and $\delta\theta_{iz}$ is driven by anisotropy torques (D) associated with δS_{iz}. Note that by $\sum_j \lambda_{ij} = \lambda_i - \sum_j J_{ij} = 0$, the uniform rotation $\delta\theta_{jz} = \text{const}$ is an eigenmode with $\omega = 0$. [This would be true even without the approximation of (2.23) so long as D is strong enough to force the equilibrium orientations of the spins into the plane.]

A noteworthy feature of (2.25) is that, in the $D \gg J$ limit, there is a one-to-one correspondence (absent in the Heisenberg case) between the true resonant modes and the eigenvalues of the Hessian (linearized Hamiltonian) operator. The latter are the modes of a "relaxational" dynamics which replaces the actual precessional dynamics by a viscous damping which is more convenient computationally.[28] The approximation of relaxational dynamics is valid only for studying the long-time relaxations, but (2.25) shows that for XY spins, known numerical results for the density of states, etc., of the relaxation modes[23] can be directly translated to results for the true resonance modes. In the case of Heisenberg systems, such a correspondence can be justified in the long-wavelength limit.[26] Note that a similar result was obtained in Ref. 20.

III. SPIN STIFFNESS AND ANISOTROPY CONSTANT ($D \gg J$)

From the preceding section one can obtain the eigenmodes in the long-wavelength limit. We will treat only the case $D \gg J$. Consider (following Ginzburg[26])

$$\delta\theta_{jz}^{(k)} = u_{jz}^{(k)} e^{i\mathbf{k}\cdot\mathbf{R}_j} \tag{3.1}$$

as a possible solution to (2.25) as $k \to 0$. Note that

$$u_{jz}^{(k)} = a + k_\alpha \Psi_{\alpha j} + \tfrac{1}{2} k_\alpha k_\beta \Phi_{\alpha\beta j} + \cdots \tag{3.2}$$

with

$$\Psi_{\alpha j} \equiv \left.\frac{\partial u_{jz}^{(k)}}{\partial k_\alpha}\right|_{\mathbf{k}=0}, \quad \Phi_{\alpha\beta j} \equiv \left.\frac{\partial^2 u_{jz}^{(k)}}{\partial k_\alpha \partial k_\beta}\right|_{\mathbf{k}=0}$$

is not a constant, so that k is not a complete label for the eigenfunctions, since $\Psi_{\alpha j}$ and $\Phi_{\alpha\beta j}$ must be determined. Multiplying (2.25) by $\delta\theta_{iz}^{(k)*}$ and summing on i yields

$$D\delta\theta_i^{(k)*}\lambda_{ij}\mathbf{S}_i^{(0)}\cdot\mathbf{S}_j^{(0)}\delta\theta_j^{(k)} = \omega_k^2 \delta\theta_i^{(k)*}\delta\theta_j^{(k)} \quad (D \gg J) . \tag{3.3}$$

Expanding in powers of k_α, we see that the left-hand side has no terms of $O(k^0)$ or $O(k^1)$, by $\sum_j \lambda_{ij} = 0$. Thus the left-hand side is of $O(k^2)$. As a consequence,

$$\omega_k^2 = c^2 k^2 , \tag{3.4}$$

where

$$c^2 \equiv \frac{\hat{\mathbf{k}}_\alpha \hat{\mathbf{k}}_\beta \tilde{\Psi}_{\alpha i}^* D\lambda_{ij}\mathbf{S}_i^{(0)}\cdot\mathbf{S}_j^{(0)}\tilde{\Psi}_{\beta j}}{|a|^2} \quad (D \gg J) ,$$
$$\tilde{\Psi}_{\beta j} \equiv \Psi_{\beta j} - iaR_{\beta j} . \tag{3.5}$$

Since, on the average, the system is spatially isotropic, and here α and β are spatial indices, we have

$$\tilde{\Psi}_{\alpha i}^* \lambda_j \mathbf{S}_i^{(0)}\cdot\mathbf{S}_j^{(0)}\tilde{\Psi}_{\beta j} = \tfrac{1}{3}\delta_{\alpha\beta}\tilde{\Psi}_{\gamma i}^* \lambda_{ij}\mathbf{S}_i^{(0)}\cdot\mathbf{S}_j^{(0)}\tilde{\Psi}_{\gamma j} . \tag{3.6}$$

Moreover, since the actual magnetic susceptibility is given by

$$\chi_{zz} = m_z/H_z = \frac{(\gamma/V)\sum_i S_{iz}}{H_z} \approx (\gamma^2 N/V)D^{-1} , \tag{3.7}$$

with the spin stiffness ρ_s defined by

$$\rho_s \equiv c^2 \chi_{zz} , \tag{3.8}$$

we find that

$$\rho_s \approx \gamma^2 (N/V)\frac{\tilde{\Psi}_{\gamma i}^* \lambda_{ij}\mathbf{S}_i^{(0)}\cdot\mathbf{S}_j^{(0)}\tilde{\Psi}_{\gamma j}}{3|a|^2} \quad (D \gg J) . \tag{3.9}$$

Note that (3.4) implies that the spin waves have linear dispersion, in agreement with the numerical calculations.[20–24]

Let us now include a weak random microscopic anisotropy within the plane. In (1.1) we must thus employ

$$H_{\text{an}} = -\tfrac{1}{2}\sum_{ij} D_{ij\alpha\beta}^{\perp} S_{i\alpha} S_{j\beta} . \tag{3.10}$$

(We will take $D^\perp$ to be the characteristic magnitude for the anisotropy $D_{ij\alpha\beta}^\perp$'s, analogous to J for the exchange.) Note that in (3.10) we have taken H_{an} to be bilinear in the spin variables; as for the exchange term, normally the most important types of anisotropy are of this form. These may be interactions between spins ($i \neq j$), the most important examples of which are the dipolar interactions and the (effectively random) anisotropic exchange interactions—namely, the DM and pseudodipolar interactions.[29,30] They might also be single-spin interactions ($i = j$), such as random uniaxial anisotropies.

In addition, however, there may be nonrandom, crystal-field anisotropies in the easy plane, very weak compared to D and J. In the most common crystal structures these will be fourfold or sixfold anisotropies. As noted in Ref. 31, such anisotropies can yield a macroscopic anisotropy just as do random microscopic anisotropies. [The key observation needed to explain this fact is that, although to first order in $D^\perp/J$ there may be no correlation energy between the spins and the microscopic anisotropy, to second order such a correlation will exist, thus leading to a net macroscopic anisotropy. It is not only random spins and random microscopic anisotropy[29,30,11,13] that can lead to a macroscopic anisotropy; this can also occur for random spins and nonrandom microscopic anisotropy,[31] and (according to Ref. 13) nonrandom spins and random microscopic anisotropy.] Furthermore, in the presence of a nonzero magnetization, there is a contribution of $O(D^\perp)$ which couples to $\mathbf{m}^2$, just as in other magnetic systems; however, since $\mathbf{m}$ is typically quite small in spin glasses, this term is expected to be dominated by the terms of $O[(D^\perp)^2/J]$ (to be discussed below), which are independent of $\mathbf{m}$. Note that randomness in z does not effect the system, since the strong planar anisotropy keeps the equilibrium spin orientations away from the $\hat{\mathbf{z}}$ axis.

With H_{an}, uniform rotations about the z axis can cost energy, so that (2.22) develops a modified value for λ_{ij} and the equilibrium mean-field λ_i:

$$\lambda_{ij}\delta_{\alpha\beta}\equiv(\lambda_i\delta_{ij}-J_{ij})\delta_{\alpha\beta}\rightarrow\lambda_{ij\alpha\beta}$$
$$\equiv(\lambda_i'\delta_{ij}\delta_{\alpha\beta}-J_{ij}\delta_{\alpha\beta}-D^{\perp}_{ij\alpha\beta}) , \tag{3.11}$$
$$\lambda_i S^{(0)}_{i\alpha}\equiv\sum_j J_{ij}S^{(0)}_{j\beta}\rightarrow\lambda_i' S^{(0)}_{i\alpha}$$
$$\equiv\sum_j (J_{ij}\delta_{\alpha\beta}+D^{\perp}_{ij\alpha\beta})S^{(0)}_{j\beta} . \tag{3.12}$$

Then (2.25) becomes, for the $k=0$ mode,

$$\omega_0^2\delta\theta^{(0)}_{iz}=D\sum_j[D^{\perp}_{ij\alpha\beta}(S^{(0)}_{i\beta}S^{(0)}_{j\alpha}+S^{(0)}_{i\alpha}S^{(0)}_{j\beta})$$
$$-D^{\perp}_{ij\alpha\alpha}\mathbf{S}^{(0)}_i\cdot\mathbf{S}^{(0)}_j]\delta\theta^{(0)}_{jz} . \tag{3.13}$$

To lowest order in $D^{\perp}$, we may take $\delta\theta_{jz}$ to be given by the $D^{\perp}=0$ solution [i.e., $\delta\theta_{jz}(0)=a$]. Then, with the anisotropy constant K (an energy density) defined by

$$\gamma^2K\equiv\omega_0^2\chi_{zz} , \tag{3.14}$$

(3.13) and (3.7) yield

$$K\equiv(1/V)\sum_{ij}D^{\perp}_{ij\alpha\beta}[(S^{(0)}_{i\beta}S^{(0)}_{j\alpha}+S^{(0)}_{i\alpha}S^{(0)}_{j\beta})-\mathbf{S}^{(0)}_i\cdot\mathbf{S}^{(0)}_j\delta_{\alpha\beta}] . \tag{3.15}$$

It is useful to define

$$A_{\beta'\gamma\mu\mu'}\equiv(1/V)S^{(0)}_{i\beta'}D^{\perp}_{ij\gamma\mu}S^{(0)}_{j\mu'} , \tag{3.16}$$

which, because of the symmetry of the system, must have the following form:

$$A_{\beta'\gamma\mu\mu'}=A_1\delta^{\perp}_{\beta'\mu}\delta^{\perp}_{\gamma\mu}+A_2\delta^{\perp}_{\beta'\gamma}\delta^{\perp}_{\mu\mu'}+A_3\delta^{\perp}_{\beta'\mu}\delta^{\perp}_{\mu'\gamma} , \tag{3.17}$$

where $\delta^{\perp}_{\beta\mu}=\delta_{\beta\mu}-(\hat{\mathbf{z}})_\beta(\hat{\mathbf{z}})_\mu$ is the projector into the plane. Combining (3.15)–(3.17) then yields

$$K=4(A_2+A_3) . \tag{3.18}$$

Note that if the $S^{(0)}_{i\beta}$ do not correlate with the $D^{\perp}_{ij\alpha\beta}$ (as for the ground state when $D^{\perp}=0$), the terms in (3.15) are uncorrelated and therefore the sum goes to zero. Since the correlation is produced by the random anisotropy acting against the exchange, the correlation angle is of $O(D_r/J)$, so that K is of $O(D_r^2/J)$, as in the Heisenberg case.[32,11,12] (Here we have replaced $D^{\perp}$ by D_r to remind ourselves that this anisotropy is associated with randomness.) Since χ_{zz} is of $O(D^{-1})$, (3.14) yields the result that $\omega_0\sim D_r(D/J)^{1/2}$.

One can learn a great deal by studying the anisotropy torque as a function of (finite) angle of rotation θ about the z axis. This has been done experimentally by static torque measurements,[33–36] by transverse susceptibility,[37,38] and by measuring the magnetization as a function of the strength and orientation of the external field (e.g., the asymmetric hysteresis loop in Heisenberg spin glasses with weak anisotropy, which is a signature of the unidirectional anisotropy[39–41]). In the present case, the theory can be employed to find the symmetry properties of the macroscopic anisotropy. To do this, we imagine that the spins rotate rigidly and uniformly from their "unperturbed" orientations. Then the anisotropy torque per unit volume takes the form

$$\Gamma^{\rm an}_\alpha=(\gamma/V)\epsilon_{\alpha\beta\gamma}S_{i\beta}H^{\rm (an)}_{i\gamma}$$
$$=(1/V)\epsilon_{\alpha\beta\gamma}S_{i\beta}D^{\perp}_{ij\gamma\mu}S_{j\mu}$$
$$=(1/V)\epsilon_{\alpha\beta\gamma}R_{\beta\beta'}R_{\mu\mu'}S^{(0)}_{i\beta'}D^{\perp}_{ij\gamma\mu}S_{j\mu'} , \tag{3.19}$$

where $S^{(0)}_{i\beta'}$ gives the equilibrium orientations (in the plane) and $R_{\beta\beta'}$ is the rotation matrix about the z axis:

$$R_{\beta\beta'}=\delta_{\beta\beta'}\cos\theta+(\hat{\mathbf{z}})_\beta(\hat{\mathbf{z}})_{\beta'}(1-\cos\theta)-\epsilon_{\beta\beta'\lambda}(\hat{\mathbf{z}})_\lambda\sin\theta . \tag{3.20}$$

In writing this, we assume that we know what three-dimensional rotation the system undergoes. Furthermore, we do not permit the system to undergo local rearrangements. In contrast, for a real experimental system the rotation is specified only to the extent that the remanence is rotated, which could be performed by any of a continuous infinity of three-dimensional rotations. Moreover, the system has time to undergo local rearrangements which lower its energy. This leads to a decrease in the anisotropy torque and, therefore, in the effective anisotropy constant. On the other hand, in the limit of low temperatures such relaxation is inhibited and (3.20) becomes more quantitatively valid. In any event, the argument that follows is expected to give the correct symmetries of the macroscopic anisotropy torque.

There are three terms in each of $R_{\beta\beta'}$, $R_{\mu\mu'}$, and $A_{\beta'\gamma\mu\mu'}$, so in principle $\Gamma^{\rm an}_\alpha$ can have as many as 27 terms:

$$\Gamma^{\rm an}_z=\epsilon_{z\beta\gamma}[\delta_{\beta\beta'}\cos\theta+\delta_{\beta z}\delta_{\beta' z}(1-\cos\theta)-\epsilon_{\beta\beta' z}\sin\theta]$$
$$\times[\delta_{\mu\mu'}\cos\theta+\delta_{\mu z}\delta_{\mu' z}(1-\cos\theta)-\epsilon_{\mu\mu' z}\sin\theta]$$
$$\times(A_1\delta^{\perp}_{\beta'\mu}\delta_{\gamma\mu}+A_2\delta^{\perp}_{\beta'\gamma}\delta^{\perp}_{\mu'\mu}+A_3\delta^{\perp}_{\beta'\mu}\delta^{\perp}_{\mu'\gamma}) . \tag{3.21}$$

However, this expression is overly general, since arbitrary rotation axes are permitted, whereas only rotations about $\hat{\mathbf{z}}$ are of interest to us. Restricting ourselves to this case, we find that β' and μ' must be normal to $\hat{\mathbf{z}}$, so the middle terms in $R_{\beta\beta'}$ and $R_{\mu\mu'}$ cannot appear [cf. (3.20)]. As a consequence, the number of possible terms reduces to 12. Now, note that the term A_1 correlates the spin indices with one another and the spatial indices with one another, so that the spin and the spatial coordinates remain uncoupled; as a consequence, this should yield no anisotropy torque, an assertion which is verified by explicit calculation. This reduces the number of possible terms to eight: four for A_2 and four for A_3.

In considering the A_2 terms, note that the $\delta^{\perp}_{\mu'\mu}$ factor prevents the $\epsilon_{\mu\mu' z}$ terms from contributing. This leaves the following terms in $\Gamma^{\rm an}_z$, due to A_2:

$$\epsilon_{z\beta\gamma}A_2\delta^{\perp}_{\beta'\gamma}\delta^{\perp}_{\mu'\mu}(\delta_{\beta\beta'}\cos\theta-\epsilon_{\beta\beta' z}\sin\theta)\delta_{\mu\mu'}\cos\theta=A_2(\cos^2\theta\epsilon_{z\beta\gamma}\delta^{\perp}_{\beta\gamma}-\sin\theta\cos\theta\epsilon_{z\beta\gamma}\epsilon_{\beta\gamma z})\delta^{\perp}_{\mu\mu}=-4A_2\sin\theta\cos\theta .$$

The A_3 terms give the following contribution to $\Gamma^{\rm an}_z$:

$$\epsilon_{z\beta\gamma}A_3\delta^{\perp}_{\beta'\mu}\delta^{\perp}_{\mu'\gamma}(\delta_{\beta\beta'}\cos\theta-\epsilon_{\beta\beta'z}\sin\theta)(\delta_{\mu\mu'}\cos\theta-\epsilon_{\mu\mu'z}\sin\theta)$$
$$=A_3[\cos^2\theta\epsilon_{z\beta\gamma}\delta^{\perp}_{\beta\gamma}+\sin^2\theta\epsilon_{z\beta\gamma}\epsilon_{\beta\mu z}\epsilon_{\mu\gamma z}-\sin\theta\cos\theta(\epsilon_{z\beta\gamma}\epsilon_{\beta\gamma z}+\epsilon_{z\beta\gamma}\epsilon_{\beta\gamma z})]=-4A_3\sin\theta\cos\theta\ .$$

Thus, collecting terms and using (3.18), we have

$$\Gamma^{\rm an}_z=-4(A_2+A_3)\sin\theta\cos\theta=-\tfrac{1}{2}K\sin2\theta\ . \tag{3.22}$$

This is periodic in π, in contrast to the case of anisotropy for Heisenberg spin glasses, where a DM interaction gives a $\sin\theta$ torque.[11–13,16,29] Note that, since

$$\Gamma^{\rm an}_z=-\frac{\partial\epsilon^{\rm an}}{\partial\theta_z}\ ,$$

(3.22) implies that (with $\theta\to\theta_z$)

$$\epsilon^{\rm an}=-\tfrac{1}{2}K\cos^2\theta_z\ . \tag{3.23}$$

So far, our analysis has not made any assumptions about the nature of the microscopic anisotropy. All forms of microscopic anisotropy must satisfy $D_{ij\alpha\beta}=D_{ji\beta\alpha}$. However, the most common anisotropies also satisfy the following condition:

$$D_{ij\alpha\beta}=\pm D_{ij\beta\alpha}\ , \tag{3.24}$$

where the plus holds for the dipolarlike case (as well as for single-ion interactions) and the minus holds for the DM-like case. If we apply this condition to (3.16), then we see that

$$A_{\beta'\gamma\mu\mu'}=\pm A_{\beta'\mu\gamma\mu'}\ . \tag{3.25}$$

Neglecting A_1 (since it is irrelevant to the anisotropy torque) we see that (3.25) and (3.17) imply that

$$A_2=\pm A_3\ . \tag{3.26}$$

As a consequence, a pure DM-like interaction ($A_2=-A_3$) will yield no macroscopic anisotropy torque ($K=0$). Only a dipolarlike interaction will yield a nonzero macroscopic anisotropy torque, and that will be proportional to $\sin2\theta$. It would be of considerable interest to verify this result experimentally.

It is not difficult to see why a DM interaction cannot contribute to the macroscopic anisotropy. The DM interaction is proportional to the cross product $\mathbf{S}_i\times\mathbf{S}_j$, which points in the $\hat{\mathbf{z}}$ direction because the spins lie in the XY plane. Under rigid rotations in the XY plane this vector is left unchanged, so that the energy (both exchange and anisotropy) is unchanged.

Moreover, it is not difficult to see why the system has a macroscopic anisotropy which is unchanged under rotations by π. The Hamiltonian is invariant under time reversal and, for a planar system, time reversal has the effect of rotating the system by π. On the other hand, this does not constrain the macroscopic anisotropy to be uniaxial. In fact, in the case of p-fold crystal fields (with p even), it is straightforward to show that the macroscopic anisotropy is proportional to $\cos p\theta$. However, it should be remarked that highly symmetric crystals field lead to high values of p: for square and hexagonal symmetry (this latter is the case for the systems studied in Refs. 1 and 2) one has $p=4$ and $p=6$, whereas for rectangular symmetry one has $p=2$. (Note that θ is not the same as θ_m, the angle of the magnetization measured relative to the crystal axes, but rather is the orientation of θ_m relative to the cooling field direction. Thus, it should be possible to distinguish the familiar $\cos p\theta_m$ anisotropy from the $\cos p\theta$ anisotropy, by repeated measurements with the cooling field in the XY plane at various angles with respect to the crystal axes.)

IV. MACROSCOPIC DYNAMICS

From Sec. III we have been able to determine the properties of the normal modes in the long-wavelength limit, using a microscopic approach. Let us now consider a macroscopic approach. The magnetization is defined via

$$m_\alpha(\mathbf{r})\equiv\frac{\gamma}{V}\sum_{i\in R}S_{i\alpha}\ , \tag{4.1}$$

where R is a region of volume V and containing N spins. For large uniaxial anisotropy ($D\gg J$), the macroscopic angle $\theta_z(\mathbf{r})$ defined by (2.12) becomes

$$\theta_z(\mathbf{r})\equiv\frac{1}{N}\sum_{i\in R}\theta_{iz}\quad(D\gg J)\ . \tag{4.2}$$

Here we have utilized (3.7) and the fact that in (2.12), $\chi_{ijz\beta}\approx\delta_{ij}\delta_{z\beta}D^{-1}$ for $D\gg J$ [from (2.20) and (2.21)]. Thus the angles are equally weighted (unlike the case for the Heisenberg spin glass[13]). [Note that, although (2.12) is valid only for small angles, (4.2) is sensible also for large rotation angles. This does not happen in the Heisenberg case, where three-dimensional rotations do not commute. Moreover, not all definitions of the two-dimensional rotation angle would share with (4.2) the commutative property.]

Associated with $\mathbf{m}$ and θ are the internal-energy densities

$$\epsilon_D=\frac{m_z^2}{2\chi_{zz}}\ , \tag{4.3}$$

$$\epsilon_{XY}=-\tfrac{1}{2}K\cos^2\theta_z+\tfrac{1}{2}\rho_s(\nabla\theta_z)^2\ . \tag{4.4}$$

Here, ϵ_D arises from the strong planar anisotropy and the terms in ϵ_{XY} arise from the weak random anisotropy in the plane and from the exchange energy; K and ρ_s were calculated in Sec. III. Thus the total energy density is

$$\epsilon=\epsilon_D+\epsilon_{XY}-\mathbf{m}\cdot\mathbf{H}\ , \tag{4.5}$$

where we have included the Zeeman term also.

From microscopic theory it is well known that

$$\dot m_\alpha=-\gamma\frac{\partial\epsilon}{\partial\phi_\alpha}\ , \tag{4.6}$$

where $\mathbf{m}\to\mathbf{m}+\delta\boldsymbol{\phi}\times\mathbf{m}$ and $\theta\to\theta+\delta\boldsymbol{\phi}$ under the rotation $\delta\boldsymbol{\phi}$. To find the equation of motion for θ_z, we may apply the Appendix of Ref. 13, which yields

$$\frac{d\theta_z}{dt}=\gamma(h_z-H_z)\ , \tag{4.7}$$

if the commutator $[m_\alpha,\theta_z]=i\gamma V^{-1}\delta_{\alpha z}$, as is the case here. In (4.7), H_z is the applied field (previously taken to be zero) and h_z is the field due to the internal interactions, for small m_z given by

$$h_z=\frac{1}{\chi_{zz}}m_z\ . \tag{4.8}$$

Thus, for $H_z\neq 0$ the equilibrium solution is $h_z=H_z$, so $m_z=\chi_{zz}H_z$. Note that (4.7) may also be written

$$\frac{d\theta_z}{dt}=\gamma\frac{\partial\epsilon}{\partial m_z}\ , \tag{4.9}$$

if (4.5), (4.7), and (4.8) are employed. Finally, observe that for the nonuniform case, (4.6) generalizes to

$$\dot{m}_\alpha=-\gamma\frac{\delta\epsilon}{\delta\phi_\alpha}\equiv-\gamma\left[\frac{\partial\epsilon}{\partial\phi_\alpha}-\nabla\cdot\frac{\partial\epsilon}{\partial\nabla\phi_\alpha}\right]\ . \tag{4.10}$$

The normal modes about the $\theta_z=0$ equilibrium then follow from (4.9) and (4.10), with ϵ given by (4.3)–(4.5):

$$\dot{\theta}_z=\gamma\frac{m_z}{\chi_{zz}}\ , \tag{4.11}$$

$$\dot{m}_z=-\gamma(K\theta_z-\rho_s\nabla^2\theta_z)\ . \tag{4.12}$$

Letting θ_z, $m_z\sim e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}$, (4.11) and (4.12) lead to

$$\omega^2=\frac{\gamma^2(K+\rho_s k^2)}{\chi_{zz}}\ , \tag{4.13}$$

in agreement with what was to be expected from Sec. III.

For $k=0$, (4.13) reduces to (3.14), or

$$\omega_0^2=\gamma^2\frac{K}{\chi_{zz}}\ . \tag{4.14}$$

As discussed after (3.18), $K\sim D_r^2/J$ and $\chi_{zz}\sim D^{-1}$, so that $\omega_0\sim\gamma D_r(D/J)^{1/2}$. Since D_r cannot be due to DM interactions, we expect it to be smaller (by perhaps an order of magnitude) for planar spin glasses than for Heisenberg spin glasses. On the other hand, D/J may be of the order of ten, so that the net effect should be only a slight lowering of the resonance frequency in the planar case (as opposed to the Heisenberg case).

V. SUMMARY

We have considered certain of the properties of planar spin glasses from a microscopic viewpoint, building around the microscopic definition of the macroscopic rotation angle. It was established analytically that, in the absence of random in-plane anisotropy, the normal modes in the long-wavelength limit have a linear dispersion relation (in agreement with numerical calculations[20–24] and a macroscopic calculation[27]) and an expression for the spin-wave velocity was derived. In addition, the effect of random in-plane anisotropy was considered and it was shown that for bilinear interactions a macroscopic uniaxial anisotropy results, with no contribution from DM interactions. As a consequence, the macroscopic anisotropy in the planar case can be expected to be smaller than in the Heisenberg case, where the DM interactions dominate.[30] The normal modes were also studied using a macroscopic formulation. It is concluded that the longitudinal resonance will not be tunable with field normal to the plane if there is no remanence. A companion paper, which is more phenomenological, considers the effect of remanence and shows that the longitudinal resonance becomes field dependent when the field is in the plane.[25]

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