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Plasma resonance in the high-frequency conductivity of a superlattice

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Employing the random-phase-approximation density-density correlation function, we calculate the real and imaginary parts of the high-frequency electron conductivity due to remote and background impurity scatterings in a type-I superlattice, which is composed of an infinite number of periodically arranged quantum wells of finite width. The relaxation time and the effective-mass correction, calculated as functions of frequency, show strong resonance around the plasma frequency of the system.

Recently, intensified interest in electronic transport in a multilayer system has been focused not only on its dc linear and high-field mobility,[1-3] but also on its high-frequency conductivity.[4] In a closely packed multilayer system the carriers within each layer form a quasi-two-dimensional (2D) interacting electron gas and they are also coupled with those in different layers via the long-range Coulomb interaction. Furthermore, these carriers are scattered not only by the impurities within and in the immediate vicinity of the same layer but also by the charged impurities located around the other layers. It is well known that the Coulomb interaction between carriers in a two-dimensional system has a profound effect on their transport properties. For superlattices, such an effect is even more significant because of the coupling between interlayer carriers and the scattering by the impurities in different layers. We find that this interlayer Coulomb-interaction manifests itself saliently in plasma resonant behavior of the linear high-frequency conductivity, reflecting the distinction of a superlattice from a single quasi-2D layer and from a 3D bulk gas. Tzoar and Zhang[4] recently investigated the high-frequency conductivity of a superlattice composed of infinitely thin layers of 2D electron gas due to remote impurity scattering in some detail. Unfortunately, in their discussion and calculation, the important contribution of the electron collective modes seems to be ignored, so that they predicted a relatively structureless behavior of the relaxation time as a function of frequency. In this Communication we report a study of the linear high-frequency conductivity of type-I superlattices due to both remote and background impurity scatterings at temperature $T=0$ K. The full contribution of the electron collective modes is taken into account within the random-phase approximation, and the effects of finite well width and the other superlattice geometrical parameters are examined.

The superlattice discussed consists of an infinite number of periodically arranged quantum wells of width a, and d is the spatial period or the distance between the two centers of adjacent layers. An $Al_xGa_{1-x}As$-GaAs-$Al_xGa_{1-x}As$ multilayer structure, in which the mobile electrons are in GaAs well regions, can be approximately described by such a model. We assume that (1) the potential wells are deep enough that the tunneling can be neglected and the electrons are confined to just one well, and (2) the width of the well is narrow and the electron area density is not too high so that only the lowest subband is occupied. The wave function of the electron in the lth well is $\psi_{l\mathbf{k}}(\mathbf{r},z)=e^{i\mathbf{k}\cdot\mathbf{r}}\zeta(z-ld)$, with energy $\epsilon_{\mathbf{k}}=\hbar^2k^2/2m$ and $\zeta(z)=(2/a)^{1/2}\cos(\pi z/a)$ for $-a/2<z<a/2$, and $\zeta(z)=0$ elsewhere. Here m is the electron band effective mass, $\mathbf{r}=(x,y)$ and $\mathbf{k}=(k_x,k_y)$ are 2D vectors in coordinate and momentum space, respectively, $k=|\mathbf{k}|$.

When a uniform ac electric field of frequency ω is applied parallel to the layer plane, the steady-state linear conductivity $\sigma(\omega)$ for each layer of the system can be expressed by means of the memory function[5] $M(\omega)$ as

$$\sigma(\omega)=i\frac{N_se^2}{m}\frac{1}{\omega+M(\omega)}, \tag{1}$$

where e is the electron charge and N_s is the carrier density per layer. The expression for $M(\omega)$ can be derived as the linear limit of a frequency-dependent generalization of the nonlinear balance-equation theory of transport.[6,7] In the case of an infinite homogeneous medium this reduces to the well-known results of Ron and Tzoar.[8] The derivation at hand involves the analysis of the density-density correlation function of the quantum-well superlattice and an average over impurity sites. We consider two different kinds of impurities: remote impurities and background impurities. The former are located in thin planar layers at a distance s from the center of each quantum well with sheet density N_r, and the latter are distributed uniformly within the well region with area density N_b per layer. We assume that there is no interference effect between different kinds of scatterers, so that the contributions to the frictional force due to the remote and background impurity scatterings are additive and the average over these two kinds of impurities can be done separately. In either case, we essentially deal with the quantity

$$A=\left\langle\sum_{\alpha,\beta}e^{i\mathbf{q}\cdot\mathbf{r}_{m\alpha}+i\mathbf{q}'\cdot\mathbf{r}_{n\beta}}\right\rangle. \tag{2}$$

Here $\mathbf{r}_{m\alpha}$ is the x-y position of the αth impurity in the mth sheet (remote impurity) or mth quantum well (background impurity). The average is over all possible configurations of the impurity distribution in the mth and nth sheets (or quantum wells). The difference between the two cases is that for background impurity scattering we need also to average the impurity distribution in the z direction within the width of the well. This is easily done since the quantity treated is a smooth nonoscillatory function of z. If the impurity distribution is random within each sheet (well) and there is no correlation between different sheets (wells), the

quantity in Eq. (2) is a two-dimensional extension of the averaging discussed by Kohn and Luttinger in the 3D case.[9] Therefore, to the lowest order of N_I ($N_I = N_r, N_b$ is the impurity sheet density), $A = N_I \delta_{\mathbf{q},-\mathbf{q}'}$, independent of the layer indices m and n. The possibility of correlation between the impurity distribution in different layers, however, can be included in the following way. The superlattice system structure enables us to assume that the quantity in Eq. (2) is a function of $m-n$ after configuration averaging:

$$\left\langle \sum_{\alpha,\beta} e^{i\mathbf{q}\cdot\mathbf{r}_{m\alpha}+i\mathbf{q}'\cdot\mathbf{r}_{n\beta}} \right\rangle = N_I \delta_{\mathbf{q},-\mathbf{q}'} g(m-n) \ . \tag{3}$$

The function $g(m)$ ($m=0, \pm 1, \ldots$) can be expressed as a Fourier coefficient:

$$g(m) = \frac{d}{2\pi} \int_{-\pi/d}^{\pi/d} dq_z g(q_z) e^{iq_z md} \ , \tag{4}$$

and the corresponding inverse series is

$$g(q_z) = \sum_m g(m) e^{-iq_z md} \ . \tag{5}$$

The formula for the memory function $M(\omega)$ due to impurity scattering can be obtained in these terms as

$$M(\omega) = \frac{1}{mN_s\omega} \left(\frac{e^2}{2\epsilon_0 \kappa} \right)^2 \left(\frac{d}{2\pi} \right) \int_{-\pi/d}^{\pi/d} dq_z \sum_{\mathbf{q}} \frac{q_x^2}{q^2} \tilde{N}(q,q_z) [\hat{\Pi}(q_z,\mathbf{q},0) - \hat{\Pi}(q_z,\mathbf{q},\omega)] g(q_z) \ , \tag{6}$$

with κ being the dielectric constant of GaAs. $\hat{\Pi}(q_z,\mathbf{q},\omega)$ is the electron density-density correlation function of the superlattice,[3] which can be expressed in the random-phase approximation in the form

$$\hat{\Pi}(q_z,\mathbf{q},\omega) = \frac{\Pi(\mathbf{q},\omega)}{1 - V(q,q_z)\Pi(\mathbf{q},\omega)} \ , \tag{7}$$

in which the effective Coulomb interaction is (neglecting the image-charge contribution)

$$V(q,q_z) = \frac{e^2}{2\epsilon_0 \kappa q} [H(q) + S(q,q_z)], \tag{8}$$

where $S(q,q_z)$ comes from the interlayer carrier interaction:

$$S(q,q_z) = \frac{\cos(q_z d) - \exp(-qd)}{\cosh(qd) - \cos(q_z d)} \exp(qa) [I(q)]^2 \ . \tag{9}$$

$H(q)$ and $I(q)$ are form factors depending on the electron wave function within the layer ($u \equiv qa$):

$$H(q) = 3\frac{1-\exp(-u)}{u^2+4\pi^2} + \frac{u}{u^2+4\pi^2} - \frac{1-\exp(-u)}{(u^2+4\pi^2)^2} \times (u^2-4\pi^2) + \frac{2}{u}\left[1 - \frac{1-\exp(-u)}{u}\right] , \tag{10}$$

$$I(q) = 4\pi^2 [1-\exp(-u)]/[u(u^2+4\pi^2)] \ . \tag{11}$$

In Eq. (7) $\Pi(\mathbf{q},\omega)$ is the two-dimensional density-density correlation function for a single sheet of electrons in the absence of the Coulomb interaction:

$$\Pi(\mathbf{q},\omega) = 2\sum_{\mathbf{k}} \frac{f(\epsilon_{\mathbf{k}+\mathbf{q}}) - f(\epsilon_{\mathbf{k}})}{\hbar\omega + \epsilon_{\mathbf{k}+\mathbf{q}} - \epsilon_{\mathbf{k}} + i\delta} \ , \tag{12}$$

in which $f(\epsilon) = \{\exp[(\epsilon-\epsilon_f)/k_B T]+1\}^{-1}$ is the Fermi-Dirac function and ϵ_f is the chemical potential for electrons in a single quantum well. In Eq. (6) $\tilde{N}(q,q_z)$ is an effective impurity density:

$$\tilde{N}(q,q_z) = N_r Z_r^2 \left| \frac{\sinh[q(d-s)] + \exp(iq_z d)\sinh(qs)}{\cosh(qd) - \cos(q_z d)} \right|^2 \exp(qa)[I(q)]^2 + N_b Z_b^2 \left[\frac{\cos(q_z d) - \exp(-qd)}{\cosh(qd) - \cos(q_z d)} \frac{\exp(qa)-1}{qa} I(q) + K(q) \right]^2 \ , \tag{13}$$

with ($u \equiv qa$)

$$K(q) = \frac{8\pi^2}{(4\pi^2+u^2)u} \left[1 + \frac{u^2}{4\pi^2} - \frac{1-\exp(-u)}{u} \right] . \tag{14}$$

Z_r and Z_b are equivalent charge numbers of the remote and background impurities.

For a completely uncorrelated distribution of the impurities $g(m)=1$ and $g(q_z) = 2\pi\delta(q_z d)$, whence Eq. (6) reduces to

$$M(\omega) = \frac{1}{mN_s\omega} \left(\frac{e^2}{2\epsilon_0\kappa} \right)^2 \sum_{\mathbf{q}} \frac{q_x^2}{q^2} \tilde{N}(q,0) [\hat{\Pi}(0,\mathbf{q},0) - \hat{\Pi}(0,\mathbf{q},\omega)] \ . \tag{15}$$

On the other hand, if we take $g(q_z)=1$, independent of q_z, the remote part of Eq. (6) is equivalent to the formula given by Tzoar and Zhang.[4] However, $g(q_z)=1$ means $g(m)=\delta_{m0}$, i.e., $g(m-n)$ is zero unless $m=n$. This may be reasonable if the impurities are freely mobile in the plane. For conventional impurities (which are spatially localized), however, this corresponds to an impurity distribution in different sheets that is so strongly correlated that the contribution to the averaged quantity associated with any two different sheets always vanishes—a difficult condition to fulfill experimentally.

Writing $M(\omega)$ in terms of its real part and imaginary part, $M(\omega) = M_1(\omega) + iM_2(\omega)$, we can easily see that in the zero-frequency limit Eq. (1) reduces to the linear expression for the isothermal dc conductivity:[7]

$$\frac{1}{\sigma(0)}=\frac{m}{N_s e^2}M_2(0)=\left(\frac{e}{2\epsilon_0\kappa N_s}\right)^2\left(\frac{d}{2\pi}\right)\int dq_z g(q_z)\sum_{\mathbf{q}}\frac{q_x^2}{q^2}\tilde{N}(q,q_z)\frac{\partial}{\partial\omega}\hat{\Pi}_2(q_z,\mathbf{q},\omega)\Big|_{\omega=0}. \tag{16}$$

Therefore, $[M_2(0)]^{-1}$ is a direct measure of the isothermal dc transport relaxation time τ_0 or dc mobility μ_0: $M_2(0)=1/\tau_0=e/m\mu_0$. At high frequency, the conductivity expression [Eq. (1)] is conventionally written as a Drude-type formula:

$$\sigma(\omega)=i\frac{N_s e^2}{m^*}\frac{1}{\omega+i/\tau}, \tag{17}$$

in which $m^*=m[1+M_1(\omega)/\omega]$ and $\tau^{-1}=M_2(\omega)/[1+M_1(\omega)/\omega]$. Generally, the effective-mass correction $\Delta m/m=(m^*-m)/m=M_1(\omega)/\omega$ is much less than 1, such that the relaxation time is determined essentially only by the imaginary part of the memory function: $\tau^{-1}\cong M_2(\omega)$.

By means of the well-known expression for $\Pi(\mathbf{q},\omega)$ at zero temperature,[10] the real and the imaginary parts of the memory function $M(\omega)$ can be calculated directly from Eq. (6). In calculation, however, care must be taken not to neglect the contribution of the electron collective modes. These collective modes of the electrons are well-defined excitations when the real part of the dielectric function $\epsilon(q_z,\mathbf{q},\omega)=1-V(q,q_z)\Pi(\mathbf{q},\omega)$ has a zero (plasma pole) in a region where its imaginary part vanishes. For a 3D bulk system the contribution of plasma poles to $M_2(\omega)$ was discussed by Ron and Tzoar.[8] They found a relatively weak (15%) effect. In a superlattice, however, the effect is significantly enhanced. The dispersion relation of the electron plasmons in a superlattice at $T=0$ K has been discussed in detail in the literature.[11,12] The relevant information is employed in our computation of the high-frequency conductivity.

In the following we first discuss the case of an uncorrelated impurity distribution. We have calculated $M_1(\omega)$ and $M_2(\omega)$ from Eq. (15) at zero temperature for several different geometrical parameters of the GaAs-$Al_xGa_{1-x}As$ superlattice due to remote and background impurity scatterings. In Fig. 1 we plot the calculated normalized inverse relaxation time $M_2(\omega)/M_2(0)$ and effective-mass correction $\Delta m/m$ as functions of normalized frequency $\hbar\omega/4\epsilon_F$ (ϵ_F is the Fermi energy at zero temperature) for a quantum-well superlattice with $a=100$ Å, $d=200$ Å, and $N_s=2.3\times10^{11}$ cm^{-2}. In this figure $\Delta m/m$ is scaled in units of

$$\frac{e^2}{2\pi\hbar\sigma(0)}=\frac{3.9\times10^{-5}}{\sigma(0)(1\ \Omega)}=0.024\left(\frac{10^{15}\ \mathrm{m}^{-2}}{N_s}\right)\left(\frac{10\ \mathrm{m}^2/\mathrm{V\ s}}{\mu_0}\right). \tag{18}$$

In calculation we assume the equality of the contributions of remote and background impurity scatterings to dc resistivity and the remote impurities are located at $s=75$ Å from the center of each well. The material parameters used are the electron-band effective mass $m=0.07m_e$ (m_e is the free-electron mass) and the GaAs dielectric constant $\kappa=12.9$. For comparison we also show $M_2(\omega)/M_2(0)$, calculated by neglecting the plasma pole contribution, as discrete points in Fig. 1. These should correspond to the curves given in Ref. 4, if the background impurity scattering is excluded. The enhancements in $M_2(\omega)/M_2(0)$ and $\Delta m/m$ due to collective modes are very pronounced, resulting in the most striking feature in the frequency dependence of the conductivity: a strong plasma resonance.

The sharp rise of $M_2(\omega)$ at the bulk plasmon frequency $\omega_p=(e^2N_s/\epsilon_0\kappa md)^{1/2}$ of the system is the characteristic of a *closely packed* infinite-number-layer system, in which the electrons in one layer are scattered by the charged impurities located at all different layers, giving rise to a $(q,d)^{-1}$ divergence of $N(q,0)$ at small q. This jump just comes from the long-wavelength ($q\sim0$ and $q_z=0$) plasmon contribution at $\omega\cong\omega_p$ and is easily obtained from the behavior of $\tilde{N}(q,0)$ and $\epsilon(0,\mathbf{q},\omega)$ at small q as

$$\Delta M_2(\omega_p)=\frac{1}{12\pi\hbar^2}\left(\frac{e^2}{\epsilon_0\kappa}\right)^{3/2}\frac{m^{1/2}}{(N_s d)^{3/2}}(N_r Z_r^2+N_b Z_b^2). \tag{19}$$

In Fig. 2 we plot the calculated results of $M_2(\omega)/M_2(0)$ for five GaAs-$Al_xGa_{1-x}As$ superlattice systems, all composed of quantum wells of width $a=50$ Å and carrier density $N_s=2.3\times10^{11}$ cm^{-2}, but with different layer separations: $d=60$, 100, 200, 400, and 1600 Å, respectively. The variation of the high-frequency conductivity with the layer separation is clearly seen. As anticipated, for the $d=1600$ Å system we have almost a quasi-two-dimensional (single quantum well) behavior.

It is worth noting that the assumption that the impurity distribution is random within each layer and has no correlation from one layer to another is relevant to the sharp edge in the plasma resonance of the conductivity in a superlattice. This assumption generally seems reasonable. Nevertheless,

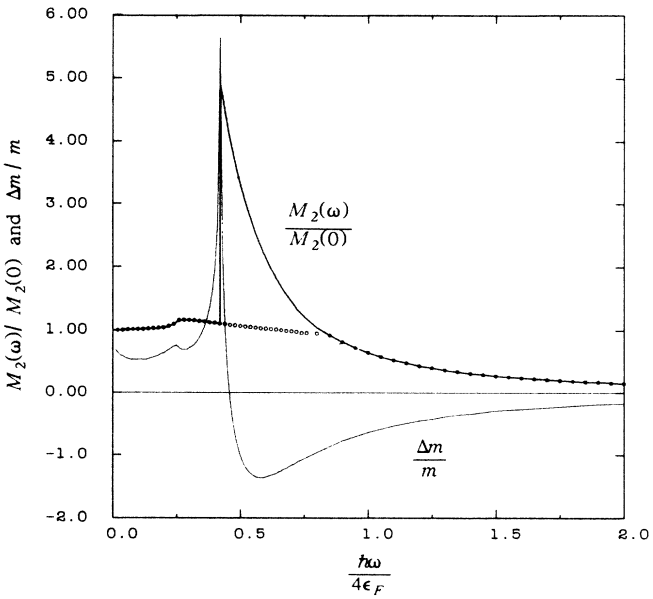

FIG. 1. Calculated $M_2(\omega)/M_2(0)$ and $\Delta m/m$ [in units of $e^2/2\pi\hbar\sigma(0)$] are shown as functions of $\hbar\omega/4\epsilon_F$ for a quantum-well superlattice with $a=100$ Å, $d=200$ Å, and $N_s=2.3\times10^{11}$ cm^{-2}. The scatterings are due to remote impurities and background impurities (1:1 in contribution to dc resistivity). The former are located at a distance $s=75$ Å from the center of each quantum well. The discrete points are $M_2(\omega)/M_2(0)$ calculated by neglecting the plasma-pole contribution.

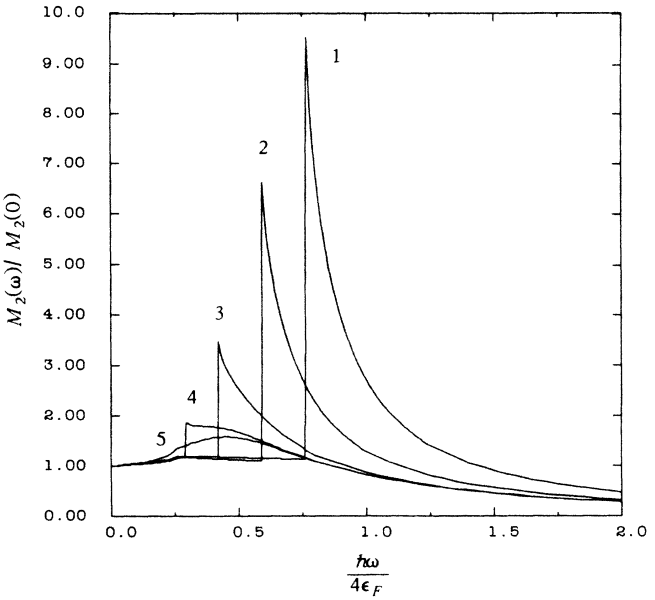

FIG. 2. Calculated $M_2(\omega)/M_2(0)$ for five superlattice systems, all composed of quantum wells of $a = 50$ Å and $N_s = 2.3\times10^{11}$ cm^{-2}, but with different layer separation d: 1, 60 Å; 2, 100 Å; 3, 200 Å, 4, 400 Å; 5, 1600 Å. The scatterings are due to remote and background impurities (1:1 in contribution to dc resistivity). $s = 30$ Å for $d = 60$ Å system and $s = 40$ Å for the other systems.

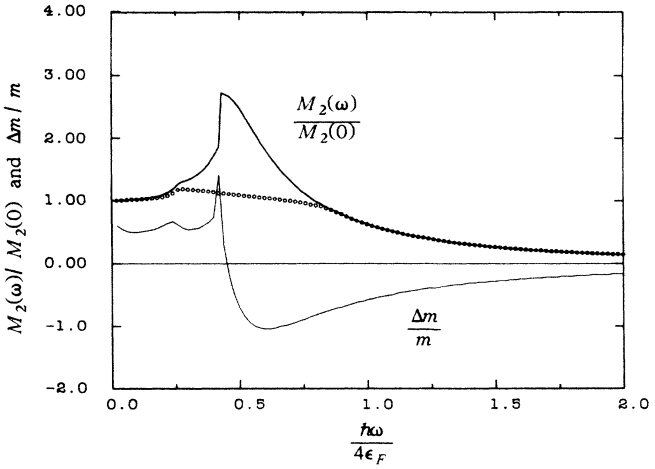

FIG. 3. $M_2(\omega)/M_2(0)$ and $\Delta m/m$, calculated from Eq. (6) with $g(q_z) \sim 1+\cos(q_z d)$, are shown as functions of $\hbar\omega/4\epsilon_F$ for the same system as described in Fig. 1.

to see how such a correlation affects the high frequency conductivity of a superlattice we examine a model function $g(q_z) \sim 1+\cos(q_z d)$ and calculate the real part and the imaginary part of the memory function from Eq. (6) for the same system as described in Fig. 1. The results are plotted in Fig. 3. The plasmon contribution begins at lower frequency (due to the modes of $q_z \neq 0$) and the peak at $\omega \geq \omega_p$ is diminished.

For a finite-number-layer system, in which $\tilde{N}(q,0)$ is finite at small q and the $q_z \neq 0$ components also contribute to the conductivity (even if the impurity distribution is uncorrelated), the rise of $M_2(\omega)$ is expected to occur gradually over a range of frequencies and the peak will, of course, be rounded off.

In summary, we have calculated the real part and the imaginary part of the high-frequency conductivity of a superlattice composed of an infinite number of quantum wells. Both the relaxation time and the effective-mass function show strong resonance around the bulk plasmon frequency of the system. Although finiteness of the layer number and correlation of the impurity distribution between different layers may smear the sharpness of the resonant edge, the plasma resonance emerges as an essential and striking feature of the high-frequency conductivity in these closely packed multilayer systems.

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