Ordering in ferromagnets with random anisotropy

E. M. Chudnovsky
Prospekt Gagarina 199, Apt. 17, Kharkov 310080, U.S.S.R.

W. M. Saslow
Department of Physics, Texas A&M University, College Station, Texas 77843

R. A. Serota
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 22 April 1985)

We summarize and extend our study (using real-space response and correlation functions) of the properties of a continuous-symmetry ferromagnet with random anisotropy, distinguishing between the cases of weak and strong random anisotropy. For the weak-anisotropy case we find three different magnetic regimes, according to the strength of the external magnetic field H. In zero H, the net magnetization is zero, although the ferromagnetic correlation length (FCL) is large. We call a ferromagnet in this first regime a correlated spin glass (CSG). It has a very large magnetic susceptibility, and hence a relatively small coherent anisotropy converts it into a nearly typical ferromagnetic domain structure. Also, a relatively small magnetic field nearly aligns the CSG, producing the second regime, which we call a ferromagnet with wandering axis (FWA). The FWA is a slightly noncollinear structure in which the tipping of the magnetization with respect to the field varies over the system. The tipping angle is correlated over a (field-dependent) correlation length which is smaller than the FCL of the CSG. As the field increases the correlation length in the FWA decreases, until the third regime is reached, wherein the tipping angles (which are smaller than in the FWA) are completely uncorrelated from site to site. We obtain the magnetization or susceptibility (as appropriate) for each of these three regimes. We also show that the temperature dependence of the (single-ion) random anisotropy strength can provide a plausible explanation for certain classes of reentrant phenomena and susceptibility cusps observed in magnetization studies. Neutron scattering studies appear to be consistent with the predicted $H^{-1/2}$ dependence of the FCL in the FWA regime, and display the expected rise of the FCL in the CSG regime as the random anisotropy strength decreases with increasing temperature.

I. INTRODUCTION

A. Defining the system

Imry and Ma[1] showed that random fields, no matter how small, destroy long-range order for spatial dimension $d \leq 4$ in systems with a continuous symmetry order parameter and a bending energy density proportional to the square of the gradient of that order parameter. They employed both real-space domain arguments and k-space fluctuation arguments.

Since the paper of Imry and Ma, a considerable amount of work has been done on that system and on the related one (which will be the subject of this paper), where the randomness is due to random anisotropy. To help define the discussion which follows, consider the macroscopic energy density

$$\epsilon = \tfrac{1}{2}\alpha(\nabla_i M_\mu)(\nabla_i M_\mu) - \tfrac{1}{2}\beta_r(\mathbf{M}\cdot\hat{\mathbf{n}}_r)^2 - \mathbf{M}\cdot\mathbf{H} , \quad (1.1)$$

where the magnetization $\mathbf{M}$ is assumed to be of fixed length M_0 (determined by the temperature and short-range exchange constants), the constant α is proportional to Ja^2 (where J is a microscopic exchange constant and a is an interatomic separation), and the constant β_r is proportional to a microscopic anisotropy D_r. The quantities α and β_r are taken to be fixed (although α is expected to increase with the magnetic impurity concentration x, and β_r is expected to decrease with the temperature T, as will be discussed in Sec. VI). The element of randomness comes in when one permits the anisotropy axis $\hat{\mathbf{n}}_r$ to point in arbitrary directions and to change significantly over a spatial scale R_a.

For the initial discussion, we will neglect the possibility of a coherent (i.e., nonrandom) anisotropy, such as, the uniaxial form

$$\epsilon_c = -\tfrac{1}{2}\beta_c(\mathbf{M}\cdot\hat{\mathbf{N}})^2 , \quad (1.2)$$

where $\hat{\mathbf{N}}$ does not vary. (Such a term could be due to magneto-elastic effects caused by the tensile stresses which inevitably occur during the process of preparing amorphous ferromagnets.) It is to be contrasted with the case of single-ion random anisotropy [the second term in (1.1)], for which the directions of the anisotropy axes are correlated only over a length R_a of several atomic spacings a. (However, observe that $R_a \gg a$ for the random anisotropy associated with a sample composed of polycrystallites.) The effects of including a coherent anisotropy will be considered in Sec. IV.

Note that random fields correspond to a onefold axis, and random anisotropy corresponds to a twofold axis. It

is straightforward to generalize to the case of a p-fold axis,[2] but it turns out that, within the phenomenological approach employed here, little depends on p. (However, the temperature dependence of the anisotropy is strongly p dependent, see Sec. VI.) Therefore, the $p=2$ results that we describe in what follows are appropriate (with only minor changes) for any p. Another point to be mentioned is that the results do not depend strongly on the number n of the spin components, as long as $n>2$ (to give the spins a continuous degree of freedom).[3] Although $n=3$ is the experimentally relevant case, many of the results we describe have been obtained for $n=2$. Finally, we note that the spatial dimensionality d is quite significant, because the response functions show a strong dependence on d. Most of the results we will describe have been obtained for the physically relevant case $d=3$, but other dimensionalities will also be considered.

B. Properties of the system: Summary

It will be useful to introduce a few definitions which permit us to associate a characteristic magnetic field with each of the terms in (1.1) and (1.2). These definitions will be useful for future reference. To the exchange term in (1.1) we associate the exchange field

$$H_{\rm ex}\equiv\alpha M_0/R_a^2 , \tag{1.3}$$

to the random anisotropy term in (1.1) we associate the random anisotropy field

$$H_r\equiv\beta_r M_0 , \tag{1.4}$$

and to the coherent anisotropy term of (1.2) we associate the coherent anisotropy field

$$H_c\equiv\beta_c M_0 . \tag{1.5}$$

When the random anisotropy is large ($H_r>H_{\rm ex}$), each spin is directed almost along the random anisotropy axis at its site. A representation of the magnetic state as a set of arrows ("arrow representation") would be the same as for a spin glass (SG). The magnetic susceptibility χ in this case is very small ($\chi\sim M_0/H_r$), and only in a very large magnetic field does reorientation occur, with the spins pointing in the hemisphere defined by the field. The system exhibits a finite coercivity and hysteretic behavior, which was studied in detail by Callen *et al.*[4] Collective behavior is only a secondary phenomenon in this case. (Note that in Coey's terminology[5] this system is called a speromagnet.)

When the random anisotropy is weak ($H_r<H_{\rm ex}$), the system displays clear remnants of its collective behavior in the absence of random anisotropy. Early work for $d=3$ was done by Alben, Becker, and Chi,[6] who employed Imry-Ma real-space domain arguments to obtain a characteristic correlation length $\sim R_a(H_{\rm ex}/H_r)^2$ (over which the spins retain a local ferromagnetic order), and the susceptibility of the system in the low-field limit. A study by Pelcovits *et al.*[7] employed renormalization-group arguments to show that the random anisotropy destroys the long-range order for $d\leq 4$. Fahnle and Kronmuller[8] performed a k-space fluctuation study of the transverse magnetization in the presence of a large field H, establishing that for $d=3$ the square of the transverse magnetization varies as $H^{-1/2}$, and therefore that the magnetization deviates from saturation as $H^{-1/2}$. This paper implicitly assumed that $H_r\ll H_{\rm ex}$ and $H_{\rm ex}\gg H$, and thus did not apply for $H_{\rm ex}\leq H$. In the latter case, the results of Ref. 4, predicting an approach to saturation as H^{-2} for large H_r, also apply for small H_r.

More recently, two of the present authors undertook a study emphasizing weak random anisotropy, using the real-space analog of the Imry-Ma k-space fluctuation arguments.[9-11] This involved consideration of real-space response and correlation functions, whose properties have helped to further reveal the behavior of this complex and rich system. The remainder of this section will be devoted to summarizing what has already been learned from the real-space correlation function approach, and to indicating new results that have been obtained in this paper.

In addition to confirming the semiquantitative calculations of the correlation length, susceptibility, and saturation field for the low-field state, the real-space correlation function approach has yielded the macroscopic anisotropy constant K (due to a rigid rotation of the spin system).[11] If one assumes that the macroscopic rotation angle is a valid dynamical variable (by which we mean that the system actually undergoes a motion involving a nearly uniform rotation in the long-wavelength limit), one finds a triply-degenerate set of normal modes,[11] quite similar to those expected for an isotropic Heisenberg spin glass with weak random anisotropy.[12-14] This resonance frequency, like the inverse of the susceptibility, the saturation field, and the macroscopic anisotropy, varies as $H_r^4/H_{\rm ex}^3$, a very small quantity for $(H_r/H_{\rm ex})\ll 1$. Because this system has no net magnetization, but only a local ferromagnetic magnetization which changes direction significantly as one moves by a distance on the order of $R_F\sim R_a(H_{\rm ex}/H_r)^2$ [which we call the ferromagnetic correlation length (FCL)], we have given it the name correlated spin glass (CSG).[9] Note that, like ordinary spin glasses (whose three-dimensionality is due to random exchange rather than random anisotropy, and whose correlation length is of the order of atomic dimensions), the CSG also is expected to have multiple energy minima which are separated by rather small energy barriers.

It is clear that a moderate field $H>H_s$, where

$$H_s\equiv H_r^4/H_{\rm ex}^3 , \tag{1.6}$$

produces a significant moment. Part of our earlier work focused on the intermediate field regime, where the system is characterized by an approach to saturation which varies as $H^{-1/2}$ [because the spins tip from the field by an amount $\theta\sim(H/H_s)^{-1/4}$]. In this paper, we extend our studies of the intermediate field regime, showing that (for any perturbation) the system is characterized by the correlation length $R_F^{\perp}\sim R_a(H_{\rm ex}/H)^{1/2}$. We characterize the system in this intermediate field regime as a ferromagnet with wandering axis (FWA). A semiquantitative argument is given which relates θ and $R_F^{\perp}$. It has been predicted that the FWA should support both a transverse and a longitudinal resonance in ESR.[15]

C. Organization of the paper

This paper is organized as follows. In Sec. II we treat the low-field (CSG) regime, first using semiquantitative real-space arguments, and then employing quantitative response function arguments. For simplicity we consider $d=3$ and $n=2$, but the $d=3$ and $n=3$ results are also given. In addition to summarizing the nature and results of this approach, we also discuss the complications associated with divergent response functions in lower dimensionalities. In Sec. III we consider the intermediate-field regime (FWA), using both semiquantitative real-space arguments and quantitative response function arguments. The physical case of $d=3$ and $n=3$ is considered explicitly. In Sec. IV we provide a brief summary of results for the large field regime, which is included largely for completeness. In Sec. V we consider the effects of uniform (or "coherent") anisotropy, showing that in the intermediate-strength regime the spins "wander," as in the FWA, although the system does successfully mimic an ordinary ferromagnet in many ways.

In Sec. VI we point out that the anisotropy constant β_r is not, in fact, constant, but rather is expected to decrease as the temperature is raised. This has significant implications for the interpretation of magnetization versus temperature data, and may provide an explanation for certain classes of reentrant phenomena and susceptibility cusps. In particular, for the case of strong random anisotropy at low temperatures, the decrease of anisotropy with temperature could lead to a crossover from a magnetic phase with an SG-like arrow representation and a small magnetic susceptibility (the speromagnet), to a CSG with a large FCL and a very large susceptibility, which might be interpreted (incorrectly) as being an ordinary ferromagnet. (One of the central themes of this section will be to show how one determines whether or not one has a ferromagnet with random anisotropy which is mimicking an ordinary ferromagnet.) For the CSG regime, neutron scattering studies appear to support a temperature-dependent anisotropy, and for the FWA regime they appear to support the predicted $H^{-1/2}$ correlation length. Finally, in Sec. VII we provide a summary and discussion, emphasizing what roles the diverse experimental techniques of low-field susceptibility, high-field magnetization, ESR, neutron scattering, Mössbauer effect, and nuclear orientation may play in elucidating the properties of ferromagnets with random anisotropy.

II. LOW-FIELD REGIME—CORRELATED SPIN GLASS

A. Overview

When the random anisotropy is weak ($H_r \ll H_{ex}$), one may estimate the ferromagnetic correlation length, R_F, for the zero-field case using the arguments of Refs. 1 and 6. From (1.1), when $H=0$, if the system is distorted over the spatial scale R_F, the energy density averaged over the corresponding volume is approximately given by

$$\epsilon \approx \tfrac{1}{2}\alpha M_0^2 R_F^{-2} - \tfrac{1}{2}\beta_r M_0^2 (R_F/R_a)^{-d/2} . \tag{2.1}$$

Minimizing this leads to

$$R_F \sim R_a (H_r/H_{ex})^{2/(d-4)} . \tag{2.2}$$

This shows that 4 is the marginal dimensionality, above which long-range ferromagnetic order is not destroyed. It also predicts a large FCL for $d<4$. This estimate of the FCL, however, can be applied straightforwardly only for $d>2$. As will be discussed in Sec. II B, the analytical calculations encounter complications caused by divergent Green functions. This point is not revealed in the above argument, which implicitly assumes that only spins within a radius R_F contribute.

We now estimate the zero-field magnetic susceptibility for the physically relevant case $d=3$. Then (2.2) yields

$$R_F \sim R_a (H_{ex}/H_r)^2 \quad (d=3) . \tag{2.3}$$

From (2.1) and (2.3), the energy associated with the formation of a FCL in $d=3$ is of the order of

$$M_0^2 D_r (R_F/R_a)^{-3/2} \sim M_0 (H_r^4/H_{ex}^3) . \tag{2.4}$$

Now observe that the magnetization induced by a small magnetic field is of the order of the tipping angle $\delta\theta$ towards the field. Since the exchange and anisotropy energies are minimized for $\delta\theta=0$, their energy change on tipping is proportional to $(\delta\theta)^2$ and the FCL formation energy. Thus, the total energy change due to tipping is approximately given by

$$\delta\epsilon \approx -M_0 H \delta\theta + M_0 (H_r^4/H_{ex}^3)(\delta\theta)^2 , \tag{2.5}$$

and minimization gives

$$\delta\theta \sim H/(H_r^4/H_{ex}^3) . \tag{2.6}$$

Thus, we obtain

$$\chi \sim M_0 \delta\theta/H \sim M_0/(H_r^4/H_{ex}^3) , \tag{2.7}$$

which is very large.[16] An immediate consequence is that the CSG exists only for fields, such as,

$$H < M_0/\chi \sim (H_r^4/H_{ex}^3) = H_s , \tag{2.8}$$

where H_s is defined in (1.6). A field stronger than H_s aligns the CSG, leading to the "FWA" state, which will be treated in detail in Sec. III.

B. Analytical results

For $H=0$, analysis[2,17] shows that the spin-spin correlation function decays exponentially over the FCL, which means that the magnetization undergoes smooth stochastic rotations over the sample. The net magnetization is zero, whereas the ferromagnetic order persists over any length scale smaller than R_F. There are no sharp boundaries between the ferromagnetically ordered regions, unlike the typical ferromagnetic domain structure in which domains whose spins are aligned in opposite directions are separated by a thin domain wall. In fact, the subdivision into ferromagnetic regions in the present case has a somewhat arbitrary character.

In our previous papers[9-11] we called the above type of magnetic order a correlated spin glass. A ferromagnet can be converted into a multitude of different CSG struc-

tures with the same FCL by weak random anisotropies—a situation typical for a SG. Note that although the spins are ferromagnetically correlated over a FCL, an arrow representation of the system as a whole is that of a SG, which accounts for the name CSG.

For simplicity we consider the case $n=2$ (the planar ferromagnet), for which it is convenient to introduce the angles θ and ϕ via

$$\mathbf{M}(x)=M_0\{\cos\theta(x),\sin\theta(x)\}\ ,$$
$$\mathbf{n}_r(x)=\{\cos\phi(x),\sin\phi(x)\}\ . \tag{2.9}$$

The random anisotropy is assumed to be correlated over R_a due to short-range structural order. This implies (on averaging over the disorder) that the random angle ϕ may be taken to satisfy

$$\langle \sin 2\phi(x')\sin 2\phi(x'')\rangle=\tfrac{1}{2}\exp(-|x'-x''|/R_a)\ . \tag{2.10}$$

The specific form given here for the spatial decorrelation of the anisotropy is chosen for technical convenience. Any other reasonable form with the same characteristic length R_a would be acceptable.

The dimensionless quantity (H_r/H_{ex}) appears in nearly all subsequent considerations. In the case of weak random anisotropy, it is small. Minimizing the energy functional of (1.1) for $H=0$, we obtain

$$\nabla_d^2\theta(x)=\frac{(H_r/H_{ex})}{2R_a^2}\sin\{2[\theta(x)-\phi(x)]\}\ , \tag{2.11}$$

where the index d stands for the dimensionality of space.

At this point we should emphasize that, unlike Ref. 18, we do not solve Eq. (2.11) by iterations. For zero field, because we expect that there are an infinite number of solutions, each with the magnetization undergoing smooth stochastic rotations in all directions, we do not attempt to produce any particular solution to (1.1), but rather, we study the way in which correlations of the magnetization fall off with distance. Therefore, we employ the d-dimensional Green function $G_d(x)$ of the Laplace equation to rewrite (2.11) in integral form, and then proceed to compute the angle-angle correlation function:

$$\langle[\theta(x_2)-\theta(x_1)]^2\rangle=\left[\frac{(H_r/H_{ex})}{2R_a^2}\right]^2\int\int[G_d(x_2-x')-G_d(x_1-x')][G_d(x_2-x'')-G_d(x_1-x'')]$$
$$\times\langle\sin\{2[\theta(x')-\phi(x')]\}\sin\{2[\theta(x'')-\phi(x'')]\}\rangle d^dx'd^dx''\ . \tag{2.12}$$

We first consider $d=3$ and $d=4$. The qualitative considerations given in the Introduction indicate that θ decorrelates over a much larger spatial scale (the FCL) than does ϕ, so that, to lowest order in $(H_r/H_{ex})^2$,

$$\langle\sin\{2[\theta(x')-\phi(x')]\}\sin\{2[\theta(x'')-\phi(x'')]\}\rangle\simeq\langle\sin[2\phi(x')]\sin[2\phi(x'')]\rangle\ . \tag{2.13}$$

Substitution of (2.13) into (2.11) yields[9]

$$\langle[\theta(x_2)-\theta(x_1)]^2\rangle=\begin{cases}\frac{1}{4}(H_r/H_{ex})^2(|x_2-x_1|/R_a) & (d=3)\ ,\\ \frac{3}{8}(H_r/H_{ex})^2\ln(|x_2-x_1|/R_a) & (d=4)\ .\end{cases} \tag{2.14}$$

We now employ the recent argument[2] that the system under consideration satisfies the relation

$$\langle\mathbf{M}(x_2)\cdot\mathbf{M}(x_1)\rangle=M_0^2\exp\{-\tfrac{1}{2}\langle[\theta(x_2)-\theta(x_1)]^2\rangle\}\ . \tag{2.15}$$

Provided that (2.15) is true, the magnetization correlation function can be expressed as

$$\langle\mathbf{M}(x_2)\cdot\mathbf{M}(x_1)\rangle=\begin{cases}M_0^2\exp\{-|x_2-x_1|/[8(H_r/H_{ex})^{-2}R_a]\} & (d=3)\ ,\\ M_0^2(R_a/|x_2-x_1|)^{(3/8)(H_r/H_{ex})^2} & (d=4)\ ,\end{cases} \tag{2.16}$$

in agreement with the qualitative predictions of (2.4). In particular, Eq. (2.16) gives a power-law (rather than exponential) decay in the marginal dimensionality $d=4$.

We may define the ferromagnetic correlation length as the characteristic distance over which exponential decay of the correlations takes place. From (2.16) it is given by

$$R_F=8(H_r/H_{ex})^{-2}R_a\quad(n=2,\ d=3)\ . \tag{2.17}$$

The calculation for three-dimensional spins ($n=3$) gives a slight renormalization of the FCL:[2,9]

$$R_F=\tfrac{15}{2}(H_r/H_{ex})^{-2}R_a\quad(n=3,\ d=3)\ . \tag{2.18}$$

This renormalization (16 going to 15) on going from $n=2$ to $n=3$ holds for many phenomena to be considered in what follows. To make our results more convenient for comparison with experiment, we hereafter present our results for the case $n=3$. Clearly, the specific value of n is not significant except to produce a slight renormalization

of certain coefficients. Note that (2.4) is in qualitative agreement with (2.17) and (2.18).

Eqs. (2.17) and (2.18) give $(H_r/H_{ex})^{crit} \approx 1-3$ as a criterion to distinguish between the weak- and strong-anisotropy limits. As mentioned in the Introduction, the case of strong anisotropy

$$(H_r/H_{ex}) > (H_r/H_{ex})^{crit}, \quad R_F \sim R_a$$

was studied by Callen *et al.*[4] This case will be discussed in more detail in Sec. V.

To compute the susceptibility in the CSG regime,[10] one must include the magnetic field term in (1.1). This modifies (2.11) so that it now reads

$$\nabla_d^2\theta(x) - \delta_H^{-2}\sin[\theta(x)] = \frac{(H_r/H_{ex})}{2R_a^2}\sin\{2[\theta(x)-\phi(x)]\}\ , \tag{2.19}$$

where

$$\delta_H^{-2} \equiv (H/H_{ex})R_a^{-2}\ . \tag{2.20}$$

Treating the field as a perturbation, and looking for a solution to (2.20) in the form

$$\theta(x) = \theta_0(x) + \delta\theta(x)\ , \tag{2.21}$$

where $\theta_0(x)$ is a solution to (2.11) [the $H=0$ version of (2.19)], one finds that $\delta\theta(x)$ satisfies

$$\nabla_d^2\delta\theta(x) = \delta_H^{-2}\sin[\theta_0(x)] + \frac{(H_r/H_{ex})}{R_a^2}\cos\{2[\theta_0(x)-\phi(x)]\}\delta\theta(x)\ . \tag{2.22}$$

Although $\sin\{2[\theta(x)-\phi(x)]\}$ becomes correlated to $\nabla_d^2\theta(x)$ through (2.19), we expect that $\cos\{2[\theta_0(x)-\phi(x)]\}$ remains basically random. Therefore, to lowest order in H it does not correlate to $\delta\theta$, and hence the second term on the right-hand side of (2.22) may be neglected. Thus, the solution to (2.22) is given by

$$\delta\theta(x) = \delta_H^{-2}\int d^3x' G(x-x')\sin[\theta_0(x')]\ . \tag{2.23}$$

From this the susceptibility is obtained from

$$\begin{aligned}\chi &= (M_0/H)\langle\cos[\theta(x)]\rangle = -(M_0/H)\langle\sin[\theta_0(x)]\delta\theta(x)\rangle \\ &= -(M_0/H)\delta_H^{-2}\int d^3x' G(x-x')\langle\sin[\theta_0(x')]\sin[\theta_0(x)]\rangle \\ &= \left[\frac{32M_0}{(H_r^4/H_{ex}^3)}\right] = \frac{1}{2\beta_r}\left[\frac{4}{(H_r/H_{ex})}\right]^3 \quad (n=2,\ d=3)\ .\end{aligned} \tag{2.24}$$

(See also Refs. 6, 17, and 19.) Here we have employed the relation

$$\langle\sin\theta_0(x')\sin\theta_0(x'')\rangle = \tfrac{1}{2}\exp(-|x'-x''|/R_F)\ , \tag{2.25}$$

which follows from (2.16). For $n=3$ and $d=3$ one can show that

$$\chi = \frac{1}{2\beta_r}\left[\frac{15}{4(H_r/H_{ex})}\right]^3 \quad (n=3,\ d=3)\ . \tag{2.26}$$

The susceptibility given in (2.26) can be quite large.[16,20]

We now consider the anisotropy constant K for the CSG state, with a view toward finding the normal-mode frequencies in the long-wavelength limit.[12] In this case, true $n=2$ spins have no dynamics,[4] so our calculations are only appropriate to $n=3$, where the frequency is given by[12,13,21]

$$\omega = \gamma(K/\chi)^{1/2}\ , \tag{2.27}$$

where γ is the gyromagnetic ratio. Rather than compute K for the $n=3$ CSG, we will consider the $n=2$ CSG, where we will only make an estimate. Specifically,

$$\begin{aligned}K &\equiv \langle\partial^2\epsilon/\partial\theta^2\rangle = \beta_r M_0^2\langle\cos[2(\theta-\phi)]\rangle \\ &\sim \alpha M_0^2(\partial_i\theta)^2 \sim \alpha M_0^2/R_F^2 \\ &\sim \frac{M_0 H_r^4}{H_{ex}^3} \sim \frac{M_0}{\chi}\ .\end{aligned} \tag{2.28}$$

By way of contrast, note that $K \sim H_r^2/H_{ex}$ for spin glasses; hence, the result embodied in (2.28) cannot be considered to be an obvious one.[13,14,21-23] Combining (2.27) and (2.28) yields[11]

$$\omega \sim \frac{\gamma M_0}{\chi} \sim \frac{\gamma H_r^4}{H_{ex}^3}\ . \tag{2.29}$$

For $H_r \ll H_{ex}$ this can be very low, and one can question that this mode can actually be observed, on a number of accounts. First, the rotational degree of freedom associated with this mode may not be a dynamical one. Note that, in the superficially similar case of the SG, the rotational degree of freedom is defined by the state produced exclusively by the exchange interactions, and the anisotropy is a perturbation; whereas for the CSG one must employ the microscopic anisotropy to produce the very three dimensionality which makes the rotation a degree of freedom that is independent of the magnetization. (The different dependence of K on H_r and H_{ex} is surely an indication of this difference.) Another (and perhaps equivalent) point is that, due to the inhomogeneous nature of the CSG, the resonance might have a significant inhomogeneous linewidth. Moreover, this oscillation frequency is so low that homogeneous broadening (i.e., T_1 relaxation) may make the mode unobservable. For wavelengths smaller than the FCL ($k^{-1} < R_F$) the normal modes are expected to be ferromagneticlike magnons with $\omega = \gamma M_0 \alpha k^2$.[11]

C. Ordering in lower dimensionalities

As shown in Ref. 9, and outlined in Sec. II A, $d=2$ is the marginal dimensionality for the straightforward implementation of the qualitative arguments due to Imry and Ma.[1] Using $G_2(x)$ and the results of Sec. I, one finds that the angle-angle correlation function diverges logarithmically in an infinite system.[9] Specifically, for a system of dimension L, we find that[24]

$$\langle[\theta(x_2)-\theta(x_1)]^2\rangle = \frac{(H_r/H_{ex})^2}{4}\frac{|x_2-x_1|^2}{R_a^2}\ln\left[\frac{L}{|x_2-x_1|}\right]. \quad (2.30)$$

It should be emphasized that the divergence in (2.30) is not due to the approximation leading to (2.13), but rather to the logarithmically diverging Green function. This divergence, familiar from electrostatics in $d=2$, signals that the more remote spins enter Eq. (2.12) with larger weights. Thus, the spins under consideration are extremely sensitive to the directions of the spins at infinity, which causes the system to be totally unstable. We believe that for the same reason one finds divergent time-dependent single-site spin-spin correlation functions.[25]

The presence of coherent anisotropy, however, provides a natural infrared cutoff in Eq. (2.30): the Bloch domain wall width δ_B. Furthermore, because of the slow logarithmic divergence, even a very weak coherent anisotropy (compared to actual experimental situations) restores the Imry-Ma picture [cf., Eq. (2.4)] with an FCL $\sim(H_r/H_{exp})^{-1}R_a$.[24] This result does not contradict the conclusion of Cardy and Ostlund[26] that for $p^2>8$, a pure XY phase exists over a finite temperature range. Indeed, at lower temperatures (where we expect a phenomenological approach to be valid) they find only short-range correlations of the magnetization. However, as pointed out by Goldschmidt and Schaub,[25] a vortex-unbinding mechanism might be equally important for understanding the low-temperature phase. We hope to extend our analysis to finite temperatures, including the effect of vortices, elsewhere.

As follows from Eqs. (2.12) and (2.13), when one employs the $d=1$ Green function (which varies as $|x|$), the angle-angle correlation function diverges linearly with the size of the system.[27] A CSG with a FCL of $\sim(H_r/H_{ex})^{-2/3}R_a$ is recovered in this case [cf., Eq. (2.4)] by imposing boundary conditions in a very large but finite system: for example, one can fix the spins at the boundaries. However, the details of the system would be very sensitive to the actual boundary conditions imposed on the spins.

A similar idea has been exploited by Vinokur *et al.*[28] However, the correlation function they find is not the true spin-spin correlation function: although their γ_N is indeed the orientation of the Nth spin in the chain of $N+1$ spins, their approach allows all the preceding spins to relax, in order to adjust to the spin which has been most recently accounted for. Thus, their γ_0 is not the adjusted orientation of the spin at the origin.

Finally, we remark that the above discussion of the divergences which occur in lower dimensionalities is in disagreement with the work of Villain and co-workers, who predict a power-law decay in $d=3$.[27,29] Such a decay, possessing no intrinsic spatial scale, is in disagreement with the Imry-Ma argument, which is generally accepted as giving the characteristic spatial scale for the decay of correlations in these systems.

III. INTERMEDIATE FIELD REGIME: FERROMAGNET WITH WANDERING AXIS

A. Overview

According to (2.8), in a field on the order of H_s, the system becomes nearly aligned. Clearly, the assumption that the system has no significant moment is no longer applicable for $H \gtrsim H_s$, and thus we must turn to a new analysis. In this case, it is appropriate to consider that the system is basically aligned, with the random anisotropy providing a perturbation about the aligned state. For the intermediate field regime ($H_s < H < H_{ex}$), this phase retains its alignment due to the combined effects of both the applied field and the exchange energy. Nevertheless, the system is not completely ordered, since the random anisotropy causes the local magnetization axis to wander slightly as one moves about the system. For this reason, we describe the system in this regime as a ferromagnet with wandering axis.

Thus, the FWA is a slightly noncollinear structure in which the deviation of the magnetization with respect to the field varies over the system. The deviation angle θ is correlated over a (field-dependent) correlation length $R_F^{\perp}$. To see this, one minimizes the energy, which is written as a sum of each of its three contributions, expanded about the angle θ made by the magnetization with respect to the field, and with the characteristic scale over which the magnetization varies taken to be R:

$$\epsilon \approx \left[\frac{\alpha}{2R^2}\right](M_0^2\theta^2) - \tfrac{1}{2}(\beta_r M_0^2\theta)(R_a/R)^{3/2} + M_0 H\theta^2/2 . \quad (3.1)$$

The first and third terms give the increase in the exchange and Zeeman energies due to the noncollinearity, and have no terms linear in θ because $\theta=0$ will minimize each of these terms. The second term gives the decrease in the random anisotropy energy arising from the adjustment of the local magnetization to the spatial fluctuations of the anisotropy axis orientations; it has a term linear in θ because its minimum is not at $\theta=0$.

Since all three terms in θ are coupled by the minimization conditions, they are of the same order of magnitude. Consideration only of the θ^2 terms in the exchange and Zeeman energies thus leads to the characteristic length

$$R_F^{\perp} \equiv \left[\frac{\alpha M_0}{H}\right]^{1/2} = R_a(H_{ex}/H)^{1/2} . \quad (3.2)$$

[Note that the result (3.2) is independent of the nature of the perturbation to the system.] As for R_F of the CSG, $R_F^{\perp}$ also has the meaning of a ferromagnetic correlation length because the FWA, although aligned along the field,

preserves a nearly rigid ferromagnetic order only over a length on the order of $R_F^{\perp}$. On the other hand, in the preceding section on the CSG, the quantity equivalent to $R_F^{\perp}$ was called δ_H, and it was not the ferromagnetic correlation length for the CSG. Also, note that $R_F^{\perp} \approx R_F$ for $H \approx H_s$, but $R_F^{\perp} < R_F$ for larger fields H. Thus as H increases, $R_F^{\perp}$ decreases. When $R_F^{\perp}$ becomes comparable to R_a, one has $H \approx H_{ex}$, and for larger H the system maintains its stability about $\theta \approx 0$ dominantly due to the presence of the field.

The full minimization of (3.1) leads to (3.2) and the characteristic tipping angle

$$\theta \sim (R_a/R_F^{\perp})^{3/2}(H_r/H) \sim H_r/(H_{ex}^3 H)^{1/4} \sim (H_s/H)^{1/4} . \tag{3.3}$$

The characteristic tipping angle θ gives the magnetization deviation via

$$\delta M \approx (M_0/2)\theta^2 \sim M_0 H_r^2/(H_{ex}^3 H)^{1/2} \sim M_0 (H_s/H)^{1/2} \tag{3.4}$$

in the approach to saturation.

B. Analytical results

It is much simpler to obtain exact results for the FWA than for the CSG, since the starting point (a nearly collinear system) is so much less complex. To include field effects[10] we include the Zeeman term $-\mathbf{M}\cdot\mathbf{H}$ in the energy functional (1.1). For sufficiently large fields, the deviation angle θ is small, which allows us to treat the problem perturbatively. Minimization of (1.1) for $n=3$, subject to the constraint that $|\mathbf{M}| = M_0$, gives

$$\mathbf{0} = \mathbf{M} \times \frac{\delta \epsilon}{\delta \mathbf{M}} = \mathbf{M} \times [-\mathbf{H} - \beta_r \hat{\mathbf{n}}_r (\mathbf{M}\cdot\hat{\mathbf{n}}_r) - \alpha \nabla^2 \mathbf{M}] . \tag{3.5}$$

In the intermediate field regime, for which $\mathbf{M} \approx M_0 \hat{\mathbf{H}} + \mathbf{M}_{\perp}$, (3.5) becomes (on dropping the subscript on $\hat{\mathbf{n}}_r$)

$$[\nabla^2 - (R_F^{\perp})^{-2}]\mathbf{M}_{\perp} \approx -\frac{(H_r/H_{ex})M_0}{R_a^2}(\mathbf{C}_{\perp}) , \tag{3.6}$$

where

$$\mathbf{C}_{\perp} \equiv (M_{||}\hat{\mathbf{n}}_{\perp} - \mathbf{M}_{\perp}\hat{\mathbf{n}}_{||})(M_{||}\hat{\mathbf{n}}_{||} + \mathbf{M}_{\perp}\cdot\hat{\mathbf{n}}_{\perp})/M_0^2 . \tag{3.7}$$

It solution is given by

$$\mathbf{M}_{\perp}(x) = \frac{M_0 H_r}{4\pi R_a^2 H_{ex}} \int d^3x' \frac{\exp(-|x-x'|/R_F^{\perp})}{|x-x'|}(\mathbf{C}_{\perp}') . \tag{3.8}$$

From (3.8) it then follows that

$$\langle \mathbf{M}_{\perp}(x)\cdot\mathbf{M}_{\perp}(x+y)\rangle = \left[\frac{M_0 H_r}{4\pi R_a^2 H_{ex}}\right]^2 \int d^3x' d^3x'' \frac{\exp[-(|x-x'| - |x+y-x''|)/R_F^{\perp}]}{|x-x'|\,|x+y-x''|}\langle \mathbf{C}_{\perp}'\cdot\mathbf{C}_{\perp}''\rangle . \tag{3.9}$$

If the anisotropy decorrelates over a distance R_a, then it is reasonable to assume that the average on the right-hand side of (3.9) amounts to its value for $x'=x''$ times an exponential decorrelation factor:

$$\langle \mathbf{C}_{\perp}'\cdot\mathbf{C}_{\perp}''\rangle \approx \frac{2}{15}\exp(-|x'-x''|/R_a) . \tag{3.10}$$

[In writing this, we neglect a factor of $(1-|\mathbf{M}_{\perp}|^2/2M_0^2)$, which we take to be nearly equal to unity in the intermediate field regime.] Then, since $R_a \ll R_F^{\perp}$, (3.10) permits us to set $x''=x'$ in $|x+y-x''|$ of (3.9). The integral over x'' is then straightforward, so that (3.9) becomes

$$\langle \mathbf{M}_{\perp}(x)\cdot\mathbf{M}_{\perp}(x+y)\rangle \approx \left[\frac{M_0 H_r}{4\pi R_a^2 H_{ex}}\right]^2 \left[\frac{16\pi R_a^3}{15}\right] \int d^3x' \frac{\exp[(-|x-x'| - |x+y-x'|)/R_F^{\perp}]}{|x-x'||x+y-x'|} \tag{3.11}$$

For $y=0$, (3.11) yields

$$\begin{aligned}\langle |\mathbf{M}_{\perp}(x)|^2\rangle &\approx \left[\frac{M_0 H_r}{4\pi R_a^2 H_{ex}}\right]^2 \left[\frac{16\pi R_a^3}{15}\right] \int d^3x' \frac{\exp[-2|x-x'|/R_F^{\perp}]}{|x-x'|^2} \\ &= \tfrac{2}{15} M_0^2 \left[\frac{H_s}{H}\right]^{1/2} \quad (H_s \equiv H_r^4/H_{ex}^3) \\ &= \tfrac{2}{15}(H_r/H_{ex})^{3/2}\left[\frac{H_r}{H}\right]^{1/2} M_0^2 .\end{aligned} \tag{3.12}$$

Thus, the magnetization deviation is given by

$$(\delta M/M_0) \equiv (M_0 - M)/M_0 \approx \frac{1}{2M_0^2}\langle |\mathbf{M}_{\perp}(x)|^2\rangle = \tfrac{1}{15}(H_r/H_{ex})^{3/2}\left[\frac{H_r}{H}\right]^{1/2} = \frac{1}{15}\left[\frac{H_s}{H}\right]^{1/2} . \tag{3.13}$$

Reference 8 was the first to obtain an approach to saturation going as $H^{-1/2}$. However, Ref. 10 was the first to establish that this behavior is true only for weak random anisotropy, and that (3.13) applies only in the intermediate field regime. This approach to saturation has been observed in recent experiments, after the ever present background term linear in H was subtracted from the data.[30,31] Note that for $d=3$ and $n=2$, the factor of 15 in (3.13) is replaced by a factor of 16.[10] The analysis for $d=2$ and $n=2$ is given in Ref. 24, where it was found that for the intermediate regime the approach to saturation goes as H^{-1}, rather than as $H^{-1/2}$.

Note that it follows from (3.13) that the differential susceptibility in the FWA regime is given by

$$\chi=\partial M/\partial H=\frac{1}{30}\frac{H_r^2}{(H_{ex}H)^{3/2}}M_0 \ . \tag{3.14}$$

The interplay of the different parameters (random anisotropy, coherent anisotropy, exchange stiffness, magnetic field, and short-range structural order) is studied in detail in Ref. 16. In Fig. 1 we present a typical magnetization curve for a ferromagnet with weak random anisotropy, as follows from Eqs. (2.26) and (3.13), assuming that (3.13) holds throughout the FWA regime.

For $y\neq 0$ (3.11) can be employed to provide a detailed rationale for the FWA terminology. First, note that the first exponential in (3.11) makes $|x-x'|\leq R_F^{\perp}$. Thus, for $y\geq R_F^{\perp}$, the second exponential in (3.11) forces $\langle \mathbf{M}_{\perp}(x)\cdot\mathbf{M}_{\perp}(x+y)\rangle$ to fall off as $\exp(-y/R_F^{\perp})$. Hence (3.11) and (3.12) yield that the transverse magnetization correlation satisfies

$$\langle \mathbf{M}_{\perp}(x)\cdot\mathbf{M}_{\perp}(x+y)\rangle \approx \langle |\mathbf{M}_{\perp}(x)|^2\rangle \exp(-y/R_F^{\perp}) \quad (R_a\ll R_F^{\perp}) \ . \tag{3.15}$$

Clearly, Eq. (3.15) implies that true ferromagnetic order holds only over a field-dependent correlation length $R_F^{\perp}$. As was mentioned in the Introduction, the FWA has some similarity to the CSG, in that the spin adjustments to the spatial fluctuations in the orientations of the random anisotropy axes are of great importance. In the FWA each individual spin does not "feel" its individual random anisotropy, as is also the case for the CSG.

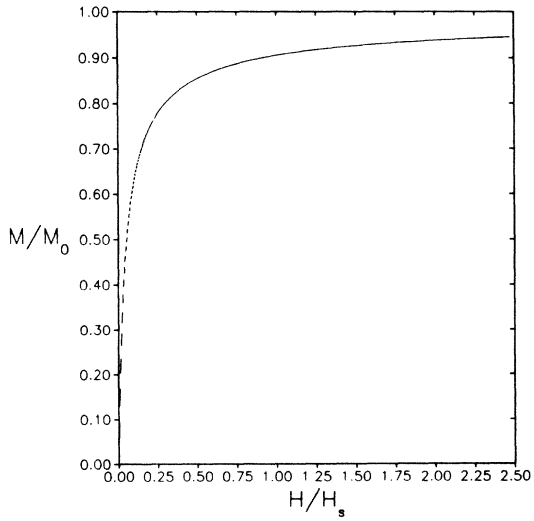

FIG. 1. Reduced magnetization versus reduced field. $H_s\equiv H_r^4/H_{ex}^3$.

IV. LARGE-FIELD REGIME

In the large-field regime ($H>H_{ex}$), the noncollinear structure closes even further toward the field: each spin is only slightly tipped from the applied field by the (random) anisotropy at its site. The tipping angle is then of the order of H_r/H and, hence, the magnetization deviation in the approach to saturation is proportional to $(H_r/H)^2$ (this result applies also to the case of strong random anisotropy, where $H_r>H_{ex}$, provided that the field satisfies $H\gg H_r$). More specifically, it has been shown[10,11] that

$$\delta M/M_0=\tfrac{1}{15}[H_r/(H+H_{ex})]^2 \quad (H_{ex}\ll H) \ . \tag{4.1}$$

We repeat that (4.1) is also true when the random anisotropy is strong relative to exchange ($H_r\gg H_{ex}$), so long as it is still weak compared to the field.[4] This is not surprising, since in both cases we deal with what is essentially a single-site problem.

V. EFFECT OF COHERENT ANISOTROPY

We now consider the effects of coherent uniaxial anisotropy [Eq. (1.2)]. Noting that $H_c(\hat{\mathbf{M}}\cdot\hat{\mathbf{N}})\hat{\mathbf{N}}$ is like an applied magnetic field, and that for $H_c=0$ and $H<H_s$ the system is a CSG with $M=\chi H$, it is clear that for $H=0$ and $H_c<H_s$ the system is a CSG whose "arrow representation" is skewed toward both $+\hat{\mathbf{N}}$ and $-\hat{\mathbf{N}}$, and has no net magnetic moment. In the presence of a field $H<H_s$ along, say, $+\hat{\mathbf{N}}$, one has a skewing of the CSG toward $+\hat{\mathbf{N}}$, due to H, but H_c still tends to skew both $\pm\hat{\mathbf{N}}$. Thus for the case of weak random anisotropy and $(H+H_c)\ll H_s$ one has a CSG with magnetization approximately given by

$$M=\chi H \quad [(H+H_c)\ll H_s\ll H_{ex}] \ , \tag{5.1}$$

where χ is given by (2.26). Note that the FCL continues to be given by (2.18).

On the other hand, for $H_c>H_s$ the system is very like a FWA, with either $+\hat{\mathbf{N}}$ or $-\hat{\mathbf{N}}$ as the preferred direction, and a coherence length given by $\delta_c\sim R_a(H_{ex}/H_c)^{1/2}$. This has the same form as the (Bloch) domain wall width (in the absence of random anisotropy) of an ordinary uniaxial ferromagnet. The difference between this case and the ordinary case is that, although both have $\pm\hat{\mathbf{N}}$ domains (with the same domain wall width), in the presence of random anisotropy the magnetization wanders even within a domain, on the spatial scale δ_c, by a characteristic angle on the order of $(H_s/H_c)^{1/4}$ [which is the obvious generalization of (3.3)]. In the presence of a field $H>H_c$ along $\pm\hat{\mathbf{N}}$, so that all domains align with the field, one needs to simply replace H by $(H+H_c)$. In this regime the magnetization satisfies

$$\delta M/M_0=\frac{1}{15}\frac{H_r^2}{[H_{ex}^3(H+H_c)]^{1/2}}$$
$$[H_s \ll (H+H_c) \ll H_{ex}] . \quad (5.2)$$

Moreover, for this same regime the FCL satisfies

$$R_F^{\perp}=R_a\left[\frac{H_{ex}}{(H+H_c)}\right]^{1/2} . \quad (5.3)$$

Finally, for a field $H>H_c$ such that $H_{ex}\ll(H+H_c)$, the obvious generalization of (4.1) is given by

$$\delta M/M_0=\tfrac{1}{15}[H_r/(H+H_c+H_{ex})]^2$$
$$[H_{ex}\ll(H+H_c+H_{ex})] . \quad (5.4)$$

VI. TEMPERATURE-DEPENDENT ANISOTROPY AND ITS EXPERIMENTAL IMPLICATIONS

The length R_a, assumed to be of several atomic spacings, over which the random anisotropy axes are correlated, can be ascribed to whatever short-range crystalline order persists in amorphous magnets. This "residual" crystallinity accounts for the fact that the strength of the random anisotropy is indeed of the order of the strength of the crystalline anisotropy in the crystalline counterpart of an amorphous magnet.[32] It is well known, however, that crystalline anisotropy can depend on high powers of the magnetization.[33] Therefore, starting at low temperature, even a slight decrease of the magnetization with increasing temperature might result in a sharp decrease of the random anisotropy strength $\beta_r(T)$. Specifically, it follows from Ref. 33 that $\beta_r\sim M^{[p(p+1)/2]-2}$. For the present case of $p=2$, we have $\beta_r\sim M^{\lambda}$, with $\lambda=1$. (Note that the result of Ref. 33 is most rigorously established only at low temperatures. Certainly, near T_c it is not known how β_r behaves. Nevertheless, it is to be expected that the anisotropy β_r decreases as one increases the temperature, and we will accept the result of Ref. 33 for the sake of argument. Therefore certain of the results which follow may only have a qualitative validity.) Since $\chi\sim M_0\beta_r^{-4}$ for the CSG [cf. (2.24)], if we take the mean-field result that $M(T)/M(0)\approx(1-T/T_c)^{1/2}$, we would then have

$$\chi(T)/\chi(0)\approx(1-T/T_c)^{-3/2} . \quad (6.1)$$

The mechanism proposed above is probably operative for rare-earth-based compounds with strong single-ion anisotropy.[30] In particular, we consider the case where, at $T=0$, $(H_r/H_{ex})_0>(H_r/H_{ex})^{crit}$. Then, as T increases, there is the possibility of a crossover to the weak-anisotropy regime, where $(H_r/H_{ex})_T\ll(H_r/H_{ex})^{crit}$. In other words, the apparently random spin-glass–like structure[4] (or speromagnet[5]) described in the Introduction becomes transformed into a CSG. The latter, due to its large FCL (which increases as T increases), and its ability both to align in relatively weak fields and to form domains when subject to a relatively weak coherent anisotropy, might very successfully mimic an ordinary ferromagnet.[30] In contrast to the anisotropy-induced speromagnetic "state," which is very unpolarizable due to the strong anisotropy $[\chi(0)\sim\beta_r^{-1}]$, the CSG is very soft

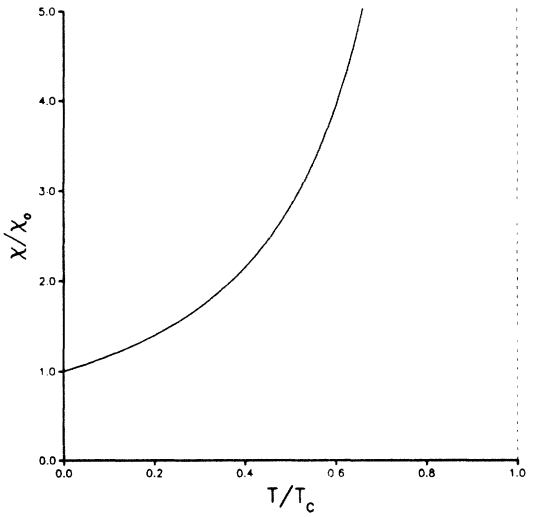

FIG. 2. Reduced susceptibility versus reduced temperature.

$(\chi(T)\sim\beta_r^{-1}(T)[(H_r/H_{ex})_T]^{-3}$ and $(H_r/H_{ex})_T\ll 1)$. This can be summarized in Fig. 2, which is a plot of χ versus T [based upon (6.1)], which is in qualitative agreement with the susceptibility cusps which have been seen experimentally.

Since the approach employed in this paper assumes an almost fixed magnitude for the magnetization, it is strictly expected to apply only at relatively low temperatures (and, thus, only far from criticality). Nevertheless, simple estimates within the present framework show that the FCL ξ of the typical paramagnetic-ferromagnetic (PM-FM) transition is much smaller in the critical region than the FCL of the CSG. To see this, note that near T_c, (2.18) yields

$$R_F\sim(H_r/H_{ex})_0^{-2}(1-T/T_c)^{-2\beta\lambda} , \quad (6.2)$$

where λ may no longer be the mean-field value of unity, and the subscript 0 denotes the $T=0$ value. We have also employed the critical exponent β for the magnetization in the critical region. On the other hand,

$$\xi\sim(1-T/T_c)^{-\nu} , \quad (6.3)$$

where ν is the critical exponent for the ordinary FCL. Taking $\lambda=1$ for the sake of argument, and noting that $2\beta\approx0.7$ and $\nu\approx0.7$ for the Heisenberg model ($n=3$) in $d=3$, we see that R_F/ξ is nearly temperature independent; since $R_F/\xi\gg1$ at $T=0$, it should also be true near the critical region. This leads to the conclusion that the PM-CSG transition might well appear to be a typical PM-FM transition, since the randomness does not destroy the local ferromagnetic order until that ferromagnetic order has been able to define itself. Recent neutron scattering results for $H=0$ (i.e., the CSG regime) indicate that the correlation length needed to fit the energy-integrated intensity $I(Q)$ as a sum of a Lorentzian and a Lorentzian squared increases significantly as the temperature is raised toward T_c, in qualitative agreement with (6.2).[34] This rise is observed far from T_c, and is resolution limited in the vicinity of T_c. For the FWA regime the neutron scattering results yield a correlation length which, for large H, decreases approximately as $H^{-1/2}$.[34] Moreover, for inter-

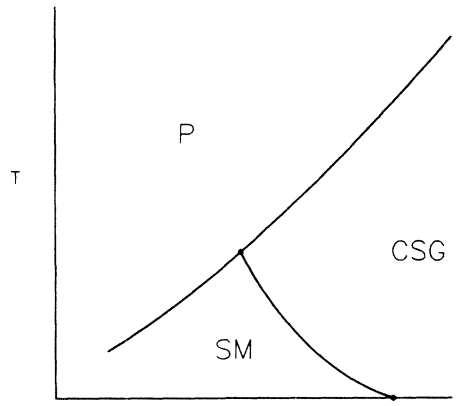

FIG. 3. Schematic phase diagram, accounting for the concentration dependence of the exchange and the temperature dependence of the anisotropy.

mediate fields the measured correlation length displays small but definite hysteresis effects, an indication of the metastability of the system.

Note that for single-ion anisotropy, the strength of the random anisotropy β_r is expected to depend on T but not on the concentration x of the magnetic ions. On the other hand, the exchange stiffness α should be independent of T, but should depend on x. Namely, one expects that $\alpha(x)$ increases as x increases. Incorporating the temperature dependence of β_r and the concentration dependence of α gives a condition on the reentrant transition temperature, which we denote by T_F:

$$\beta_r[T_F(x)]/\alpha(x) \approx (H_r/H_{\rm ex})^{\rm crit}/R_a^2 . \tag{6.4}$$

It is clear from (6.4) that, as x increases (so that α also increases), so does $\beta_r[T_F(x)]$. As a consequence, $T_F(x)$ decreases, as is observed experimentally (see, for example, Ref. 35). Using the results of Secs. I and III, we can give a qualitative description of the resulting phase diagram; see Fig. 3. (Note that this description is based only upon considerations of randomness in the anisotropy; randomness in the exchange, which certainly can occur, is completely ignored in the present discussion. This does not mean that we do not think its effects can be considerable, we simply wish to follow the implications of the model to their logical conclusions.)

Finally, it should be noted that, even above T_c, if there is a large field-induced magnetization, there can be a significant anisotropy, since the relationship $\beta_r \sim M^\lambda$ is expected to hold for all T. Thus one can also explain why magnetization experiments in large fields for $T > T_c$ need a nonzero anisotropy for their interpretation.[36]

VII. SUMMARY AND DISCUSSION

We have studied the properties of ferromagnets with random anisotropy in two limits. For $(H/H_{\rm ex}) \ll (H_r/H_{\rm ex})^4$, the system has no long-range order, although it preserves ferromagnetic order for lengths $R < R_F = R_a(H_{\rm ex}/H_r)^2$. Over larger distances it decorrelates, resembling a spin glass for lengths $R \gg R_F$. This has been called the correlated spin glass. For $(H/H_{\rm ex}) > (H_r/H_{\rm ex})^4$, the system is nearly aligned by the external field, but the spins "wander" about the field direction making a characteristic tipping angle $\theta \sim (H_r^4/H_{\rm ex}^3 H)^{1/4}$, whose orientation is locally preserved for lengths $R \ll R_F^\perp = R_a(H_{\rm ex}/H)^{1/2}$. This has been called the ferromagnet with wandering axis.

There are at least six types of experiments which may yield useful information about these systems: magnetic susceptibility ($H \to 0$), magnetization in large fields, ESR, neutron scattering, Mössbauer effect, and nuclear orientation. We will discuss each of them in turn.

(1) Magnetic susceptibility χ. For the CSG, measurements of χ can yield $(H_r^4/H_{\rm ex}^3)$, via (2.26), if M_0 is known.[37]

(2) Magnetization in large fields. For the FWA, magnetization deviation measurements can also yield $(H_r^4/H_{\rm ex}^3)$ by (3.13).[30]

(3) ESR in the CSG takes place for $\omega \approx \gamma(H_r^4/H_{\rm ex}^3)$, a frequency so low that it is probably masked by damping and spectrometer limitations. On the other hand, in the FWA there is a transverse resonance, with a shift which has not been calculated from a microscopic theory, and (with less certainty) a longitudinal resonance, whose position also has not yet been calculated from a microscopic theory.

(4) Neutron scattering. For the FWA, it is possible to study the transverse correlation function using neutron scattering. Such measurements can yield both $R_F^\perp$ and $(H_r^4/H_{\rm ex}^3)$, by (3.15), (3.12), and (3.2).[34] Moreover, it is also possible to study the correlation length R_F [see (2.18)] for the CSG.

(5) Mössbauer effect. This technique can be used to study the noncollinearity of a system.[38] This may be particularly useful in studying the FWA regime.

(6) Nuclear orientation. As in the case of the Mössbauer effect, this technique can be used to study the noncollinearity of a system, and should be particularly relevant to the FWA. The authors of Ref. 39 obtained a field-dependent parameter α for the angular spread of the magnetization orientation. An analysis of the field dependence of α was not given, but it was in qualitative agreement with the dependence of (3.3).

Our discussion has not exhausted the possibilities associated with ferromagnets with random anisotropy. For example, we have not considered the re-orientation time associated with the system "learning" a new direction for the applied field, nor have we considered the question of the linewidths for the transverse and longitudinal resonances. It should be clear, however, that this system is rich with possibilities and, due to the large number of materials from which such systems can be fabricated, there should be a wide variety of systems to which the considerations of our paper apply.

ACKNOWLEDGMENTS

We would like to acknowledge valuable conversations with S. M. Bhagat, S. Dodds, C. L. Henley, C.-R. Hu, J. J. Rhyne, D. L. Sellmyer, and S. E. Schultz. R.A.S. would like to acknowledge numerous illuminating discussions with P. A. Lee. W. M. S. gratefully acknowledges the support of the National Science Foundation, through Grant No. DMR-82-09577.

[1]Y. Imry and S.-K. Ma, Phys. Rev. Lett. **35**, 1399 (1975).
[2]E. M. Chudnovsky (unpublished).
[3]Since spin is an antisymmetric tensor (rather than a vector) one must treat these generalizations with some care. See W. M. Saslow, S. A. Fulling, and C.-R. Hu, Phys. Rev. B **31**, 363 (1985).
[4]E. Callen, Y. Liu, and J. R. Cullen, Phys. Rev. B **16**, 263 (1977).
[5]J. M. D. Coey, J. Appl. Phys. **49**, 1646 (1978).
[6]R. Alben, J. J. Becker, and M. C. Chi, J. Appl. Phys. **49**, 1653 (1978).
[7]R. E. Pelcovits, E. Pytte, and J. Rudnick, Phys. Rev. Lett. **40**, 476 (1978).
[8]M. Fahnle and H. Kronmuller, J. Magn. Magn. Mater. **8**, 149 (1978).
[9]E. M. Chudnovsky and R. A. Serota, Phys. Rev. B **26**, 2697 (1982).
[10]E. M. Chudnovsky and R. A. Serota, J. Phys. C **16**, 4181 (1983).
[11]E. M. Chudnovsky and R. A. Serota, J. Magn. Magn. Mater. **47**, 48 (1984).
[12]A. F. Andreev, Zh. Eksp. Teor. Fiz. **14**, 186 (1978) [Sov. Phys.—JETP **47**, 411 (1978)].
[13]C. L. Henley, H. Sompolinsky, and B. I. Halperin, Phys. Rev. B **25**, 5849 (1982).
[14]C. G. Morgan-Pond, Phys. Rev. Lett. **51**, 490, 1307(E) (1983).
[15]W. M. Saslow, Phys. Rev. Lett. **50**, 1320 (1983).
[16]E. M. Chudnovsky and R. A. Serota, IEEE Trans. Magn. **MAG-20**, 1400 (1984).
[17]A. Aharony and E. Pytte, Phys. Rev. B **27**, 5872 (1983).
[18]K. B. Efetov and A. I. Larkin, Zh. Eksp. Teor. Fiz. **72**, 2350 (1977) [Sov. Phys.—JETP **45**, 1236 (1977)].
[19]V. S. Dotsenko and M. V. Feigelman, J. Phys. C **16**, L803 (1983).
[20]M. J. O'Shea, S. G. Cornelison, Z. D. Chen, and D. J. Sellmyer, Solid State Commun. **46**, 313 (1983).
[21]B. I. Halperin and W. M. Saslow, Phys. Rev. B **16**, 2154 (1977).
[22]P. M. Levy, C. Morgan-Pond, and A. Fert, J. Appl. Phys. **53**, 1168 (1982).
[23]W. M. Saslow, Phys. Rev. Lett. **48**, 505 (1982); Phys. Rev. B **27**, 6873 (1983).
[24]E. M. Chudnovsky, J. Magn. Magn. Mater. **40**, 21 (1983).
[25]Y. Goldschmidt and B. Schaub, Nucl. Phys. B **251**, 77 (1985).
[26]J. Carly and S. Ostlund, Phys. Rev. B **25**, 6899 (1982).
[27]J. Villain and B. Semeria, J. Phys. Lett. **44**, L889 (1983).
[28]V. M. Vinokur, M. B. Mineev, and M. V. Feigelman, Zh. Eksp. Teor. Fiz. **81**, 2142 (1981) [Sov. Phys.—JETP **54**, 1138 (1982)].
[29]J. Villain and J. F. Fernandez, Z. Phys. B **54**, 139 (1984).
[30]D. J. Sellmyer and S. Nafis, J. Appl. Phys. **57**, 3584 (1985).
[31]P. Garoche and A. P. Malozemoff, Phys. Rev. B **29**, 226 (1984).
[32]H. Kronmuller, J. Magn. Magn. Mater. **24**, 159 (1981).
[33]H. B. Callen and E. Callen, J. Phys. Chem. Solids **27**, 1271 (1966).
[34]J. J. Rhyne, IEEE Trans. Magn. Mag. (to be published).
[35]D. J. Webb and S. M. Bhagat, J. Magn. Magn. Mater. **42**, 109 (1984).
[36]G. R. Gruzalski and D. J. Sellmyer, Phys. Rev. B **20**, 194 (1979).
[37]B. Barbara and B. Dieny, Physica **130B**, 245 (1985).
[38]R. A. Brand, J. Lauer, and W. Keune, Phys. Rev. B **31**, 1630 (1985); S. M. Dubiel, Ch. Sauer, and W. Zinn, *ibid.* **31**, 1643 (1985).
[39]A. Kettschau, J. Boysen, W. D. Brewer, and I. A. Campbell, J. Magn. Magn. Mater. **37**, L1 (1983).