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Structure of a solid film on an imperfect surface

E. M. Chudnovsky
Prospekt Gagarina 199, Apt. 17, Kharkov 310080, U.S.S.R.

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Two-dimensional solid films physisorbed on a substrate having quenched random imperfections are studied within the continuum elastic theory. Correlation functions describing positional and orientational order at zero temperature are found. In the presence of arbitrarily weak disorder in a substrate, only the short-range positional order survives in a film. Orientational order is long-ranged or quasi-long-ranged depending on the strength of the substrate-generated orientational field.

I. INTRODUCTION

There is now considerable theoretical and experimental interest in structures of two-dimensional (2D) solids.[1] As is known,[2–4] thermal fluctuations destroy crystalline long-range order (LRO) in a 2D solid. Although this has been known for 50 years, significant progress in the understanding of the structure and thermodynamics of 2D solids has been achieved during the last decade. In particular, it has been realized that the phase diagrams of 2D solids can be even richer than the phase diagrams of three-dimensional (3D) solids. Berezinskii[5] and Kosterlitz and Thouless (KT)[6] originally proposed that the unbinding of dislocation pairs with increasing temperature would drive a continuous melting of a 2D solid. Building on this prediction, Halperin and Nelson (HN),[7] Nelson,[8] and Young[9] developed a comprehensive theory of two-dimensional phases. The major points of this theory are as follows.

Periodic density modulations in a crystalline solid correspond to broken translational invariance. It is measured via the positional order parameter

$$\rho_{\mathbf{G}}(\mathbf{r})=\exp[i\mathbf{G}\cdot\mathbf{u}(\mathbf{r})]\ , \tag{1.1}$$

where $\mathbf{G}$ is a reciprocal-lattice vector and $\mathbf{u}(\mathbf{r})$ is the phonon displacement field. Global rotational invariance is also broken in a crystal with positional LRO. It can be broken, however, even if there is only positional short-range order (SRO). This state would be characterized by extended correlations in the orientations of locally defined crystallographic axes. For a triangular 2D lattice the orientational order parameter is

$$\psi(\mathbf{r})=\exp[6i\theta(\mathbf{r})]\ , \tag{1.2}$$

where $\theta(\mathbf{r})$ is the angle which one of the crystalline axes makes with some fixed direction at the point $\mathbf{r}$. When $T<T_{\mathrm{KT}}$ the crystal has orientational LRO and positional quasi-LRO, that is, correlations between $\rho_{\mathbf{G}}(\mathbf{r})$ at two points decay algebraically with the distance between the points. For $T_{\mathrm{KT}}<T<T_{\mathrm{HN}}$ the dislocation pairs dissociate into free dislocations and the solid melts into a hexatic phase with orientational quasi-LRO and positional SRO. When $T>T_{\mathrm{HN}}$ the dislocations dissociate into free disclinations which transforms the hexatic phase into a true liquid with both orientational and positional SRO.

Nelson[10] argued that the phase state of a 2D solid should depend also on quenched randomness in the system. In particular, quenched random impurities transform positional LRO at $T=0$ into quasi-LRO. Note that quenched randomness has been the subject of many recent theoretical investigations.[11] In part, they were inspired by the work of Imry and Ma[12] who showed that in systems with a continuous-symmetry order parameter arbitrarily weak symmetry-violating random fields destroy LRO for $D\leq 4$. An example of such a system is an amorphous ferromagnet with weak local random anisotropy.[13] Consideration of crystalline order in a solid with quenched defects[10] is not based on the Imry-Ma theorem, however. Indeed, in the presence of quenched defects the continuum elastic free energy of a (isotropic for simplicity) crystalline solid is[14]

$$F=\frac{1}{2}\int d^Dr\,(2\mu u_{ij}^2+\lambda u_{kk}^2)+\int d^Dr\,\sigma_{ij}u_{ij}\ , \tag{1.3}$$

where $u_{ij}(\mathbf{r})$ is the strain tensor

$$u_{ij}=\frac{1}{2}[\partial_i u_j(\mathbf{r})+\partial_j u_i(\mathbf{r})]\ , \tag{1.4}$$

μ and λ are Lamé elastic constants, and σ_{ij} is the stress due to the defects. Even in the presence of defects the free energy (1.3) remains invariant under a uniform translation

$$\mathbf{u}(\mathbf{r})\rightarrow\mathbf{u}(\mathbf{r})+\mathbf{u}_0\ , \tag{1.5}$$

which is an obvious consequence of the fact that quenched defects are allowed to move with the solid matrix. That is why $D=2$, but not $D=4$, turns out to be a lower dimension where arbitrarily small concentrations of quenched defects first destroy positional LRO in crystalline solids at $T=0$.[10]

We study the low-temperature phase of a 2D solid film physisorbed on a flat inhomogeneous surface. The inhomogeneity is due to the quenched randomness in the substrate. The structure of the film is defined by two factors. Firstly, atoms of the film seek relative positions which minimize the energy of their elastic interaction. Secondly, they seek positions on the substrate with smaller substrate-adsorbate—interaction energies. Within the framework of continuum elastic theory, the latter factor can be taken into account in terms of a substrate-generated random force acting in the plane of the film with a surface density $\mathbf{f}(\mathbf{r})$. The corresponding elastic free energy can be written as

$$F=\tfrac{1}{2}\int d^2r\,(2\mu u_{ij}^2+\lambda u_{kk}^2)-\int d^2r\, f_i u_i\ . \qquad (1.6)$$

Note that $f_i(\mathbf{r})$ is not reduced to $\partial_k\sigma_{ik}$; therefore, Eq. (1.6) cannot be reduced to Eq. (1.3) by integrating by parts in the second term. We shall assume a Gaussian distribution for probabilities of given configurations of $\mathbf{f}(\mathbf{r})$,

$$P[\mathbf{f}(\mathbf{r})]\propto\exp\left[-\frac{1}{2\sigma}\int d^2r[\mathbf{f}(\mathbf{r})]^2\right]\ . \qquad (1.7)$$

Since the free energy (1.6) is now not invariant under the transformation (1.5), one should expect that the system satisfies the Imry-Ma theorem. For the decay of positional order at $T=0$, we find

$$\langle\rho_{\mathbf{G}}(\mathbf{R})\rho_{\mathbf{G}}^*(\mathbf{0})\rangle=\exp[-(k_{\parallel}^2R_{\parallel}^2+k_{\perp}^2R_{\perp}^2)\ln(L/R)]\ , \qquad (1.8)$$

where $R_{\parallel}$ and $R_{\perp}$ are the longitudinal and transverse components of $\mathbf{R}$ with respect to $\mathbf{G}$, L is the linear size of the 2D solid, and both $k_{\parallel}$ and $k_{\perp}$ are proportional to $\sqrt{\sigma}$. Extending this calculation to higher dimensionalities, one finds

$$\langle\rho_{\mathbf{G}}(\mathbf{R})\rho_{\mathbf{G}}^*(\mathbf{0})\rangle=\exp(-kR)\ ,$$

with $k\propto\sigma$ for $D=3$, algebraic decay of $\langle\rho_{\mathbf{G}}(\mathbf{R})\rho_{\mathbf{G}}^*(\mathbf{0})\rangle$ for $D=4$, and positional LRO for $D>4$, which is in accordance with the Imry-Ma theorem.

The orientational field $\theta(\mathbf{r})$ is locked at low temperatures to the 2D curl of the phonon displacement field,[7]

$$\theta(\mathbf{r})=\tfrac{1}{2}\nabla\times\mathbf{u}(\mathbf{r})\ . \qquad (1.9)$$

For decay of orientational order, we find

$$\langle\psi(\mathbf{R})\psi^*(\mathbf{0})\rangle=\left[\frac{a}{R}\right]^{9\sigma/2\pi\mu^2}\ , \qquad (1.10)$$

where a is the average interatomic distance.

Possible systems which could produce the kind of disorder studied here include monolayers physisorbed on a glassy substrate and incommensurate monolayers physisorbed on an imperfect crystalline surface.[15] The latter systems are characterized by "orientational epitaxy," that is, the ground-state energy of a 2D crystal is minimized when its crystalline axes have a definite orientation with respect to the substrate.[16] This effect can be expressed in terms of the substrate-generated orientational field h. Nonzero h provides the appearance of additional characteristic length $\delta=(K_A/h)^{1/2}$, where K_A is the Frank elastic constant.[7] For $R\ll\delta$, $\langle\psi(\mathbf{R})\psi^*(\mathbf{0})\rangle$ is still defined by Eq. (1.10), while for $R>\delta$ we find

$$\langle\psi(\mathbf{R})\psi^*(\mathbf{0})\rangle=\exp\left[-\frac{9\sigma}{2\pi\mu^2}\ln\left[\frac{\delta}{a}\right]\right]\ . \qquad (1.11)$$

In Sec. II we consider positional order in a 2D solid on a substrate with quenched randomness. Orientational order is studied in Sec. III. Section IV is devoted to the effect of the orientational epitaxy. The relevance of our results to the experiments on physisorbed monolayers is discussed in Sec. V.

II. POSITIONAL ORDER

At low temperatures the statistical mechanics associated with the free energy (1.6) must be dominated by extremal configurations of the displacement field $\mathbf{u}(\mathbf{r})$ satisfying

$$(\lambda+\mu)\nabla(\nabla\cdot\mathbf{u})+\mu\nabla^2\mathbf{u}+\mathbf{f}=0\ . \qquad (2.1)$$

The solution of this equation for a given configuration $\mathbf{f}(\mathbf{r})$ is

$$u_i(\mathbf{r})=\int d^2r'\,G_{ij}(\mathbf{r}-\mathbf{r}')f_j(\mathbf{r}')\ , \qquad (2.2)$$

where $G_{ij}(\mathbf{r})$ is the Green's function for the two-dimensional equation (2.1) (see the Appendix):

$$G_{ij}(\mathbf{r})=\alpha n_i n_j+\beta\delta_{ij}\ln r,\quad \mathbf{n}=\mathbf{r}/r \qquad (2.3)$$

$$\alpha=\frac{\lambda+\mu}{4\pi\mu(\lambda+2\mu)},\quad \beta=-\frac{\lambda+3\mu}{4\pi\mu(\lambda+2\mu)}\ . \qquad (2.4)$$

To describe positional order in the system we shall calculate the correlation function

$$\langle\rho_{\mathbf{G}}(\mathbf{R})\rho_{\mathbf{G}}^*(\mathbf{0})\rangle=\langle\exp\{i\mathbf{G}\cdot[\mathbf{u}(\mathbf{R})-\mathbf{u}(\mathbf{0})]\}\rangle\ , \qquad (2.5)$$

where we have used definition (1.1) for the positional order parameter. With the help of Eq. (2.2) we have

$$u_i(\mathbf{R})-u_i(0)=\int d^2r\,[G_{ij}(\mathbf{r}-\mathbf{R})-G_{ij}(\mathbf{r})]f_j(\mathbf{r})\ . \qquad (2.6)$$

The angular brackets $\langle\ \rangle$ in Eq. (2.5) mean an average with respect to possible configurations of $\mathbf{f}(\mathbf{r})$ which occur with the probabilities given by Eq. (1.7). Thus the correlation function (2.5) is defined by the path integral

$$\langle\rho_{\mathbf{G}}(\mathbf{R})\rho_{\mathbf{G}}^*(\mathbf{0})\rangle=\frac{\int D[\mathbf{f}(\mathbf{r})]\exp\left[i\int d^2r\,K_j(\mathbf{G},\mathbf{R},\mathbf{r})f_j(\mathbf{r})-\frac{1}{2\sigma}\int d^2r[f_j(\mathbf{r})]^2\right]}{\int D[\mathbf{f}(\mathbf{r})]\exp\left[-\frac{1}{2\sigma}\int d^2r[f_j(\mathbf{r})]^2\right]}\ , \qquad (2.7)$$

where

$$K_j=G_i[G_{ij}(\mathbf{r}-\mathbf{R})-G_{ij}(\mathbf{r})]\ . \qquad (2.8)$$

Integration over $\mathbf{f}(\mathbf{r})$ gives[17]

$$\langle \rho_{\mathbf{G}}(\mathbf{R})\rho^*_{\mathbf{G}}(\mathbf{0})\rangle = \exp\left[-\frac{\sigma}{2}\int d^2r[K_j(\mathbf{G},\mathbf{R},\mathbf{r})]^2\right] . \tag{2.9}$$

For K_j^2, with the use of Eq. (2.3), we obtain

$$K_j^2=\alpha^2[(\mathbf{G}\cdot\mathbf{N})^2+(\mathbf{G}\cdot\mathbf{n})^2-2(\mathbf{G}\cdot\mathbf{N})(\mathbf{G}\cdot\mathbf{n})(\mathbf{N}\cdot\mathbf{n})]+\beta^2G^2\ln^2\left[\frac{|\mathbf{r}-\mathbf{R}|}{r}\right]+2\alpha\beta[(\mathbf{G}\cdot\mathbf{N})^2-(\mathbf{G}\cdot\mathbf{n})^2]\ln\left[\frac{|\mathbf{r}-\mathbf{R}|}{r}\right] , \tag{2.10}$$

where $\mathbf{N}=(\mathbf{r}-\mathbf{R})/|\mathbf{r}-\mathbf{R}|$. Two remarks should be made on the integration in Eq. (2.9). Firstly, no shifts $\mathbf{r}\rightleftarrows\mathbf{r}-\mathbf{R}$ are allowed in separate terms under the integral if they do not go to zero when $r\rightarrow\infty$. Secondly, at large r the integral must be cut off by the linear size of the film L. Then a somewhat tedious but straightforward calculation gives

$$\int d^2r\,K_j^2=\pi(\alpha^2+\beta^2-\alpha\beta)G^2R^2\ln(L/R)+2\pi\alpha\beta(\mathbf{G}\cdot\mathbf{R})^2\ln(L/R) . \tag{2.11}$$

After substitution of Eq. (2.11) into Eq. (2.9), the positional order correlation function can be presented in the form

$$\langle \rho_{\mathbf{G}}(\mathbf{R})\rho^*_{\mathbf{G}}(\mathbf{0})\rangle = \exp\left[-\left[k_{\parallel}^2\frac{G_iG_j}{G^2}+k_{\perp}^2\left[\delta_{ij}-\frac{G_iG_j}{G^2}\right]\right]R_iR_j\ln\left[\frac{L}{R}\right]\right] , \tag{2.12}$$

where

$$k_{\parallel}^2=\frac{\sigma G^2}{32\pi}\left[\frac{1}{\mu^2}+\frac{3}{(\lambda+2\mu)^2}\right] , \tag{2.13}$$

$$k_{\perp}^2=\frac{\sigma G^2}{32\pi}\left[\frac{3}{\mu^2}+\frac{1}{(\lambda+2\mu)^2}\right] . \tag{2.14}$$

In our model the strength of the substrate-generated disorder is approximated by the parameter σ. Since positional correlations decay exponentially with increasing R, irrespective of how small σ is, we should emphasize that only positional SRO survives in a 2D solid on an imperfect substrate at $T=0$. It should be noted, however, that for a weak imperfection, the domains where the crystalline order persists can be rather large. Indeed, taking into account that $\ln(L/R)$ cannot be too large, one can estimate the longitudinal and transverse correlation lengths of the positional order as $\xi_{\parallel}\sim k_{\parallel}^{-1}$ and $\xi_{\perp}\sim k_{\perp}^{-1}$. Both $\xi_{\parallel}$ and $\xi_{\perp}$ diverge as $1/\sqrt{\sigma}$ when $\sigma\rightarrow 0$. Note also that the R dependence of the correlation function (2.12) is similar to that of the spin-spin correlation function for a 2D ferromagnet with quenched random anisotropies.[18]

III. ORIENTATIONAL ORDER

Now we shall find the local rotation (1.9) caused in a 2D solid by the substrate-generated force $\mathbf{f}(\mathbf{r})$. With the help of Eq. (2.2), we have

$$\theta(\mathbf{r})=\tfrac{1}{2}\epsilon_{ik}\partial_iu_k=\tfrac{1}{2}\epsilon_{ik}\int d^2r'\frac{\partial}{\partial r_i}G_{kj}(\mathbf{r}-\mathbf{r}')f_j(\mathbf{r}') , \tag{3.1}$$

where G_{kj} is given by Eq. (2.3), and ϵ_{ik} is the unit antisymmetric tensor, $\epsilon_{xy}=1$. Further calculation gives

$$\theta(\mathbf{r})=\tfrac{1}{2}(\alpha-\beta)\epsilon_{ij}\int d^2r'\frac{r_i'-r_i}{(\mathbf{r}'-\mathbf{r})^2}f_j(\mathbf{r}') . \tag{3.2}$$

The orientational order in the system is described by the correlation function

$$\langle \psi(\mathbf{R})\psi^*(\mathbf{0})\rangle=\langle \exp\{6i[\theta(\mathbf{R})-\theta(\mathbf{0})]\}\rangle , \tag{3.3}$$

where we have used the definition of the orientational order parameter (1.2). Substituting Eq. (3.2) into Eq. (3.3), we obtain

$$\langle \psi(\mathbf{R})\psi^*(\mathbf{0})\rangle=\left\langle \exp\left[i\int d^2r\,Q_j(\mathbf{R},\mathbf{r})f_j(\mathbf{r})\right]\right\rangle , \tag{3.4}$$

where

$$Q_j=3(\alpha-\beta)\epsilon_{ij}\left[\frac{r_i}{r^2}-\frac{r_i-R_i}{(\mathbf{r}-\mathbf{R})^2}\right] . \tag{3.5}$$

The averaging in Eq. (3.4) with respect to $f(\mathbf{r})$ configurations is analogous to that made in Sec. II. The result of the averaging is

$$\langle \psi(\mathbf{R})\psi^*(\mathbf{0})\rangle=\exp\left[-\tfrac{1}{2}\sigma\int d^2r[Q_j(\mathbf{R},\mathbf{r})]^2\right] . \tag{3.6}$$

For Q_j^2, with the help of Eqs. (2.4) and (3.5), we find

$$Q_j^2=\left[\frac{3}{2\pi\mu}\right]^2\frac{R^2}{r^2(\mathbf{r}-\mathbf{R})^2} . \tag{3.7}$$

To perform the integration in Eq. (3.6) one should recall that there is a minimal resolution in the spatial variables $\mathbf{r}$ and $\mathbf{R}$ brought about by the atomic structure of a solid. Thus the integrals of the form $\int dr(1/r)$ should be cut off at small r by the average interatomic distance a. Integration over $\mathbf{r}$ then gives

$$\int d^2r\,Q_j^2=\frac{9}{\pi\mu^2}\ln\left[\frac{R}{a}\right] . \tag{3.8}$$

Substituting Eq. (3.8) into Eq. (3.6), we finally find

$$\langle \psi(\mathbf{R})\psi^*(\mathbf{0})\rangle=\left[\frac{a}{R}\right]^{9\sigma/2\pi\mu^2} . \tag{3.9}$$

The algebraic decay of the orientational order given by Eq. (3.9) is weaker than the exponential decay of the positional order studied in Sec. II. Therefore, for a 2D solid on an imperfect substrate, correlations in the orientations of locally defined crystallographic axes should be much more extended than correlations in the positions of atoms.

For weak disorder in the substrate, i.e., small σ, orientational quasi-LRO could be hardly experimentally distinguished from a true LRO.

IV. ORIENTATIONAL EPITAXY

Here we shall take into account the effect of a substrate-generated orientational field. For this purpose note that Eq. (3.2) can be written in the form

$$\theta(\mathbf{r})=\pi(\alpha-\beta)\epsilon_{ji}\int d^2r'\frac{\partial}{\partial r_i}\tilde{G}(\mathbf{r}-\mathbf{r}')f_j(\mathbf{r}') , \qquad (4.1)$$

where $\tilde{G}(\mathbf{r})=(1/2\pi)\ln r$. Substituting α and β from Eq. (2.4) into Eq. (4.1) and integrating by parts, we obtain

$$\theta(\mathbf{r})=\frac{1}{2\mu}\int d^2r'\tilde{G}(\mathbf{r}-\mathbf{r}')\epsilon_{ji}\partial_i f_j(\mathbf{r}') . \qquad (4.2)$$

Taking into account that $\tilde{\mathbf{G}}(\mathbf{r})$ is the Green's function for the 2D Laplacian, i.e., $\nabla^2\tilde{G}(\mathbf{r})=\delta(\mathbf{r})$, it can be easily seen that $\theta(\mathbf{r})$ satisfies the equation

$$2\mu\nabla^2\theta+\epsilon_{ij}\partial_i f_j=0 . \qquad (4.3)$$

This allows one to consider the effective free energy

$$F_{\text{eff}}=\frac{K_A}{2}\int d^2r(\nabla\theta)^2+\frac{K_A}{2\mu}\int d^2r\,\epsilon_{ij}f_j\partial_i\theta , \qquad (4.4)$$

which generates the variational equation (4.3). Note that only the ratio of the coefficients in front of the integrals in Eq. (4.4) is determined by the form of Eq. (4.3). We have chosen these coefficients from the requirement that the first term in Eq. (4.4) coincide with the orientational free energy in the Halperin-Nelson theory,[7] where K_A is the Franck elastic constant similar to that found in liquid crystals. In the presence of the substrate-generated orientational field, an additional term[7] $\frac{1}{2}h\int d^2r\,\theta^2$ must be included in the free energy, so that we have

$$F_{\text{eff}}=\frac{K_A}{2}\int d^2r(\nabla\theta)^2 +\frac{K_A}{2\mu}\int d^2r\,\epsilon_{ij}f_j\partial_i\theta+\frac{h}{2}\int d^2r\,\theta^2 . \qquad (4.5)$$

Variation of this functional with respect to $\theta(\mathbf{r})$ gives

$$\nabla^2\theta-\delta^{-2}\theta=\frac{1}{2\mu}\epsilon_{ji}\partial_i f_j , \qquad (4.6)$$

where we have introduced $\delta=(K_A/h)^{1/2}$. The solution of this equation can be presented as

$$\theta(\mathbf{r})=\frac{1}{2\mu}\int d^2r'\tilde{G}_h(\mathbf{r}-\mathbf{r}')\epsilon_{ji}\partial_i f_j(\mathbf{r}') , \qquad (4.7)$$

where $\tilde{G}_h(\mathbf{r})$ is the Green's function for Eq. (4.6),

$$\tilde{G}_h(\mathbf{r})=-\frac{1}{2\pi}K_0(r/\delta) , \qquad (4.8)$$

and $K_0(r)$ is the modified Hankel function. Integrating by parts in Eq. (4.7), we obtain

$$\theta(\mathbf{r})=\frac{\epsilon_{ji}}{4\pi\mu\delta}\int d^2r'K_1(|\mathbf{r}-\mathbf{r}'|/\delta)\frac{(r_i-r_i')}{|\mathbf{r}-\mathbf{r}'|}f_j(\mathbf{r}') , \qquad (4.9)$$

where we have used $K_0'=-K_1$.

Calculations analogous to those made in Sec. III, but with Eq. (4.9) instead of Eq. (3.2), give the following expression for the orientational-order correlation function:

$$\langle\psi(\mathbf{R})\psi^*(0)\rangle=\exp\left\{-\sigma\left[\frac{3}{2\pi\mu\delta}\right]^2\int d^2r\left[K_1^2(r/\delta)-K_1(r/\delta)K_1(|\mathbf{r}-\mathbf{R}|/\delta)\frac{\mathbf{r}(\mathbf{r}-\mathbf{R})}{r\,|\mathbf{r}-\mathbf{R}|}\right]\right\} . \qquad (4.10)$$

For $R \ll \delta$, integration over $\mathbf{r}$ gives

$$\langle\psi(\mathbf{R})\psi^*(0)\rangle=\exp\left[-\frac{9\sigma}{2\pi\mu^2}\ln\left[\frac{R}{a}\right]\right] , \qquad (4.11)$$

which coincides with Eq. (3.9). When $R>\delta$, the integration gives

$$\langle\psi(\mathbf{R})\psi^*(0)\rangle=\exp\left[-\frac{9\sigma}{2\pi\mu^2}\ln\left[\frac{\delta}{a}\right]\right] . \qquad (4.12)$$

An important remark should be made on the latter equation. Observing that $\langle\psi(\mathbf{R})\psi^*(0)\rangle\rightarrow$const as $R\rightarrow\infty$, one would think that an arbitrarily weak substrate-generated orientational field h would restore orientational LRO irrespective of the strength σ of the substrate disorder. Note, however, that for a mere definition of the orientational LRO, fluctuations in the orientation of any crystallographic axis must be small in comparison with the angle between two different axes. For a triangular 2D lattice this angle is $\pi/3$. Thus, orientational LRO persists in a system only if the modulus of the power in Eq. (4.12) is small. It defines the critical value of h which restores LRO for a given σ,

$$h_c\sim\frac{K_A}{a^2}\exp\left[-\frac{4\pi\mu^2}{9\sigma}\right] . \qquad (4.13)$$

For the weak disorder in a substrate, h_c is exponentially small.

V. DISCUSSION

We have shown that the structure of a 2D solid on an imperfect substrate at zero temperature is characterized by positional SRO and orientational quasi-LRO. This state of the solid can be called a quenched hexatic phase, by analogy with the high-temperature hexatic phase which results from a dislocation-unbinding phase transition.[7] The kind of disorder studied here can be realized in incommensurate monolayers physisorbed on an imperfect crystalline surface or on a glassy substrate. Consideration of the latter case, however, should also include the effect of random topography[19] because the surface of an amorphous solid could hardly be as flat as a crystalline surface.

In the case of an imperfect crystalline substrate, the effect of the substrate-generated orientational field should be taken into account. We have shown that there is a finite critical value of the orientational field which restores orientational LRO in a monolayer. Our conclusion as to the positional order is that in the presence of an arbitrarily weak quenched disorder in the substrate, positional correlations in the monolayer should decay exponentially. In the case of weak disorder the domains where positional order persists could be rather large, however.

An important realization of a 2D solid is provided by rare-gas monolayers on graphite. These systems have been the subject of intensive recent experimental study.[1] In particular, a melting transition[20] and, probably, a hexatic phase[21] have been observed in monolayer xenon on graphite using x-ray scattering. In such experiments the surface of the substrate must be as perfect as possible to ensure that corresponding effects exist as a consequence of thermal fluctuations in the adsorbate, not the substrate interaction. The inevitable weak quenched disorder in the substrate should be unessential, because at $T=0$ crystalline domains in a monolayer would be greater than the whole surface under investigation. For strong randomness in the substrate, the positional correlation length becomes small. In this case the quenched hexatic phase of the adsorbate should be observed right down to $T=0$. With increasing temperature one should expect that unbinding of dislocation pairs within the crystalline domains would lead to a further decrease of the positional correlation length. A disclination-unbinding phase transition would be necessary to complete the transition from hexatic to liquid.[7] In conclusion, note that a comprehensive consideration of the case of strong substrate disorder should also include an investigation of the possibility[22] that grain boundaries form in a 2D solid. This, however, exceeds the limits of the approach based on the continuum elastic theory.

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APPENDIX

In this appendix we will calculate the Green's function for the 2D equation of the elastic theory. Let us first consider an external force $\mathbf{f}(\mathbf{r})$ of the form $f_x(\mathbf{r})=f_x\delta(\mathbf{r})$, $f_y=0$. For $u_x(x,y)$ and $u_y(x,y)$, Eq. (2.1) then gives

$$(\lambda+\mu)\frac{\partial}{\partial x}\left[\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right]+\mu\nabla^2 u_x+f_x\delta(\mathbf{r})=0\ , \tag{A1}$$

$$(\lambda+\mu)\frac{\partial}{\partial y}\left[\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right]+\mu\nabla^2 u_y=0\ . \tag{A2}$$

With the help of the ansatz,

$$u_x=\frac{\partial\varphi}{\partial x}+\frac{\partial\psi}{\partial y},\quad u_y=\frac{\partial\varphi}{\partial y}-\frac{\partial\psi}{\partial x}\ . \tag{A3}$$

Equations (A1) and (A2) can be written as

$$(\lambda+2\mu)\nabla^2\frac{\partial\varphi}{\partial x}+\mu\nabla^2\frac{\partial\psi}{\partial y}+f_x\delta(\mathbf{r})=0\ , \tag{A4}$$

$$(\lambda+2\mu)\nabla^2\frac{\partial\varphi}{\partial y}-\mu\nabla^2\frac{\partial\psi}{\partial x}=0\ . \tag{A5}$$

Solutions of these equations are sought in the form

$$\varphi=a\frac{\partial\rho}{\partial x},\quad \psi=b\frac{\partial\rho}{\partial y}\ , \tag{A6}$$

where a,b do not depend on coordinates, and

$$\rho=r^2\ln r-r^2\ . \tag{A7}$$

Taking into account that

$$\nabla^2\rho=4\ln r,\quad \nabla^2\ln r=2\pi\delta(\mathbf{r})\ , \tag{A8}$$

substitution of Eq. (A6) into Eqs. (A4) and (A5) gives

$$a=-\frac{f_x}{8\pi(\lambda+2\mu)},\quad b=-\frac{f_x}{8\pi\mu}\ . \tag{A9}$$

For u_x and u_y, with the help of Eq. (A3), we obtain

$$u_x=-\frac{f_x}{8\pi(\lambda+2\mu)}\frac{\partial^2\rho}{\partial x^2}-\frac{f_x}{8\pi\mu}\frac{\partial^2\rho}{\partial y^2}\ , \tag{A10}$$

$$u_y=\left[\frac{f_x}{8\pi\mu}-\frac{f_x}{8\pi(\lambda+2\mu)}\right]\frac{\partial^2\rho}{\partial x\partial y}\ . \tag{A11}$$

Extending this calculation to the case of two-component $\mathbf{f}(\mathbf{r})$ of the form $f_x=f_y=\delta(\mathbf{r})$, we find that u_i caused by f_j is given by

$$G_{ij}=-\frac{1}{8\pi\mu}\left[\delta_{ij}\nabla^2\rho-\frac{\lambda+\mu}{\lambda+2\mu}\partial_i\partial_j\rho\right]\ , \tag{A12}$$

which is equivalent to Eqs. (2.3) and (2.4) is $\mathbf{f}(\mathbf{r})$ in Eq. (2.2) satisfies the condition of zero resultant force, $\int d^2r\,\mathbf{f}(\mathbf{r})=0$.

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[1] See, for references, P. Dimon, P. M. Horn, M. Sutton, R. J. Birgeneau, and D. E. Moncton, Phys. Rev. B **31**, 437 (1985).

[2] R. E. Peierls, Ann. Inst. Henri Poincaré **5**, 177 (1935).

[3] L. D. Landau, Zh. Eksp. Teor. Fiz. **7**, 627 (1937); in *Collected Papers of L. D. Landau,* edited by D. ter Haar (Gordon and Breach, New York, 1965), p. 209.

[4] N. D. Mermin, Phys. Rev. **176**, 250 (1968).

[5] V. L. Berezinskii, Zh. Eksp. Teor. Fiz. **59**, 907 (1970) [Sov. Phys.—JETP 32, 493 (1971)]; **61**, 1144 (1971) [**34**, 610 (1972)].

[6] J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973); Prog. Low Temp. Phys. **7B**, 371 (1978).

[7] B. J. Halperin and D. R. Nelson, Phys. Rev. Lett. **41**, 121 (1978); **41**, 519(E) (1978); D. R. Nelson and B. I. Halperin, Phys. Rev. B **19**, 2457 (1979).

[8] D. R. Nelson, Phys. Rev. B **18**, 2318 (1978).

[9] A. P. Young, Phys. Rev. B **19**, 1855 (1979).

[10] D. R. Nelson, Phys. Rev. B **27**, 2902 (1983).

[11] See, for references, A. Aharony, J. Magn. Magn. Mater. **31-34**, 1432 (1983).

[12] Y. Imry and S.-k. Ma, Phys. Rev. Lett. **35**, 1399 (1975).

[13] E. M. Chudnovsky and R. A. Serota, Phys. Rev. B **26**, 5272 (1982); J. Phys. C **16**, 4181 (1983).

[14] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* (Pergamon, New York, 1970).

[15] It has been recently argued that, in the presence of quenched dislocations, the crystal surface also exhibits a glassy behavior, see C. P. Bachas, Phys. Rev. Lett. **54**, 53 (1985).

[16] A. D. Novaco and J. P. McTague, Phys. Rev. Lett. **38**, 1286 (1977); J. P. McTague and A. D. Novaco, Phys. Rev. B **19**, 5299 (1979).

[17] See, e.g., R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).

[18] E. M. Chudnovsky, J. Magn. Magn. Mater. **40**, 21 (1983).

[19] S. Sadchev and D. Nelson (unpublished).

[20] E. M. Hammonds, P. Heiney, P. W. Stephens, R. J. Birgeneau, and P. M. Horn, J. Phys. C **13**, L301 (1980).

[21] T. F. Rosenbaum, S. E. Nagler, and P. M. Horn, Phys. Rev. Lett. **50**, 1791 (1983).

[22] S. T. Chui, Phys. Rev. Lett. **48**, 933 (1982); Phys. Rev. B **28**, 178 (1983).