e-e scattering in the presence of strong magnetic fields: Effect of the Landau quantization

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We study electron-electron (e-e) scattering in the presence of Landau quantization of the electron orbits. The lifetime $\tau^i_{e\text{-}e}(H)$ is evaluated for an electron in a given Landau state N_i by allowing it to scatter with all other electrons at the Fermi level $E_F(H)$ through a screened Coulomb potential. We consider the motion parallel to the magnetic field **H**, corresponding to the longitudinal geometry, where **H** is oriented parallel to **E** (the electric field). The lifetime $\tau^i_{e\text{-}e}(H)$ is obtained from the golden rule; it depends linearly on the temperature, $\tau^i_{e\text{-}e}(H)=B_i(H)T^{-1}$. The linear T dependence is due to the one-dimensional character of the scattering processes at $E_F(H)$. The coefficients $B_i(H)$ are calculated for a semiconductor InSb and for a metal Al for different magnetic fields, $N\hbar\omega < E_F(H)$, $N=1,2,\ldots,100$. The results obtained for the lifetime $\tau^i_{e\text{-}e}(H)$ are compared with the zero-field case, $\tau_{e\text{-}e}(0)$.

I. INTRODUCTION

In recent years some experimental and theoretical efforts have been made to understand the intrinsic electron scattering processes in metals and semiconductors at very low temperatures. Of particular interest is the effect of electron-electron (e-e) scattering on the transport phenomena. Its contribution to the bulk resistivity of pure crystals (e.g., metal whiskers[1]) can be observed by the use of the superconducting quantum interference effect, which allows for the measurement of very small voltages and their small changes.[2–4] In semiconductors, e-e scattering in inversion layers can be important in two different cases. In a two-dimensional electron system, deviations from ordinary conduction are observed because of the tendency towards weak localization.[5] The deviations depend on the inelastic electron scattering rate which may be dominated by e-e scattering. In addition, in very-high-mobility samples the fractional quantum Hall effect is observed at low temperatures.[6] This effect depends on the e-e Coulomb interaction.[7] In these samples one expects e-e scattering to be relevant for electric transport phenomena above the temperature regime where the new condensed phase is formed in high magnetic fields by virtue of the Coulomb interaction. In this paper we discuss the lifetime, $\tau^i_{e\text{-}e}(H)$, of an electron at the Fermi energy $E_F(H)$ of a degenerate electron gas. We assume that the external magnetic field H is sufficiently strong that the Landau quantization of the electron orbits must be taken into account. The condition $\hbar\omega_c > k_BT$, where $\omega_c = eH/m^*c$ is the cyclotron resonance frequency and m^* is the band mass, implies that the electrons at the Fermi level can only undergo "horizontal" transitions, Fig. 1, i.e., transitions where a change occurs in the momentum quantum number k_z that is oriented parallel to the magnetic field. Hence, we ignore the "vertical" transitions which become important for smaller magnetic fields $\hbar\omega_c < k_BT$. We assume spin degeneracy.

The evaluation of $\tau^i_{e\text{-}e}(H)$ is not sufficient to obtain the electrical resistivity $\rho_{e\text{-}e}(H)$. Let us recall that in the absence of an external magnetic field the resistivity is given by the equation[8]

$$\rho_{e\text{-}e}=\frac{2}{3}\frac{\pi^2}{ne^2}m_{\rm opt}\frac{\Delta}{\tau_{e\text{-}e}}\ , \tag{1}$$

where $\tau_{e\text{-}e}$ is the electron lifetime and Δ is a parameter of the order of one that accounts for the e-e umklapp processes responsible for $\rho_{e\text{-}e}$ in a one-component electron gas; n is the electron concentration and $m_{\rm opt}$ is the optical mass defined in Ref. 8. In the presence of Landau quantization, the longitudinal magnetoresistance can still be written in the form of Eq. (1), where now Δ and $\tau_{e\text{-}e}$ become H-dependent quantities. In this paper we only address $\tau_{e\text{-}e}(H)$ and not $\Delta(H)$. We mention that to obtain the transverse magnetoresistance, one must solve the equation of motion for the density matrix as discussed by Kubo[9] and by other authors,[10,11] ignoring, however, e-e scattering. In the transverse case not only the diagonal

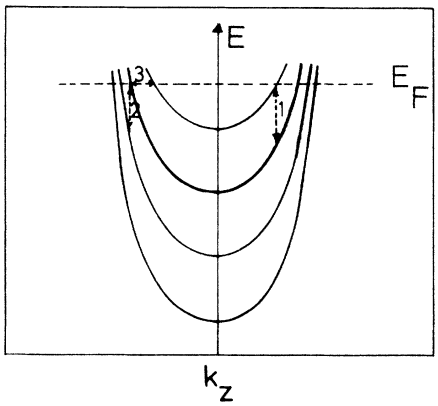

FIG. 1. Horizontal and vertical scattering processes: 1 and 2 are vertical processes and 3 is a horizontal process.

elements of the density matrix are important, as is the case in the longitudinal geometry.

The calculation of the lifetime is performed in Sec. II; the results are discussed in Sec. III.

II. ELECTRON LIFETIME

To calculate the electron lifetime we use the following equation:

$$1/\tau^1_{e\text{-}e}(H)=\sum_{N_2=0}^{N_1-1}(1/\tau_1^{N_2})+\sum_{N_2=N_1+1}^{N_{\max}}(1/\tau_1^{N_2})+1/\tau_1^{N_1}\,, \tag{2}$$

where $N_{\max}$ is the integer number determined by the Fermi energy $E_F(H)$, $N_{\max}\hbar\omega<E_F(H)<(N_{\max}+1)\hbar\omega$.

The first, second, and third terms of the above sums give the scattering rate of the electron in the quantum state N_1 with electrons in quantum states which have lower, higher, and the same Landau numbers as compared with N_1, respectively, (cf. Fig. 2). The scattering rate $1/\tau_1^{N_2}$ is defined by the equation

$$1/\tau_1^{N_2}\equiv\sum_{\substack{\text{allowed}\\ \text{scattering}\\ \text{processes}}}W(1234)\,, \tag{3}$$

where $W(1234)$ is the transition probability for the following scattering process: two electrons, in quantum states 1 and 2, interact through the Coulomb potential and scatter into the final quantum states 3 and 4.

The transition probability $W(1234)$ is calculated from the eigenstates

$$\Psi_{Nk_yk_z}\equiv\Phi_N(x-\lambda^2k_y)e^{ik_yy}e^{ik_zz}(1/L_yL_z)^{1/2}\,, \tag{4}$$

where k_y,k_z are the momentum quantum numbers, L_y,L_z are the normalizing lengths, and $\Phi_N(x-\lambda^2k_y)$ is the wave function of the harmonic oscillator in Landau state N with the zero-point coordinate $\lambda^2k_y=(c\hbar/eH)k_y$:

$$\Phi_N(x-\lambda^2k_y)=(\lambda\sqrt{\pi}2^NN!)^{-1/2}H_N((x+\lambda^2k_y)/\lambda)\times e^{-\lambda^2(x+\lambda^2k_y)^2/2}\,.$$

For $W(1234)$ the golden rule gives

$$W(1234)=(2\pi/\hbar)\delta(E_1+E_2-E_3-E_4)M^2_{1234}\,, \tag{5}$$

where

$$M(1234)\equiv\langle\psi_1\psi_2\,|\,V\,|\,\psi_3\psi_4\rangle\,. \tag{6}$$

Here, V is the interaction potential

$$V(r)=\frac{e^2}{\kappa}\frac{e^{-r/r_0}}{r}\,, \tag{7}$$

where r_0 is the Thomas-Fermi screening length and κ is the dielectric constant of the background medium.

Before evaluating the transition probability using the Eqs. (4) to (7), we discuss in some detail the allowed scattering processes on the basis of energy and momentum conservation. At the Fermi level, where the scattering processes take place, the momentum quantum number k_z has the following value:

$$|k_{zi}|\equiv|k_F^{N_i}|=(\sqrt{2m}/\hbar)[E_F-\hbar\omega(N_i+\tfrac{1}{2})]^{1/2}\,. \tag{8}$$

The Taylor expansion in terms of the parameter $x_i=(\hbar\omega/E_F)(N_i+\frac{1}{2})$ yields

$$|k_F^Ni|=\frac{\sqrt{2m}\sqrt{E_F}}{\hbar}\left[1+\tfrac{1}{2}x_i+\sum_{n=2}^{\infty}\frac{(2n-3)!!}{2n!!}x_i^n\right]\,. \tag{9}$$

This series converges for every Landau state N_i below E_F, i.e., $x_i<1$. Using Eq. (9) we calculate the increment

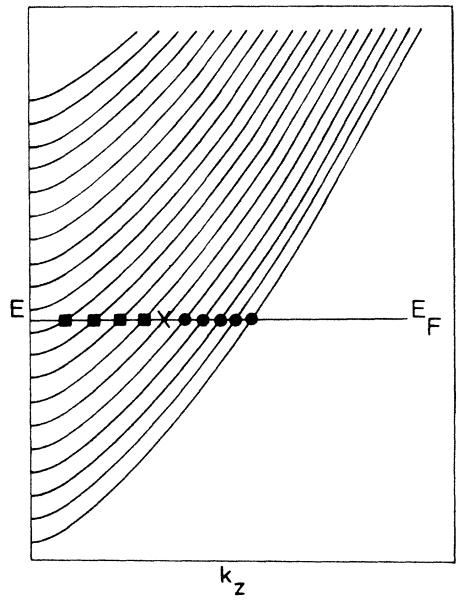

FIG. 2. Scattering partners at the Fermi level. The states labeled with ■ correspond to scattering partners with a higher Landau number, those labeled with ● have a lower Landau number, and the state indicated by an × corresponds to the scattering partners with the same Landau number as the electron under consideration.

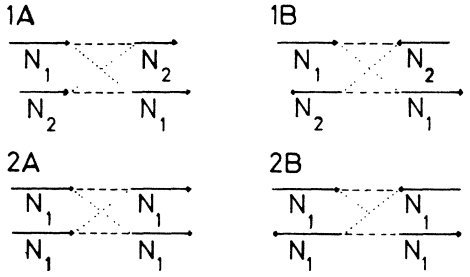

FIG. 3. Allowed scattering processes. The arrows indicate the k_z momenta. The numbers N_i label the corresponding Landau states. The direct scattering is indicated by a dashed line and the exchange processes by a dotted one.

$\Delta k_i = |k_{zi} - k_{z,i+1}|$ as we move from k_{zi} to $k_{z,i+1}$. The result is

$$\Delta k_i = \frac{\sqrt{2m}\sqrt{E_F}}{\hbar}\left[\tfrac{1}{2}(x_i - x_{i+1}) + \sum_{j=2}^{\infty}\frac{(2j-3)!!}{2j!!}(x_i^j - x_{i+1}^j)\right].$$

The dependence of Δk_i on all of the powers of the corresponding Landau numbers N_i and N_{i+1}, $x_i = (\hbar\omega/E_F)(N_i + \frac{1}{2})$, implies that none of the increments Δk_i can be obtained by a linear combination with integer coefficients from other increments Δk_j, $j \neq i$. Therefore, two electrons in the initial quantum states, $\psi_{N_1 k_{y1} k_{z1}}$ and $\psi_{N_2 k_{y2} k_{z2}}$ can only scatter into the final states $\psi_{N_1 k_{y3(4)} k_{z1}}$ and $\psi_{N_2 k_{y4(3)} k_{z2}}$. Notice that the same quantum numbers N_i and k_{zi} from the initial states appear in the final states. Figure 3 shows schematically the allowed scattering processes. Concerning these processes let us make the following comments.

(1) The *exchange* scattering processes 1A, 1B, and 2B (indicated by a dotted line) have the same initial and final k_z momenta and, therefore, they do not contribute to the scattering rate.

(2) Since the process 2a obviously does not contribute to the scattering rate, the processes which actually contribute are the *direct* scattering processes of 1A, 1B, and 2B (indicated by a broken line). For these processes we have $N_1 = N_4$ and $N_2 = N_3$.

We now proceed to calculate the lifetime of the electron in the state $1 = (N_1, k_{z1}, k_{y1})$. The matrix element M_{1234} in Eq. (6) is obtained by integrating over the two electron coordinates, taking into account that $N_1 = N_4$ and $N_2 = N_3$; the result is

$$M_{1234} = \frac{4\pi e^2}{\kappa V}\sum_{q_x}\frac{1}{q_x^2 + (k_{y1}-k_{y3})^2 + (k_{z1}-k_{z3})^2 + 1/r_0^2}\delta_{k_{z1}+k_{z2};k_{z3}+k_{z4}}\delta_{k_{y1}+k_{y2};k_{y3}+k_{y4}}$$
$$\times\{(\lambda^2/4)[q_x^2 + (k_{y1}-k_{y3})^2]\}^{N_2-N_1} C^{N_1 N_2} L_{N_1}^{N_2-N_1}\{(\lambda^2/2)[q_x^2 + (k_{y1}-k_{y3})^2]\}$$
$$\times L_{N_1}^{N_2-N_1}\{(\lambda^2/2)[q_x^2 + (k_{y2}-k_{y4})^2]\}$$
$$\times\exp\{-(\lambda^2/2)[q_x^2 + (k_{y1}-k_{y3})^2 + iq_x(k_{y1}+k_{y3}-k_{y2}-k_{y4})]\}\,. \tag{10}$$

Here, the first Kronecker δ accounts for the conservation of the k_z momenta and the second Kronecker δ in the k_y momenta accounts for the conservation of the zero-point coordinate $\lambda^2 k_y$. The number $C^{N_1 N_2}$ is equal to $2^{N_2-N_1} N_1! N_2!$. With the above result for M_{1234} we proceed to evaluate the scattering rate

$$1/\tau_1^{N_2} = (2\pi/\hbar)\sum_{k_{y2},k_{y3},k_{y4}}\sum_{k_{z2},k_{z3},k_{z4}}\delta(E_1 + E_2 - E_3 - E_4)M_{1234}^2 f(2)[1-f(3)][1-f(4)]\,, \tag{11}$$

where $f(i)$ is the Fermi-Dirac function for the electron in the state i. From the above six sums we first carry out the energy-independent ones, i.e., the k_{y2}, k_{y3}, and k_{y4} sums

$$\sum_{k_{y2},k_{y3},k_{y4}} M_{1234}^2 = \left[\frac{4\pi e^2}{\kappa V}\right]^2 \delta_{k_{z1}+k_{z2};k_{z3}+k_{z4}}$$
$$\times\sum_{k_{y3}}\sum_{q_x}\left[\frac{1}{q_x^2 + (k_{y1}-k_{y3})^2 + (k_{z1}-k_{z3})^2 + 1/r_0^2}\right]^2\left[\frac{\lambda^2}{4}[q_x^2 + (k_{y1}-k_{y3})^2]\right]^{2(N_2-N_1)}$$
$$\times\left[C^{N_1 N_2}\right]^2 e^{-\lambda^2[q_x^2 + (k_{y1}-k_{y3})^2]}(L_{N_1}^{N_2-N_1}\{(\lambda^2/2)[q_x^2 + (k_{y1}-k_{y3})^2]\})^4\,. \tag{12}$$

Equation (12) is valid for $N_2 > N_1$; for $N_1 > N_2$ we have to exchange N_2 with N_1 in this equation. Next we carry out the integrations over q_x and k_{y3}. To this end we introduce the polar coordinates θ, ρ, where $\theta = \angle(k_{y1} - k_{y3}, q_x)$ and $\rho^2 = (k_{y1}-k_{y3})^2 + q_x^2$. We find

$$\sum_{k_{y2},k_{y3},k_{y4}} M_{1234}^2 = \left[\frac{4\pi e^2}{\kappa V}\right]^2 (L_x/2\pi)(L_y/2\pi)\int_0^{2\pi} d\theta \int_0^{\infty} d\rho\,\rho e^{-\lambda^2\rho^2}(\lambda^2\rho^2/4)^{2(N_2-N_1)}[L_{N_1}^{N_2-N_1}(\lambda^2\rho^2/2)]^4$$
$$\times\left[C^{N_1 N_2}\right]^2 1/(\rho^2 + a_{k3-k1}^2)^2\,, \tag{13}$$

where

$$a_{k1-k3}^2 = (k_{z1}-k_{z3})^2 + 1/r_0^2\,,$$

To obtain a simplified expression for the above equation we introduce the quantity

$$Q_{k1-k3}^{N_1N_2} \equiv \int_0^{2\pi} d\theta \int_0^{\infty} d\rho \rho [L_{N_1}^{N_2-N_1}(\lambda^2\rho^2/2)]^4 \frac{e^{-\lambda^2\rho^2}(C^{N_1N_2})^2(\lambda^2\rho^2/4)^{2(N_2-N_1)}}{(\rho^2+a_{k1-k3}^2)^2}$$

$$=(\lambda^2\pi)\int_0^{\infty} dz[L_{N_1}^{N_2-N_1}(z/2)]^4 e^{-z}\frac{z^{2(N_2-N_1)}(N_1!)^2 4^{N_1-N_2}}{(N_{2!})^2(z+\lambda^2 a_{k1-k3}^2)^2} . \tag{14}$$

Using the above definition we rewrite Eq. (13) as

$$\sum_{k_{y2},k_{y3},k_{y4}} M_{1234}^2 = \left[\frac{4\pi e^2}{\kappa V}\right]^2 (L_x/2\pi)(L_y/2\pi) Q_{k1-k3}^{N_1N_2} . \tag{15}$$

The next step in the evaluation of the scattering rate deals with the energy-dependent integrations. We notice that the δ function in the energy variables can be written as follows:

$$\delta(E_1+E_2-E_3-E_4)$$
$$=\delta[E_{z1}+(N_1+\tfrac{1}{2})\hbar\omega+E_{z2}$$
$$+(N_2+\tfrac{1}{2})\hbar\omega-E_{z3}-(N_3+\tfrac{1}{2})\hbar\omega$$
$$-E_{z4}-(N_4+\tfrac{1}{2})\hbar\omega] .$$

For the allowed scattering processes we have $N_1=N_4$ and $N_2=N_3$ so that the right-hand side of the above equation reduces to

$$\delta(E_1+E_2-E_3-E_4)=\delta(E_{z1}+E_{z2}-E_{z3}-E_{z4}) . \tag{16}$$

Taking this equation into account, the integration over k_{z4} in Eq. (11) yields

$$1/\tau_1^{N_2}=(2\pi/\hbar)(4\pi e^2/\kappa V)^2(L_y/2\pi)(L_x/2\pi)$$
$$\times\sum_{k_{z2},k_{z3}}\delta(E_{z1}+E_{z2}-E_{z3}-\Delta_4)Q_{k1-k3}^{N_1N_2}$$
$$\times f(2)[1-f(3)][1-f(\Delta_4)] . \tag{17}$$

Here, Δ_4 means that the energy E_{z4} is being replaced by

$$E_{z4}=(\hbar^2/2m)\times(k_{z1}+k_{z2}-k_{z3})^2$$

in the corresponding functions $\delta(E_{z4})$ and $f(4)$. The δ function

$$\delta(E_{z1}+E_{z2}-E_{z3}-\Delta_4)$$

is given by

$$\delta(E_{z1}+E_{z2}-E_{z3}-\Delta_4)$$
$$=(m/\hbar^2|k_{z1}-k_{z2}|)[\delta(k_{z1}-k_{z3})+\delta(k_{z2}-k_{z3})] \tag{18}$$

for $k_{z1}\neq k_{z2}$.

For the allowed scattering processes we have $k_{z1}\neq k_{z2}$ so that the above equation can be used to proceed with the evaluation of the scattering rate, this rate, Eq. (17), becomes

$$1/\tau_1^{N_2}=(2\pi/\hbar)(4\pi e^2/\kappa V)^2(L_y/2\pi)(L_x/2\pi)(L_z/2\pi)^2(m/\hbar^2)$$
$$\times\int dk_{z2}1/|k_{z1}-k_{z2}|\int dk_{z3}Q_{k1-k3}^{N_1N_2}f(2)[1-f(3)][1-f(\Delta_4)][\delta(k_{z1}-k_{z3})+\delta(k_{z2}-k_{z3})] . \tag{19}$$

Since for the allowed scattering processes $k_{z1}\neq k_{z3}$, the above equation yields

$$1/\tau_1^{N_2}=(2\pi/\hbar)(4\pi e^2/\kappa V)^2(L_y/2\pi)(L_x/2\pi)(L_z/2\pi)^2(m/\hbar^2)$$
$$\times\int dk_{z2}(1/|k_{z1}-k_{z2}|)Q_{k1-k2}^{N_1N_2}f(2)[1-f(2)][1-f(1)] . \tag{20}$$

The integration over k_{z2} can be carried out by taking into account that the quantity $Q_{k1-k2}^{N_1N_2}$ can be taken outside the integral. This is allowed since at the Fermi level, where the scattering takes place, the k_z momenta admit only the values $\pm k_F^{N_i}$. Hence, Eq. (20) becomes for $N_1=N_2$

$$1/\tau_1^{N_1}=(2\pi/\hbar)(4\pi e^2/\kappa V)^2(L_y/2\pi)(L_x/2\pi)(L_z/2\pi)^2(m/\hbar^2)(1/|k_F^{N_1}+k_F^{N_1}|)Q_{k_F+k_F}^{N_1N_1}[1-f(1)]\int dk_{z2}f(2)[1-f(2)] , \tag{21}$$

and for $N_1\neq N_2$

$$1/\tau_1^{N_2}=(2\pi/\hbar)(4\pi e^2/\kappa V)^2(L_y/2\pi)(L_x/2\pi)(L_z/2\pi)^2(m/\hbar^2)$$
$$\times[(1/|k_F^{N_1}-k_F^{N_2}|)Q_{k_F-k_F}^{N_1N_2}+(1/|k_F^{N_1}+k_F^{N_2}|)Q_{k_F+k_F}^{N_1N_2}][1-f(1)]\int dk_{z2}f(2)[1-f(2)] . \tag{22}$$

The final integration is carried out in the Appendix. The result for the scattering rate is for $N_1=N_2$ given by

$$1/\tau_1^{N_1}=(m^2e^4/L_yL_x\pi\hbar^5\kappa^2)(1/k_F^{N_1})(1/|k_F^{N_1}+k_F^{N_1}|)Q_{k_F+k_F}^{N_1N_1}[1-f(1)]kT , \tag{23}$$

for $N_1 \neq N_2$ we have

$$1/\tau_1^{N_2} = (m^2 e^4/L_y L_x \pi \hbar^5 \kappa^2)(1/k_F^{N_2}[(1/|k_F^{N_1} - k_F^{N_2}|)Q_{k_F - k_F}^{N_1 N_2} + (1/|k_F^{N_1} + k_F^{N_2}|)Q_{k_F + k_F}^{N_1 N_2}][1 - f(1)]kT \ . \quad (24)$$

Equations (23) and (24) with the quantity $Q_{k_1 \pm k_2}^{N_1 N_2}$ numerically evaluated are used to determine the lifetime from Eq. (2). The coefficients $B_i(H)$, defined by $\tau_{e\text{-}e}^i(H) = B_i(H) T^{-1}$, are plotted for different magnetic fields. In Fig. 5 we use the electronic parameters of a semiconductor InSb, and in Fig. 6 those of a metal Al.

	InSb	Al
n (cm^{-3})	10^{18}	18.1×10^{22}
m^*/m	0.008	0.8430
κ	18	1
r_0 (cm)	3.672×10^{-8}	0.488×10^{-8}

The Fermi energy in the magnetic field is calculated from

$$E_F(H) = \tfrac{4}{9}(E_{F0}/\hbar\omega) E_{F0} \ , \quad (25)$$

where E_{F0} is the Fermi energy in the absence of a magnetic field. The above equation is taken from Ref. 12.

III. CONCLUSION

We now discuss the results based on Eqs. (2), (23), and (24) of Sec. II. The main qualitative result is that the Landau quantization leads to a linear T dependence of the e-e scattering rate $1/\tau_{e\text{-}e}^i = B_i(H)^{-1} T$, as compared with the quadratic T dependence found theoretically and experimentally in zero field, $H = 0$; i labels a Landau state at $E_F(H)$.

The linear T dependence is a direct consequence of the one-dimensional character of the allowed scattering processes at $E_F(H)$. For a longitudinal geometry, $\mathbf{H} \| \mathbf{E}$ (the electric field) $\| \hat{\mathbf{z}}$, the orbital quantization confines each conduction electron to a fixed-space region in the x-y plane and, therefore, only the transitions along the z axis are responsible for the electrical resistivity. In other words, the quasi-one-dimensional restriction of the phase space (k_z) available for scattering processes causes the linear T dependence. This dependence will not be changed if instead of a parabolic band we use a nonparabolic one (InSb); the phase-space argument remains valid. We mention that a linear T dependence of $1/\tau_{e\text{-}e}$ is discussed by other authors[13] for quasi-one-dimensional systems ($H = 0$).

The contribution of the e-e scattering processes to the longitudinal magnetoresistance is the only one that is dependent on the temperature T, at low T. The electron-photon and electron-impurity scatterings lead to temperature-independent contributions.[14]

The scattering rate $1/\tau_{e\text{-}e}^i(T)$ is plotted in Fig. 4 for a fixed magnetic field $H = 0.5248T$, and for different Landau states $i = 0, 30, 59$, as a function of temperature. The electronic parameter values used correspond to those of InSb. Together with these curves is also plotted the curve of the zero-field situation. We note the existence of two different T regions. At low temperature the scattering rate in the presence of a magnetic field is bigger than the scattering rate with zero magnetic field, whereas at higher temperatures, the opposite becomes true.

Contrary to the simple, namely linear, dependence of $1/\tau_{e\text{-}e}^i$ on the temperature, its dependence on the magnetic strength H is not determined by a simple power law, Eqs. (2), (23), and (24). The basic reason is that in the electron-electron scattering events, the eigenstates of both the "target electrons" and the "scattered electrons" are affected by the magnetic field. This is not the case for other pertinent scattering processes, such as the electron-phonon and the electron-impurity scatterings. Here, a simple power law determines the H dependence of the resistivity.[14]

We now proceed to a quantitative discussion of $\tau_{e\text{-}e}^i$. These results are plotted in Figs. 5 and 6. Figure 5 shows the lifetime coefficients $B_i(H)$ [cf. Eq. (25)] for three different magnetic fields. The magnetic fields are determined such that

$$\hbar\omega(N_{\max} - 1) < E_F(H) < \hbar\omega N_{\max} \ ,$$

where $N_{\max} = 30$, 60, and 100 are the number of Landau states filled with electrons. The abcissa refers to the Lan-

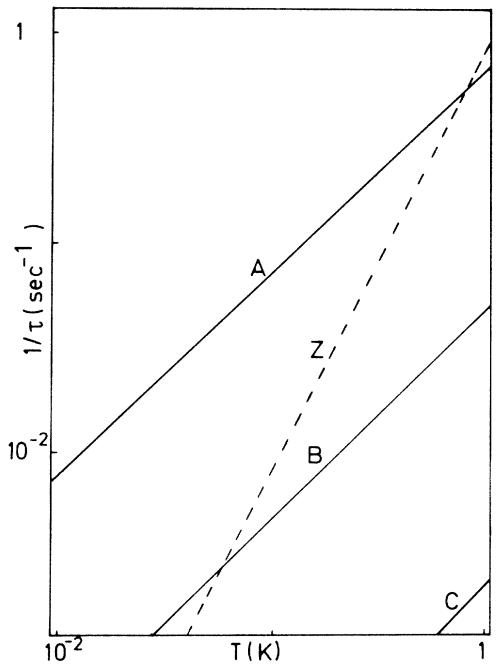

FIG. 4. Temperature dependence of the scattering rate in the presence and in the absence of a magnetic field. The electronic parameters used to calculate the scattering rates correspond to the semiconductor InSb. The curve Z indicates the scattering rate in the absence of a magnetic field. The lines labeled with A, B, and C show the scattering rate in the presence of a magnetic field $H = 0,5T$ for an electron in the states with Landau number $N = 0$, 30, and 59, respectively.

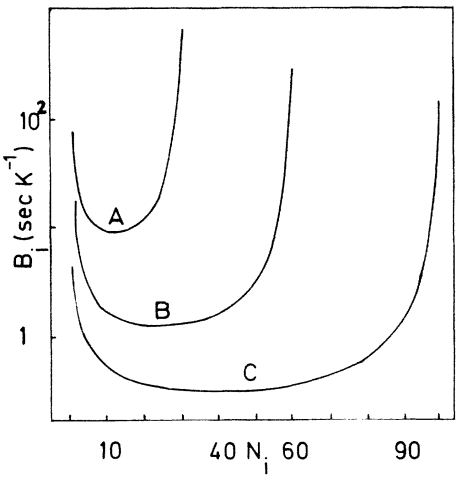

FIG. 5. Lifetime coefficients for InSb. The curve labeled with A corresponds to the magnetic field $H=1T$, the curve labeled with B to the magnetic field $H=0.5T$, and the one labeled with C to the magnetic field $H=0.3T$.

dau state at the Fermi level N_i, and the ordinate gives the lifetime coefficient $B_i(H)$ of this state. The electronic parameter values used to calculate these coefficients correspond to those of the semiconductor InSb. In Fig. 6 are drawn the equivalent curves for the metal Al.

Let us first discuss the results found in the case of InSb. This semiconductor has a small conduction-band mass, $m^*/m=0.008$, and the assumed electron density, $n=10^{18}$ cm^{-3} is also low. As a consequence of these values, the required magnetic field for 100 filled Landau states is relatively weak, $H=0.3T$. The relatively weak magnetic fields required for the Landau quantization have the following consequence: In the sequence of curves for $B_i(H)$ with $N_{\max}=30$, 60, and 100, respectively, we observe a tendency towards a constant value for B_i, independent of the Landau state i, cf. Fig. 5.

In the case of aluminium, strong magnetic fields ($H\sim 10^3T$) are required to fill even 100 Landau states. Therefore, B_i strongly depends on i.

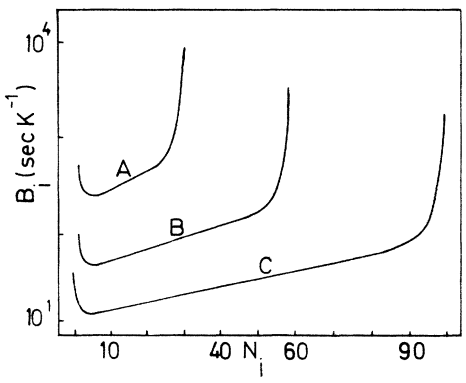

FIG. 6. Lifetime coefficients for Al. The curve labeled with A corresponds to the magnetic field $H=3360T$, the curve labeled with B to the magnetic field $H=1680T$, and the one labeled with C to the magnetic field $H=10^3T$.

Finally, let us discuss the common features of the lifetime coefficients $B_i(H)$ shown in Figs. 5 and 6. The largest values of $B_i(H)$ occur for Landau states with either very small or very large Landau quantum numbers N_i. This behavior can be understood by taking into account the momentum dependence of the scattering rate. The leading term of this dependence is given by

$$1/\tau^i_{e\text{-}e}(H)=\sum_{\substack{N=0\\(N\neq N_i)}}^{N_{\max}-1}\frac{1}{|k_F^{N_i}-k_F^N|\,|k_F^{N_i}+k_F^N|\,k_F^N}\,. \tag{26}$$

This rate is smaller the larger the differences in the k_z momenta at E_F are. For either very small or very large values of N_i, we get the largest differences and hence the largest values of B_i. The asymmetry of the values of $B_i(H)$, that the states with very large quantum numbers N_i exhibit the largest values of $B_i(H)$, can also be understood on the basis of Eq. (26). Remember that the k_z momenta at $E_F(H)$ are not uniformly distributed. There is an accumulation of the k_z states for the electrons with small quantum numbers N_i the reason being the parabolicity of the bands, cf. Fig. 1. Therefore, the scattering rate of an electron in a state with very large Landau number has more small terms in the sum as compared with an electron in a state with a small Landau number, cf. Eq. (26).

APPENDIX

We calculate the following integral

$$I=\int dk_{z2} f(2)[1-f(2)]\,,$$

where $f(2)$ is the Fermi-Dirac function. We can write I as

$$I=(\sqrt{2mkT}/2\hbar)\int_{-d}^{\infty} dx\,(e^x/(e^x+1)^2)(1/\sqrt{x+d})\,, \tag{A1}$$

where

$$x=[E_{z2}-E_F(H)+\hbar\omega/2]/kT$$

and

$$d=[E_F(H)-\hbar\omega/2]/kT\,.$$

For very low temperature $d\gg 1$ we have, therefore,

$$\begin{aligned} I&=(\sqrt{2mkT}/2\hbar)\int_{-\infty}^{\infty} dx\,(e^x/(e^x+1)^2)(1/\sqrt{x+d})\\ &\simeq\sqrt{2mkT}/2\hbar 2\sqrt{d}\\ &=(\sqrt{2m}/4\hbar)[1/(E_F(H)-\hbar\omega/2)^{1/2}]kT+o((kT)^3)\,. \end{aligned} \tag{A2}$$

Using $(\hbar k_F^{N_2})^2/2m=E_F(H)$, we finally obtain

$$I=(m/2\hbar^2)(1/k_F^{N_2})kT+o((kT)^3)\,. \tag{A3}$$

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