RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 33, NUMBER 3 1 FEBRUARY 1986

Reentrant temperature dependence of the critical current in small tunnel junctions

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(Received 22 November 1985)

We have observed an anomalous *decrease* of the measured Josephson critical current as temperature is decreased below $0.85T_c$ on small Sn-SnOx-Sn tunnel junctions. The results of analog and digital simulations over a wide range of junction noise and damping parameters suggest that this anomalous temperature dependence of the critical current could be caused by the strongly temperature-dependent damping effect of the quasiparticle tunnel resistance below the gap voltage.

According to the Ambegaokar-Baratoff (AB) theory,[1] the temperature dependence of the critical current I_c for a Josephson tunnel junction is given by $I_c=(\pi\Delta/2eR_n)\tanh(\Delta/2k_BT)$, where $\Delta(T)$ is the energy gap and R_n is the normal resistance of the junction. This expression predicts that I_c has a maximum of $\pi\Delta(0)/2eR_n$ at $T=0$ and decreases smoothly to zero as T increases to the critical temperature T_c. Although most junctions are found to follow this prediction to a semiquantitative degree at least, Buckner[2] observed that $I_c(T)$ of certain small-area Sn-SnOx-Sn junctions first increased, then decreased with decreasing temperature, with a maximum at about 2 K. Yeh and Langenberg[3] also reported a similar anomalous temperature dependence of $I_c(T)$ for certain Pb-PbOx-Pb junctions. However, no clear and accepted explanation for this remarkable behavior was given by them or subsequently to our knowledge.

In this Rapid Communication, we report new measurements showing a reentrant temperature dependence of the critical current for small Sn-SnOx-Sn tunnel junctions, including the first report of the temperature dependence of the hysteretic behavior. To gain insight into this effect, we have performed digital and analog simulations based on a modified RSJ (resistively shunted junction) model which takes account of the nonlinear quasiparticle resistance and thermal noise. From the results of simulations, it is found that the damping effect due to the subgap quasiparticle resistance, which depends strongly upon the temperature, plays an important role not only in reduction of the critical current but also in the hysteretic behavior observed with decreasing temperature.

The junctions used in the experiment were Sn-SnOx-Sn tunnel junctions fabricated by the trilayer photoresist technique reported first by Dolan[4] and Dunkleberger,[5] and used later in this group.[6] First, bridges of photoresist suspended above the surface of glass substrates were prepared, followed by the sequential procedure of metal film depositions and glow discharge oxidation without breaking the vacuum. A Sn film base electrode with thickness of 100 nm was evaporated at normal incidence to the substrate. A dc plasma discharge in pure oxygen at a pressure of 30 mtorr for 2–3 min with 11 mA of current at 1.1 kV was used to grow the oxide tunnel barrier on the base electrode. A Sn film of 300-nm thickness for a counter electrode was then deposited at an angle of 65° to the substrate normal. For these procedures, the substrates were placed on a rotatable sample holder which was cooled to liquid nitrogen temperature in a cryopumped evaporator with a base pressure of 2×10^{-7} torr. The junction area S was estimated by scanning-electron-microscopy and was typically less than 2 μm^2.

All experiments were carried out in a shielded room in a magnetically shielded cryostat. In order to isolate the sample from external noise, connections to room-temperature electronics were made through low-pass filters at the top of the Dewar and through cooled 2-kΩ resistors in the bath. The sample was simply immersed in the liquid helium, not surrounded by a cold metal can. The current bias was supplied to the junctions through a 1-MΩ resistor from a dry-battery power source.

The junction parameters obtained in the present experiment are listed in Table I. The four samples had normal resistances R_n ranging between 118–300 Ω, estimated capacitances C between 0.04–0.15 pF, and critical temperatures T_c between 3.72–3.64 K. The capacitances in Table I are estimated from the junction area and studies of Fiske steps as measured by other workers;[7,8] they are thought to be ac-

TABLE I. Experimental parameters of Sn-SnOx-Sn junctions. R_n, S, and T_c are directly measured. C is estimated from S.

Sample	R_n (Ω)	S (μm^2)	C (pF)	T_c (K)
A1	118	1.7	0.05	3.72
A2	168	5.7	0.15	3.72
A3	212	1.3	0.04	3.72
A4	300	2.0	0.06	3.64

curate to 50%. For the oxide thickness, we use 2 nm as a nominal value.[9]

Figure 1 shows typical I-V characteristics at different temperatures for the sample $A2$ with $R_n = 168\ \Omega$. All of the I-V curves show a finite initial slope and a rounded behavior of the Josephson current due to the noise fluctuations. Accordingly, the measured Josephson critical current, I_c^{m}, is *defined* as the current found by extrapolation to $V=0$ from the I-V curve at voltages below the gap voltage. As seen in the figure, I_c^{m} decreases as temperature is decreased, in contrast to the AB theory and usual observations. Note also that the I-V curve has larger hysteresis with decreasing temperature. The other three junctions listed in Table I had similar I-V curves for similar temperatures. The junctions with higher R_n than this junction showed a more rounded critical current, while the junction with the lower R_n had no rounded behavior at the lower temperatures.

The temperature dependence of the critical current for all four samples is shown in Fig. 2. All of the junctions have a similar reentrant temperature dependence of the critical current, which has a maximum in the vicinity of 3 K. Below this temperature where I_c^{m} is maximum, hysteresis appeared in the I-V curves for all the junctions, to an extent indicated in Fig. 4 below. These behaviors were reproducible after the junctions had been warmed to liquid nitrogen temperatures and recooled two times. Note that the gap voltage shows the normal increase with decreasing temperature, so that the apparent reentrant temperature dependence of the critical current is not due to weakening superconducting properties of the Sn electrodes, but must be due to the properties of the coupling in the tunnel junctions.

In order to display the anomalous temperature dependence more clearly, the ratio η of the measured critical current to the critical current predicted by the AB theory, $\eta = I_c^{\mathrm{m}}/I_c^{\mathrm{th}}$, is plotted against the reduced temperature, $t = T/T_c$, for the four different junctions in Fig. 3. η is used as a measure of deviation from the AB theoretical critical current. In the absence of fluctuation effects, η would be expected to be unity. As the temperature is decreased, however, the value of η for all four samples is observed to decrease monotonically, apparently approaching saturation at the lowest temperatures. This temperature dependence is suggestively similar to that of the quasiparticle conductance

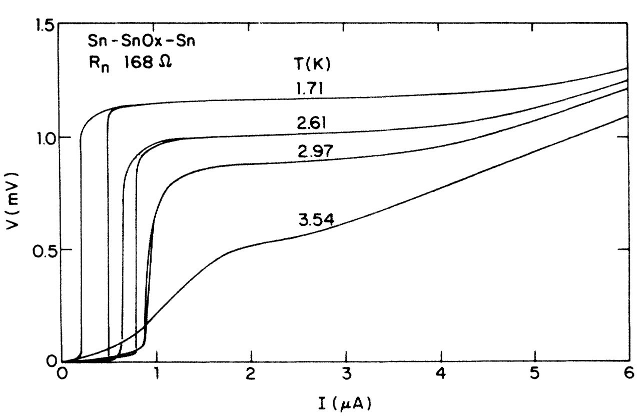

FIG. 1. I-V characteristics at different temperatures for a Sn-SnOx-Sn tunnel junction (sample A2).

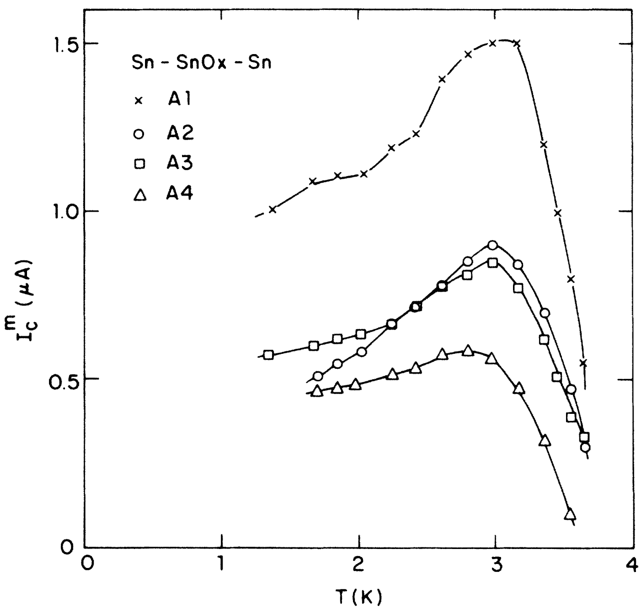

FIG. 2. Temperature dependence of measured critical current.

$\sim \exp[-\Delta(T)/k_B T]$, with a small temperature-independent leakage term.

Figure 4 shows the reduced temperature dependence of the hysteresis parameter $\alpha = I_{\min}/I_c^{\mathrm{m}}$, where $I_{\min}$ is the measured current below which a junction switches from the voltage state into the zero-voltage state. Above $t=0.85$ ($T=3.1$ K), $\alpha=1$, i.e., the junctions do not have hysteretic behaviors. Below $t=0.85$, on the other hand, α decreases in rough proportion to t, reflecting the fact that hysteresis of I-V curves appears at this temperature and becomes larger with decreasing temperature. Moreover, $t=0.85$ is also the temperature at which I_c^{m} has a maximum in the I_c^{m}-T plots

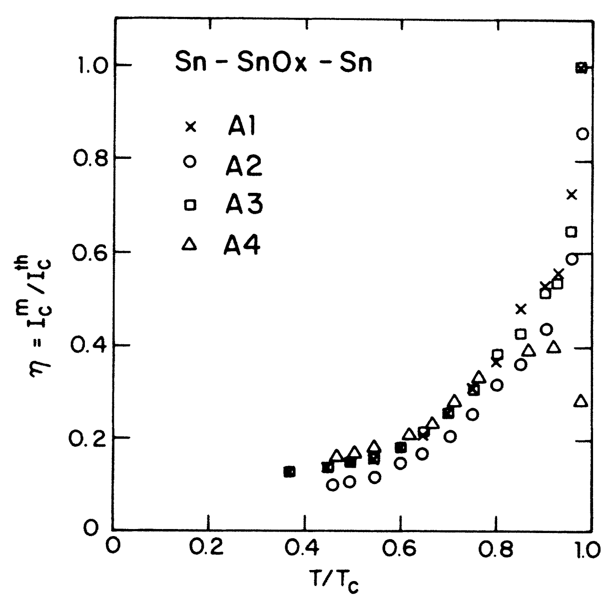

FIG. 3. Normalized temperature dependence of reduced critical current η, i.e., the ratio of measured critical current to the theoretical one obtained from the Ambegaokar-Baratoff theory.

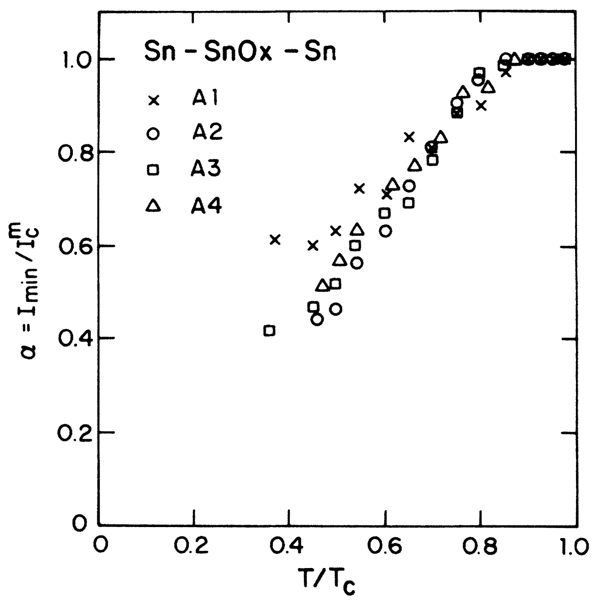

FIG. 4. Normalized temperature dependence of the hysteresis parameter α, which is the ratio of measured minimum current I_{min} to measured critical current I_c^m.

shown in Fig. 2. In addition, the leakage resistance R_l of these junctions at voltages below the gap voltage is strongly temperature dependent. These facts suggest that the anomalous temperature dependence of I_c^m might be associated with the temperature-dependent damping of the junction since the hysteretic behavior becomes larger with decreasing damping of the junction, according to the McCumber-Stewart theory.[10,11]

A Josephson tunnel junction with the critical current I_c biased at a constant current can be modeled as a particle moving in the one-dimensional "washboard" potential. In this model a junction is characterized by the damping parameter β_c and the thermal fluctuation parameter γ. Here $\beta_c = 2eI_cR^2C/\hbar$ and $\gamma = \hbar I_c/ekT$, where R and C are the junction resistance and capacitance, respectively. For the overdamped regime ($\beta_c \ll 1$) the effect of thermal fluctuations on a junction is well understood. From the Ambegaokar-Halperin theory,[12] which gives us the dependence of the I-V curve in the case of $\beta_c = 0$, we can estimate the ratio of the apparent I_c to the nonfluctuated I_c by extrapolation to $V = 0$ of I-V curves at the reduced voltage of ~ 0.25, since the measured η is taken in the same way for a rounded I-V curve. The estimated η is found to be $\eta \sim \exp(-5/\gamma)$.

In the underdamped regime ($\beta_c > 1$), on the other hand, the effects of thermal fluctuations are more complicated and extensions of the pioneer work of Kramers[13] have been discussed by many authors.[14-18] These analyses suggest that the lesser damping for $\beta_c > 0$ modifies the I-V curve of the overdamped case ($\beta_c = 0$) in such a way as to further reduce the apparent I_c. Unfortunately, however, these analyses deal with a junction with a linear resistance, which means they have not been extended to the case of a tunnel junction with the full nonlinearity of the quasiparticle I-V curve. Moreover, these analyses have not described the hysteresis of I-V curves for comparison with our results.

In order to study the damping effect on the critical current for a tunnel junction with thermal fluctuations and a nonlinear resistance, both digital and analog simulations were carried out, including a noise source.

Analog simulations were used initially to explore the γ dependence of the reduced critical current parameter η for various values of the damping parameter β_c with a linear resistance R. The results of these simulations show that as β_c is increased, η is decreased for a fixed value of γ. Assuming that the leakage resistance R_l is more appropriate than R_n as the resistance in β_c, β_c increases with decreasing temperature. Therefore, the apparent critical current would decrease with decreasing temperature if this temperature-dependent damping effect dominates over the contrary temperature dependence of I_c^{th} and the thermal noise amplitude. The physical meaning of this behavior is as follows: The particle in the potential well under the underdamping regime ($\beta_c \gg 1$) is easier to move from the well due to the fluctuations because of lower viscosity, compared to one under the overdamping regime ($\beta_c \ll 1$).

In subsequent digital simulations, the quasiparticle leakage resistance R_l at voltages below the gap voltage was changed to study its effect on the critical current for given R_n, I_c, and T. For example, the digital simulation with $R_n = R_l = 200\ \Omega$, $I_c = 6\ \mu$A, and an effective temperature of 10 K ($\gamma = 28$) yielded a rounded I-V curve which had no hysteresis, and the value of η was estimated to be 0.35. On the other hand, the simulation with $R_l = 1000\ \Omega$ *but the same values for all other parameters* yielded an I-V curve which had hysteretic behavior with the hysteresis parameter $\alpha = 0.40$ and the value of η was estimated to be 0.2. These differences qualitatively mirror those observed experimentally on cooling the junctions. These simulations, therefore, confirm that the higher quasiparticle resistance below the gap voltage plays a significant role in determining the damping of a junction with thermal fluctuations.

These comparisons suggest that the exponential T dependence of the subgap quasiparticle conductance is the basic cause of the observed reentrant $I_c^m(T)$. However, further simulations, taking into account the T dependence of I_c^{th} and of the Johnson noise term, suggest that a quantitative fit can be obtained only by adding a few degrees of extrinsic noise to the nominal (bath) noise temperature. The origin of this discrepancy is not yet resolved: It may reflect a few percent admixture of room-temperature blackbody radiation in the space surrounding the junction, coupling directly into the junction using the in-line electrodes as an antenna. On the other hand, the radically faster sweep rate in the simulation than in the experiment requires a large increase in the effective temperature, which introduces quantitative uncertainties.[6]

Attempts were also made to fit the experimental critical currents using analytic transition state theory as described in Ref. 18. If the leakage resistance is used in the damping term, the noise temperature required to fit the average I_c^m was about 5 K for the runs at $T_{bath} = 1.4$ K. The width of the switching distribution predicted by such high temperature, however, is wider than those observed experimentally, which correspond to noise temperatures closer to the bath temperature. No reentrant critical current is given by these analytical calculations, which take no account of the voltage dependence of the resistance and are only valid at low damping.

Alternative explanations for the discrepancy in noise temperatures reconsider the quantum nature of the system.

The effect of macroscopic quantum tunneling (MQT) on the system would indeed be to reduce the apparent critical current and introduce a fictitious noise temperature larger than the bath temperature of the junction. The crossover temperature for the onset of quantum behavior, however, is estimated to be at $k_B T_{\text{cross}} = \hbar\omega_p/2\pi$,[19,20] in the underdamped limit. For the junctions listed in Table I, $T_{\text{cross}} \sim 0.4$–$0.8$ K, and the effects of MQT should thus be unimportant. Widom[21] has recently predicted that for $T < \hbar/RCk_B = T_{RC}$, quantum electrodynamic Nyquist noise in the junction resistance will suppress the critical current of the junction. His theory models the junction using an RSJ model, with a linear resistance. The correct resistance to use in fitting to his model is thus in doubt, due to the nonlinearity of the quasiparticle tunneling curve. If R_n is used, then $T_{RC} \sim 0.3$–1.2 K, whereas if R_l is used, then T_{RC} is decreased into the millikelvin range. In neither case should this be a big effect. Finally, the simulations treated noise as a classical Johnson noise source. Since for the parameters of Table I, $\hbar\omega_p \sim 2k_B T$, quantum corrections[22] to the noise term as used by Sleator, Hahn, Hilbert, and Clarke[23] might be important. Work, computational and experimental, is currently in progress to test these various hypotheses.

The authors would like to acknowledge helpful discussions with A. Widom. One of the authors (H.A.) acknowledges the financial support of the Science and Technology Agency of Japan while this work was being done. This work was supported in part by National Science Foundation Grant No. DMR-8404489 and by U. S. Office of Naval Research Contract No. N00014-83-K-0383.

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