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Disorder and spin correlations in an amorphous ferromagnet

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Fixed-length spin Hamiltonian $\mathscr{H}=\frac{1}{2}\int d^3x\,[\alpha(\nabla\mathbf{S})^2-\beta(\mathbf{nS})^2]$ is considered. It is assumed that there is a short-range order in spatial distribution of local anisotropy axes $\mathbf{n}(\mathbf{x})$ characterized by the correlation function $\Gamma(x)$. For a weak anisotropy the low-temperature spin-spin correlation function is $\langle\mathbf{S}(\mathbf{x})\mathbf{S}(0)\rangle=S^2\exp[-(\Omega\beta^2/60\pi\alpha^2)|\mathbf{x}|]$ where $\Omega=\int d^3x\Gamma(x)$. This is in agreement with recent neutron scattering studies for amorphous Fe-Mn alloys.

Heisenberg spin systems with quenched random fields and anisotropies have been the subject of many recent theoretical studies.[1-6] Investigation of these systems, besides being of fundamental theoretical interest, is also necessary for understanding the nature of spin-glass ordering in solids. As is known, magnetic properties of a crystalline ferromagnet are defined by two major factors; exchange interaction, which leads to the parallel orientation of the neighbor spins, and magnetic anisotropy, which aligns spins along some preferable direction created by long-range crystalline order. One can roughly imagine an amorphous solid, as created from its crystalline counterpart by random local rotations of crystallographic axes. Since this transformation preserves short-range structural order and average interatomic distances, it does not drastically change ferromagnetic exchange and strength of the local anisotropy. The direction of the local anisotropy, however, now depends on the orientation of locally defined crystallographic axes. When anisotropy is large, compared with exchange,[7] the local crystal field orients the atomic spins practically along anisotropy axes at every site. In this case magnetic disorder trivially follows the structural disorder in the system. The case of weak random anisotropy is more subtle. Domain energy arguments,[8] as well as renormalization-group study,[9] show that weak random anisotropy destroys long-range ferromagnetic order in the absence of uniform anisotropy and external magnetic field. The atomic spins remain parallel, however, on the scale $|\mathbf{x}|\ll\xi$, where ξ is ferromagnetic correlation length (FCL). For $|\mathbf{x}|\ll\xi$, one can obtain[3] $\langle[\mathbf{S}(\mathbf{x})-\mathbf{S}(0)]^2\rangle\sim S^2|\mathbf{x}|/\xi$. Note that in the case of weak anisotropy, ξ may be much greater than the characteristic length of short-range structural order. It can be understood in terms similar to those for Brownian motion. When moving along some path through a solid, the magnetization, created by strong exchange, "feels" numerous pushes of weak random-anisotropy field. On large scales this leads to smooth stochastic rotation of the magnetization over the solid. To describe such a behavior of the magnetization, one must calculate spin-spin correlation function for arbitrary $|\mathbf{x}|$. This is done in the present paper within three-dimensional random-anisotropy model.

When considering the effects with a characteristic spatial scale, large compared to the range of exchange interaction, the random-anisotropy Heisenberg model[10] is equivalent to the continuous spin-field model with the Hamiltonian[1,3]

$$\mathscr{H}=\frac{1}{2}\int d^3x\,[\alpha(\nabla\mathbf{S})^2-\beta(\mathbf{nS})^2]\ . \tag{1}$$

Here α and β are exchange and anisotropy strengths, respectively, $[\mathbf{S}(\mathbf{x})]^2$ is a constant S^2, and $\mathbf{n}(\mathbf{x})$ is a unit vector defining the direction of local anisotropy. We will assume that there is a short-range order with a characteristic length R_s in a spatial distribution of anisotropy axes. The corresponding correlation function $\Gamma(\mathbf{x}'-\mathbf{x}'')$ rapidly goes to zero for $|\mathbf{x}'-\mathbf{x}''|>R_s$ and $\Gamma(0)=1$. To obtain the equation defining fixed-length magnetization, $\mathbf{S}(\mathbf{x})$, let us consider the functional

$$\tilde{\mathscr{H}}=\frac{1}{2}\int d^3x\,[\alpha(\nabla\mathbf{S})^2-\beta(\mathbf{nS})^2+\lambda(\mathbf{x})\mathbf{S}^2]\ , \tag{2}$$

where $\lambda(\mathbf{x})$ plays the role of local Lagrange multiplier. Variation of this functional gives

$$\alpha\nabla^2\mathbf{S}=-\beta\mathbf{n}(\mathbf{nS})+\lambda\mathbf{S}\ . \tag{3}$$

Let $\lambda(\mathbf{x})=\alpha K^2+\tilde{\lambda}(\mathbf{x})$, where K is a constant of higher than first order in β. Then Eq. (3) can be rewritten as

$$(\nabla^2-K^2)\mathbf{S}=\alpha^{-1}[\tilde{\lambda}\mathbf{S}-\beta\mathbf{n}(\mathbf{nS})]\ . \tag{4}$$

It is convenient to present Eq. (4) in the integral form

$$\mathbf{S}(\mathbf{x})=\alpha^{-1}\int d^3x'G_K(\mathbf{x}-\mathbf{x}')[\tilde{\lambda}'\mathbf{S}'-\beta\mathbf{n}'(\mathbf{n}'\mathbf{S}')]\ , \tag{5}$$

where $f'=f(\mathbf{x}')$,

$$G_K(\mathbf{x})=-\frac{e^{-K|\mathbf{x}|}}{4\pi|\mathbf{x}|} \tag{6}$$

is the Green's function of Eq. (4), satisfying the equation

$(\nabla^2 - K^2)G_K(\mathbf{x}) = \delta(\mathbf{x})$. With the help of Eq. (5) it is easy to obtain

$$\langle \mathbf{S}(\mathbf{x}_1)\mathbf{S}(\mathbf{x}_2)\rangle = \alpha^{-2}\int\int d^3x'\, d^3x'' G_K(\mathbf{x}_1-\mathbf{x}')G_K(\mathbf{x}_2-\mathbf{x}'')\langle[\tilde{\lambda}'\mathbf{S}'-\beta\mathbf{n}'(\mathbf{n}'\mathbf{S}')][\tilde{\lambda}''\mathbf{S}''-\beta\mathbf{n}''(\mathbf{n}''\mathbf{S}'')]\rangle \ . \tag{7}$$

To find the correlation function, let us consider the formal expression for $\lambda(\mathbf{x})$, which follows from Eq. (3) after multiplying it by $\mathbf{S}$,

$$\lambda(\mathbf{x}) = \alpha\frac{\mathbf{S}\nabla^2\mathbf{S}}{S^2} + \beta\frac{(\mathbf{nS})^2}{S^2} \ . \tag{8}$$

The first term in Eq. (8) is of the order of α/ξ^2, while the second term is of the order of β. Since parameter ξ^{-2}, as well as K^2, is of higher than first order in β,[8,3] then to the lowest order in β, one can replace $\tilde{\lambda}(\mathbf{x})$ in Eq. (7) by $\beta(\mathbf{nS})^2/S^2$. It gives

$$\langle \mathbf{S}(\mathbf{x}_1)\mathbf{S}(\mathbf{x}_2)\rangle = S^2\frac{\beta^2}{\alpha^2}\int\int d^3x'\, d^3x'' G_K(\mathbf{x}_1-\mathbf{x}')G_K(\mathbf{x}_2-\mathbf{x}'')\langle n_i'n_j'n_k''n_l''\sigma_j'\sigma_l''(\sigma_i'\sigma_m'-\delta_{im})(\sigma_k''\sigma_m''-\delta_{km})\rangle \ , \tag{9}$$

where we have introduced $\boldsymbol{\sigma} = \mathbf{S}/S$.

In accordance with our assumption,

$$\langle n_i(\mathbf{x}')n_j(\mathbf{x}')n_k(\mathbf{x}'')n_l(\mathbf{x}'')\rangle = \tfrac{1}{15}\delta_{ij}\delta_{kl} + \tfrac{1}{15}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\Gamma(\mathbf{x}'-\mathbf{x}'') \ , \tag{10}$$

where the tensor coefficient is found from the condition at $\mathbf{x}'=\mathbf{x}''$. Taking into account that $G_K(\mathbf{x})$ and $\boldsymbol{\sigma}(\mathbf{x})$ vary slightly on the scale R_s, and also that $[\boldsymbol{\sigma}(\mathbf{x})]^2 = 1$, we obtain from Eq. (9)

$$\langle \mathbf{S}(\mathbf{x}_1)\mathbf{S}(\mathbf{x}_2)\rangle = \frac{2}{15}S^2\Omega\frac{\beta^2}{\alpha^2}\int d^3x\, G_K(\mathbf{x}_1-\mathbf{x})G_K(\mathbf{x}_2-\mathbf{x}) \tag{11}$$

where $\Omega = \int d^3x\,\Gamma(x)$. The integral in Eq. (11), can be calculated rigorously,

$$\int d^3x\, G_K(\mathbf{x}_1-\mathbf{x})G_K(\mathbf{x}_2-\mathbf{x}) = \frac{1}{8\pi K}\exp(-K|\mathbf{x}_1-\mathbf{x}_2|) \ . \tag{12}$$

Since $\langle[\mathbf{S}(\mathbf{x})]^2\rangle = S^2$, the parameter K can be determined by setting $\mathbf{x}_1 = \mathbf{x}_2$, which finally gives

$$\langle \mathbf{S}(\mathbf{x}_1)\mathbf{S}(\mathbf{x}_2)\rangle = S^2\exp\left(-\frac{|\mathbf{x}_1-\mathbf{x}_2|}{\xi}\right) \tag{13}$$

$$\xi = K^{-1} = \frac{60\pi}{\Omega}\frac{\alpha^2}{\beta^2} \tag{14}$$

The existence of the characteristic length $L \propto (\alpha/\beta)^2$ for random-anisotropy model was pointed out by many authors.[1,8,9] It is widely accepted that L corresponds to the size typical of domains with opposite spins. Our consideration shows, however, that there are no domains separated from each other by thin domain walls, as it is in a crystalline ferromagnet. For a ferromagnet with random anisotropies, ξ characterizes smooth stochastic rotation of the magnetization over the solid. If $\Gamma(x) = \exp(-|\mathbf{x}|/R_s)$, $\Omega = 8\pi R_s^3$ and FCL is defined as[3]

$$\xi = \frac{15}{2}\frac{\alpha^2}{\beta^2 R_s^3} \ . \tag{15}$$

All the above formulas are valid when ξ is large in comparison with R_s. Thus $\Lambda = \beta R_s^2/\alpha$ is a small parameter of the theory, which applies, therefore, to solids with weak random anisotropy and not too large R_s. FCL becomes of the order of R_s at $\Lambda \sim 1$. Further increasing Λ leads to a magnetic ordering which trivially follows short-range structural order in the system, i.e., which is characterized by $\xi = R_s$. Note that since $\xi \propto R_s^{-3}$ for small R_s and $\xi = R_s$ for large R_s, it can be easily seen that $\xi(R_s)$ has a minimum $\xi_{\min} \sim (\alpha/\beta)^{1/2}$ at $R_s \sim (\alpha/\beta)^{1/2}$. It should also be noted that for small Λ, a randomly ordered magnetic state with a very large FCL may be hardly realized experimentally because of its high sensitivity to weak uniform anisotropy.[3]

We would like to emphasize that Eqs. (13) and (14), to a certain extent, are independent of the way in which randomness is introduced into the model. In particular, Eq. (13) and, with an accuracy up to the coefficient, Eq. (14) remain valid when random fields $\mathbf{hS}$ or random p-fold anisotropies $\beta(\mathbf{nS})^p$ are considered instead of the anisotropy term in the Hamiltonian (1).

Recently, conflicting results have been obtained for renormalization-group study of random-field XY model.[5,6] Our conclusion, as to the exponential decay of the spin-pair correlation function, is in agreement with the results of Ref. 5, but contradicts the results of Ref. 6, where algebraic decay of the correlation function was obtained in three dimensions.

Within spatial regions less than FCL spins are ferromagnetically correlated. In these regions well-defined spin waves with momenta $Q >> K$ must be observed in neutron scattering experiments. For small scattering vector Q, the cross section of neutron scattering must be dominated at low temperature by the Fourier transform of spin-spin correlation function (13),

$$S(Q) = \frac{8\pi K S^2}{(Q^2+K^2)^2} \ . \tag{16}$$

Such a behavior of the cross section has been observed, and interpreted as a random-field effect in recent neutron scattering studies of amorphous Fe-Mn alloys.[11] It has also been pointed out in Ref. 11 that pair correlation function (13) provides a natural explanation for many experimental features common to a wide variety of spin-glass systems. Experimental value of FCL extracted from quasielastic scattering data[11] depends on temperature and concentration of magnetic atoms. These effects could be taken into account through the effective temperature and concentration dependence of exchange and anisotropy constants, as it is often done for the crystalline ferromagnets.

The concluding remark concerns the parameter R_s. As was already noted, the direction of local anisotropy depends on the orientation of locally defined crystallographic axes.

Hence, the parameter R_s is defined by short-range orientational order,[12] rather than by translational order in the system. For a polycrystalline spin-glass system consisting of very small ferromagnetic microcrystallites, R_s coincides with the average size of a microcrystallite. The situation is less obvious, however, for amorphous systems, in particular, metallic glasses, where short-range translational order can be accompanied by extended correlation in the orientation of locally defined crystallographic axes.[12] Thus, one should be careful identifying R_s with the amorphous structure factor measured by neutron scattering.

Note added. Aharony and Pytte[13] and Aharony[14] claimed a power-law decay of correlations in amorphous ferromagnets. However, in their latest paper,[15] Aharony and Pytte used scaling arguments to derive an *exponential* decay of the spin-spin correlation function. They attribute the refutation of their earlier claim to the fact that they had been working with the leading order in D, which might not be sufficient for the renormalization-group procedure. Villain and Semeria have argued[16] that solutions showing exponential decay do not represent the true minimum of energy, which is an apparent contradiction of our results. We note, however, that their arguments are valid only for iterative solutions and do not apply to the approach developed in this paper.

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