Transverse attenuation coefficients for superconductors containing transition-metal impurities

I. M. Tang
Department of Physics, Faculty of Sciences, Mahidol University, Bangkok 4, Thailand
(Received 28 May 1985)

The attenuation of transverse sound waves in a superconductor containing transition-metal impurities whose behavior lies close to the magnetic-to-nonmagnetic transition is studied. Comparison is made between the attenuation coefficient obtained on the basis of the local spin-fluctuation (LSF) description of the system and the coefficient obtained on the basis of a high-T_K Kondo impurity description of the system. It is seen that the LSF-based transverse attenuation coefficient can give a reasonable fit to the observed attenuation in superconducting *Zn*Mn alloys (which are alloys lying very close to the magnetic-to-nonmagnetic transition), while the high-T_K Kondo-description-based coefficient cannot.

I. INTRODUCTION

Measurement of the transport properties such as the ultrasonic attenuation coefficient in a superconductor containing transition-metal (TM) impurities provide for a direct means for investigating the possible formation of local magnetic states by the impurities. In an early paper, Kadanoff and Falko[1] studied the influence of paramagnetic impurities on the ultrasonic attenuation in superconductors. Since their work was based on the Abrikosov-Gorkov (AG) treatment of the magnetic impurity, they were not able to see how the attenuation was modified by the formation of localized excited states within the energy gap. These localized states appeared when the interaction between the magnetic moments and the electrons of the host metal are treated beyond the Born approximation. Using the Shiba theory,[2] which used the T-matrix approach to treat the interaction, Machida[3] and Leon and Nagi[4] have recalculated the longitudinal attenuation (LA) coefficient for superconductors having localized states within the gap. Matsui and Masuda[5] have numerically calculated the LA coefficient for a Kondo superconductor based on the Müller-Hartmann and Zittartz[6] (MHZ) treatment of the Kondo effect. In the MHZ theory, the position of the localized state within the energy gap is a function of the temperature. This temperature dependence leads to a temperature-dependent pair-breaking parameter which in turn leads to the reentrant behavior seen in the low-T_K Kendo impurities.

This paper is concerned with the transverse attenuation (TA) coefficient for the doped superconductor when the behavior of the TM impurities lies in the transition region between the magnetic and nonmagnetic behavior regions. For superconductors lying in this region, it is not clear whether the Kondo description or the local spin-fluctuation (LSF) description of the TM is most appropriate. For example, the dilute alloy system *Zn*Mn has both Kondo-like and LSF-like properties.[7] Above 0.5 K, the normal phase resistivity exhibits the logarithmic behavior characteristic of a Kondo alloy having a $T_K=1$ K, while below 0.5 K, the resistivity exhibits the T^2-dependence characteristic of LSF alloys having $\Theta_{LSF}=2.5$ K. Furthermore, the transition temperatures of some of the *Zn*Mn superconductors lie close to or below T_K, making use of the MHZ theory inappropriate for describing the Kondo effect in these superconductors.

The ultrasonic attenuation of transverse waves in the normal phase is due to two mechanisms, the electromagnetic absorption mechanism and the collision drag mechanism. In the superconducting phase, the electromagnetic absorption term is rapidly screened out except near the transition temperature. In their study, Kadanoff and Falko discarded the electromagnetic contribution in their calculation of the TA coefficient. Since we will be interested in the region close to T_c, where we will be able to obtain TA coefficients in an analytical form, we have not dropped the term. Our work will follow closely that of Maki.[8]

To treat the situation when the TM impurities give rise to LSF, we will be using the renormalization approach of Schlottmann.[9] His approach gives results which are exact for $g=U/\pi\Gamma_d$ (U being the Coulomb repulsion energy and Γ_d being the half width of the impurity state), less than 1. To treat the case when the impurity becomes a Kondo impurity, we will be using the Matsuura, Ichinose, and Nagaoka[10] (MIN) theory for the Kondo effect in superconductors. The MIN theory introduces an interpolation scheme for the pair-breaking parameter which allows one to pass from the magnetic side of the transition to the nonmagnetic side.

In Sec. II, we present Maki's formulation[8] of the transverse attenuation coefficient in terms of various correlation functions. Like him, we then expand the propagators appearing in the correlation functions in terms of the propagators of the normal phase and the energy gap. Evaluations of the leading terms in the expansion are carried out in Secs. III and IV for the case of LSF and Kondo impurities, respectively. Discussion of the significance of the resulting expressions for the TA coefficient to the transverse attenuation data for the *Zn*Mn superconductors[11] is given in Sec. V.

 ©1986 The American Physical Society

II. FORMULATION

Using Kubo's linear response theory approach, Maki[8] showed that the attenuation coefficient for transverse ultrasonic waves is given by

$$\alpha^T = \mathrm{Re}\left[\frac{\omega^2}{i\omega\rho_{\mathrm{ion}}v_s}\langle[h_T,h_T]\rangle_{q,\omega}\right], \tag{1a}$$

where ω is the frequency of the sound wave; v_s, the speed of sound, and where

$$h_T(r,t)=(q/\omega)\overline{\tau}_{xz}(r,t)-m\overline{j}_x(r,t), \tag{1b}$$

with $\overline{\tau}_{xz}$ being a stress tensor operator and $\overline{j}_x$ being the mass current operator. The correlation function $\langle[h_T,h_T]\rangle_{q\omega}$ written above is being evaluated in a system in which the long-range current-current interactions are present. In order to be able to use the usual Gor'kov propagators in the evaluation of the correlation function, a transformation to a fictitious system in which the long-range interactions have been removed must be made (the Gor'kov propagators were formulated in such a system). Making this transformation, Maki found that the TA coefficient becomes

$$\alpha^T=\mathrm{Re}\left[\frac{q^2}{i\omega\rho_{\mathrm{ion}}v_s}\left[\langle[\tau_{xz},\tau_{xz}]\rangle_{q,\omega}-\frac{(\langle[\tau_{xz},j_x]\rangle_{q\omega})^2}{\langle[j_{x_1}j_x]\rangle_{q,\omega}}\right]\right], \tag{2}$$

where the correlation functions are now evaluated in the fictitious system.

Close to T_c, the Gor'kov propagators, $G(p,i\omega_n)$ and $F^*(p,i\omega_n)$ can be expanded in terms of the normal metal propagators and the energy gap. Following Maki, we find that the correlation functions in the complex ω_0 plane are given by

$$\langle[\tau_{xz},\tau_{xz}]\rangle_{q,\omega_0}=\frac{p_F^4}{3\pi q}(ql)^{-1}[1-g(ql)]\pi T\sum_n\left\{1-\frac{\omega_n\omega_n'}{|\omega_n||\omega_n'|}-\frac{|\Delta_{\mathrm{ph}}(r)|^2}{2}\left[\frac{\omega_n\omega_n'}{|\omega_n||\omega_n'|}\left[\frac{\gamma^2(\omega_n)}{|\tilde{\omega}_n|}+\frac{\gamma^2(\omega_n')}{|\tilde{\omega}_n'|}\right]-\frac{2\tau(\omega_n)\gamma(\omega_n')}{|\tilde{\omega}_n|\cdot|\tilde{\omega}_n'|}\right]\right\}, \tag{3}$$

$$\langle[\tau_{xz},j_x]\rangle_{q,\omega_0}=\frac{ip_F^3}{3\pi q}[1-g(ql)]\pi T\sum\left\{1-\frac{\omega_n\omega_n'}{|\omega_n||\omega_n'|}-\frac{|\Delta_{\mathrm{ph}}^{(n)}|^2}{2}\left[\frac{\omega_n\omega_n'}{|\omega_n||\omega_n'|}\left[\frac{\gamma^2(\omega_n)}{|\tilde{\omega}_n|^2}-\frac{\gamma^2(\omega_n')}{|\tilde{\omega}_n'|^2}\right]\right]\right\}. \tag{4}$$

and

$$\langle[j_x,j_x]\rangle_{q,\omega_0}=\frac{p_F^2}{3\pi q}(ql)g(ql),\pi T\sum\left\{1-\frac{\omega_n\omega_n'}{|\omega_n||\omega_n'|}-\frac{|\Delta_{\mathrm{ph}}(r)|^2}{2}\left[\frac{\omega_n\omega_n'}{|\omega_n||\omega_n'|}\left[\frac{\gamma^2(\omega_n)}{|\tilde{\omega}_n|^2}+\frac{\gamma^2(\omega_n')}{|\tilde{\omega}_n'|^2}\right]+\frac{2\gamma(\omega_n)\gamma(\omega_n')}{|\tilde{\omega}_n||\tilde{\omega}_n'|}\right]\right\}, \tag{5}$$

where $g(x)$ is the Pippard function; $\omega_n'=\omega_n-\omega_0;\tilde{\omega}_n$, the renormalized frequencies and $\gamma(\omega_n)$ is the renormalization constant connecting the energy gap to the order parameter. A further renormalization due to the averaging of the possible distribution of impurities in the metal is required. For dirty superconductors, Maki[12] has shown that this averaging could be done by the inclusion of the following correction at all of the proper vertices:

$$\eta_{q,\omega_n}[1-(\tfrac{1}{2}\gamma\,|\omega_n|)(1-\tfrac{1}{3}\tau\tau_{\mathrm{fr}}v^2q^2)]^{-1}, \tag{6}$$

where q is the external momentum associated with the order parameter, τ is the collision lifetime due to the normal scattering by the impurity, and τ_{tr} is the transport lifetime. For pure superconductors, the above expansion in powers of $|\Delta_{\mathrm{ph}}|^2$ is not permissable as was shown by Cyrot and Maki[13] They showed that the perturbative expansion method for calculating the ultrasonic attenuation coefficient for a pure superconductor in presence of an external magnetic field gave rise to unreasonable results, the coefficient of the $|\Delta_{\mathrm{ph}}|^2$ term vanished identically as $\omega\to 0$ and that the leading term would be of order $|\Delta_{\mathrm{ph}}|^4$. To determine the properties of a pure superconductor in the presence of a magnetic field, another method would have to be used. Since we will be interested in dirty superconductors, we will not be concerned with this other method. In our calculations, we have neglected the corrections to the vertices associated with the mass current and the stress tensor. Neglect of these corrections results in errors of order l/ξ_0 which tend to zero for very short electron path lengths. These corrections must be considered for the case of pure superconductors.

III. LOCAL SPIN-FLUCTUATION DESCRIPTION

Like most of the magnetic properties of TM impurities, the local spin fluctuations arise from the interplay of the Coulomb repulsion between the d electrons of opposite spins which are localized about the impurity sites and the hybridization of the d electrons with the conduction electrons of the host metal. Early treatments of the LSF were based on either the random-phase approximation or renormalized random-phase approximation treatment of the Coulomb repulsion term appearing in the Anderson model description of a TM impurity dissolved in a simple host metal. Going beyond these approximation, Iche[14] was able to develop a theory for LSF which was exact for $g=U/\pi\Gamma_d$ less than 1. Schlottmann[9] extended Iche's theory to local spin fluctuations in superconductors by dividing the diagrams for the self-energy corrections to the anomalous propagator into two subsets and then demanding that each subset obeyed multiplicative renormalization. We refer the reader to Schlottmann's paper[9] for the details of his theory.

In a recent paper,[15] the present author showed that the vertex correction $\gamma(\omega_n)$ in the Schlottmann theory is given by

$$\gamma(\omega_n)=\frac{1-n_i[N_d(0)/N(0)][1/N(0)\,|\lambda|\,]U_{\rm eff}\chi(0)}{1+n_iN_d(0)/N(0)}\eta_{q,\omega_n}\,, \tag{7}$$

where $N_d(0)$ and $N(0)$ are the densities of states of the d electrons and the conduction electrons of the host metal; n_i, $2l+1$ times the concentration of impurities; $U_{\rm eff}$, the effective Coulomb energy $[=U/(1+U/\pi\Gamma_d)]$; $\chi(0)$, the static susceptibility due to rapid spin fluctuations and where η_{q,ω_n} is the renormalization parameter given by Eq. (6).

Substituting the above into the correlation function $\langle[\tau_{xz},\tau_{xz}]\rangle_{q,\omega_0}$ we obtain

$$\begin{aligned}\langle[\tau_{xz},\tau_{xz}]\rangle_{q,\omega_0}&=\frac{p_F^4}{3\pi q}(ql)^{-1}[1-g(ql)]\pi T\\&\times\sum\left\{1-\frac{\omega_n\omega_n'}{|\omega_n||\omega_n'|}-\langle[\Delta_{\rm ph}(r)|^2\rangle\left[\frac{1-n_i/n_{\rm cr}}{1+n_iN_d(0)/N(0)}\right]^2\right.\\&\left.\times\left[\frac{1}{2}\frac{\omega_n\omega_n'}{|\omega_n||\omega_n'|}\left[\frac{1}{(|\omega_n|+\alpha)^2}+\frac{1}{(|\omega_n'|+\alpha)^2}\right]-\frac{1}{(|\omega_n|+\alpha)(|\omega_n'|+\alpha)}\right]\right\},\end{aligned} \tag{8}$$

where $n_{\rm cr}$ is the critical concentration at which the LSF can suppress the superconductivity and where

$$\alpha=\tau_{\rm tr}v^2q^2/6\,. \tag{9}$$

The appearance of $\langle\,|\Delta_{\rm ph}|^2\rangle$ in the above expression signifies that the averaging over the impurity distribution has been carried out. The summation in Eq. (8) has been performed by Maki and his results yield

$$\begin{aligned}\langle[\tau_{xz},\tau_{xz}]\rangle_{q,\omega_0}&\\=\frac{p_F^4}{3\pi q}(ql)^{-1}[1-g(ql)]\omega_0&\left\{1-\frac{\langle\,|\Delta_{\rm ph}(r)|^2\rangle}{\omega_0}\left[\frac{1-n_i/n_{\rm cr}}{1+n_iN_d(0)/N(0)}\right]^2\right.\\&\left.\times\left\{\frac{1}{2\pi T}\psi'\left[\frac{1}{2}+\frac{\omega_0}{2\pi T}+\rho\right]-\left[\frac{1}{\omega_0}+\frac{1}{\omega_0+2\alpha}\right]\left[\psi\left[\frac{1}{2}+\frac{\omega_0}{2\pi T}+\rho\right]-\psi(\tfrac{1}{2}+\rho)\right]\right\}\right\},\end{aligned} \tag{10}$$

where $\rho=\alpha/2\pi T$. Similar evaluations of the other two correlation functions yield

$$\langle[\tau_{xz},j_x]\rangle_{q,\omega_0}=\frac{ip_F^3}{3\pi q}[1-g(ql)]\omega_0 \tag{11}$$

and

$$\begin{aligned}\langle[j_x,j_x]\rangle_{q,\omega_0}=\frac{p_F^2}{3\pi q}(ql)g(ql)\omega_0&\left\{1-\frac{\langle\,|\Delta_{\rm ph}(r)|^2\rangle}{\omega_0}\left[\frac{1-n_i/n_{\rm cr}}{1+n_iN_d(0)/N(0)}\right]^2\left\{\frac{1}{2\pi T}\psi'\left[\tfrac{1}{2}+\frac{\omega_0}{2\pi T}+\rho\right]\right.\right.\\&\left.\left.+\left[\frac{1}{\omega_0}+\frac{1}{\omega_0+2\alpha}\right]\left[\psi\left[\tfrac{1}{2}+\frac{\omega_0}{2\pi T}+\rho\right]-\psi(\tfrac{1}{2}+\rho)\right]\right\}\right\}.\end{aligned} \tag{12}$$

The dependence of the order parameter on the temperature and the impurity concentration has been derived in Ref. 15 and is

$$\langle |\Delta_{\rm ph}(r)|^2\rangle = \frac{2(2\pi T_c)^2(1-T/T_c)}{-\psi''(\frac{1}{2})+\frac{n_i N_d(0)/N(0)}{1+n_i N_d(0)/N(0)}\left[\frac{\Gamma_d}{6\pi T}\right]\psi'''(\frac{1}{2})}\left[\frac{1+n_i N_d(0)/N(0)}{1-n_i/n_{\rm cr}}\right]^2 , \tag{13}$$

where $\psi'(x)$ and $\psi''(x)$ are the polygamma functions.

For low frequencies ($\omega < \pi T_{\rm co}$), the polygamma function appearing in the correlation functions can be expanded in powers of $\omega_0/2\pi T$. After this is done, the analytic continuations of the correlation functions back to the real axis can be performed. Substituting these analytically continued functions into Eq. (2), we obtain the rather simple expression for the normalized transverse attenuation coefficient near T_c

$$\frac{\alpha_s^T}{\alpha_N^T} = g(ql)\left[1-\frac{2(T_i/T)^2(1-T/T_c)[\rho^{-1}\psi'(\frac{1}{2}+\rho)-\psi''(\frac{1}{2}+\rho)]}{-\psi''(\frac{1}{2})+\frac{n_i N_d(0)/N(0)}{1+n_i N_d(0)/N(0)}\left[\frac{\Gamma_d}{6\pi T}\right]\psi'''(\frac{1}{2})}\right]$$
$$+[1-g(ql)]\left[1+\left[\frac{T_c}{\omega}\right]^2\left[\frac{4\pi(T_c/T)(1-T/T_c)\psi'(\frac{1}{2}+\rho)[\rho^{-1}\psi'(\frac{1}{2}+\rho)-\psi''(\frac{1}{2}+\rho)]}{-\psi''(\frac{1}{2})+\frac{n_i N_d(0)/N(0)}{1+n_i N_d(0)/N(0)}\left[\frac{\Gamma_d}{6\pi T}\right]\psi'''(\frac{1}{2})}\right]^2\right]^{-1} . \tag{14}$$

The above expression shows the explicit dependence of the TA coefficient on the impurity concentration and the half width of the impurity state. The dependence on the susceptibility due to the rapid spin fluctuations enters through the dependence of T_c on the susceptibility. For the low-frequency transverse waves used in Ref. 11, $g(ql)\sim 1$ and so the TA coefficient is given by the first term on the right-hand side of Eq. (14). This would also be the expression for longitudinal attenuation coefficient near T_c of a superconductor containing LSF.

IV. HIGH-T_K KONDO IMPURITY DESCRIPTION

Studies[16,17] of the low-lying excitations in a Fermi sea containing transition-metal impurities have shown that the TM impurity becomes a nonmagnetic singlet bound state at $T=0$ K and that the behavior of the impurity at very low temperatures is that of a nonmagnetic impurity. The significance of this transition into a nonmagnetic state to the problem of superconductivity was pointed out by Sakurai[18] and Matsuura *et al.*[10] MIN developed a theory for the Kondo effect in superconductors which took into account the transition of the Kondo impurity into a nonmagnetic state at very low temperatures. The main effect of the impurities at low temperatures was to cause a weakening of the BCS electron-phonon coupling constant λ due to the appearance of a repulsive interaction (due to the virtual polarization of the singlet impurity) between the Cooper pair electrons. The pair weakening mechanism leads to a new effective coupling constant

$$g = |\lambda| - n_i/4T_K N^2(0) . \tag{15}$$

To allow a crossover of their theory from one side of the magnetic-to-nonmagnetic transition region to the other, MIN proposed the following interpolation scheme for the pair-breaking parameter

$$\alpha(\omega_n) = \begin{cases} \frac{1}{\pi N(0)}\left[\frac{\pi|\omega_n|}{4T_K} - \frac{1}{2}\left[\frac{\pi|\omega_n|}{4T_K}\right]^2\right], & \frac{\pi|\omega_n|}{4T_K} > 1 \\ \frac{1}{2\pi N(0)}\,\frac{s(s+1)\pi^2}{\ln^2(\pi|\omega_n|/4T_K)+s(s+1)\pi^2}, & \frac{\pi|\omega_n|}{4T_K} < 1 . \end{cases} \tag{16}$$

The temperature dependence of the order parameter close to T_c can be obtained from the standard formula[19]

$$\langle |\Delta_{\rm ph}(r)|^2\rangle = -\frac{N(0)}{B(T_c)}\left[1+T_c\left.\frac{\partial B_0(T)}{\partial T}\right|_{T_c}\right]\left[1-\frac{T}{T_c}\right] , \tag{17}$$

where $B_0(T)$ and $B(T)$ are the two and four particle Green's functions. In the MIN theory,

$$1+T_c\frac{\partial B_0(T)}{\partial T}\bigg|_{T=T_c}=1+N(0)T_c\Bigg[\frac{\partial}{\partial T}\pi T\sum\frac{1}{|\omega_n|+n_i\alpha(\omega_n)+\tau_{\rm tr}v^2q^2/6}-\frac{n_i}{4T_KN(0)}\frac{2\Phi_1(T_c)}{1+[n_i/4T_KN(0)]\Phi_2(T_c)}\frac{\partial\Phi_1(T)}{\partial T}\bigg|_{T_c}+\left[\frac{n_i}{4T_FN(0)}\right]^2\frac{[\Phi_1(T)]^2}{\{1+[n_i/4T_KN(0)]\Phi_2(T_c)\}^2}\frac{\partial\phi_2(T)}{\partial T}\bigg|_{T_c}\Bigg], \tag{18}$$

where

$$\phi_K(T)=\pi T\sum\frac{f^K(\omega_n)}{[|\omega_n|+n_i\alpha(\omega_n)+\tau_{\rm tr}v^2q^2/6]}$$

and

$$B(T)=\pi T\sum\frac{|\omega_n|+[n_i/2\pi N(0)]\chi(\omega_n)}{[|\omega_n|+n_i\alpha(\omega_n)+\tau_{\rm tr}v^2q^2/6]^4}\left[\frac{1+[n_i/4T_nN(0)][\phi_2(T)-\phi_1(T)f(\omega_n)]}{1+[n_i/4T_KN(0)]\phi_2(T)}\right]^4, \tag{19}$$

with $\chi(\omega)$ being the susceptibility function defined in Ref. 19. For high T_K Kondo superconductors, where

$$|\omega_n|+n_i\alpha(\omega_n)\sim[1+n_i/4T_KN/(0)]\,|\omega_n|$$

and (20)

$$|\omega_n|+[n_i(2\pi N(0)]\chi(\omega_n)\sim[1+n_i/4T_KN(0)]\,|\omega_n|\ ,$$

we have

$$1+T_c\frac{\partial B_0(T)}{\partial T}\bigg|_{T_c}=\frac{[1-\rho_c\psi'(\tfrac{1}{2}+\rho_c)]}{[1+n_i/4T_KN(0)][1+n_i/4T_KN^2(0)g]^2}\ , \tag{21}$$

and

$$B(T)=\frac{1}{2(2\pi T)^2}\frac{1}{[1+n_i/4T_KN(0)]^3[1+n_i/4T_KN^2(0)g]^4}[-\tfrac{1}{2}\psi''(\tfrac{1}{2}+\rho_c)-\tfrac{1}{6}\rho_c\psi'''(\tfrac{1}{2}+\rho_c)]\ , \tag{22}$$

where $\rho_c=[(\tau_{\rm tr}v^2q^2/6)/2\pi T_c]/[1+n_i/4T_KN(0)]$. Substituting Eqs. (21) and (22) into Eq. (17), we obtain

$$\langle|\Delta_{\rm ph}(r)|^2\rangle=\left[\frac{1+n_i/4T_KN(0)}{1-n_i/4T_KN^2(0)\lambda}\right]^2\frac{2(2\pi T_c)^2[1-\rho_c\psi'(\tfrac{1}{2}+\rho_c)]}{-\tfrac{1}{2}\psi''(\tfrac{1}{2}+\rho_c)-\tfrac{1}{6}\rho_c\psi'''(\tfrac{1}{2}+\rho_c)}\left[1-\frac{T}{T_c}\right]. \tag{23}$$

The MIN theory also predicted that the critical concentration at which the high T_K Kondo impurities can suppress the superconductivity is

$$n_{\rm cr}\sim T_KN^2(0)\ . \tag{24}$$

The effects of the averaging the products of two one-particle propagators over the distribution of impurities were incorporated into Eqs. (18) and (19).

As we pointed out in Sec. II the effects of this averaging on the correlation functions can be incorporated by a further renormalization of the vertex functions by Eq. (6). Performing this additional renormalization, we find that the proper vertex correction for a high-T_K Kondo superconductor is given by

$$\frac{\gamma(\omega_n)}{|\tilde{\omega}_n|}=\frac{1}{[1+n_i/4T_KN(0)]\,|\omega_n|+\tau_{\rm tr}v^2q^2/6}\left[1-\frac{n_i}{4T_KN^2(0)\lambda}\left[\frac{4T_K}{\pi}\right]^2\frac{1}{(|\omega_n|+4T_K/\pi)^2}\right]. \tag{25}$$

Substituting Eq. (25) into Eqs. (3)–(5) leads to rather complicated expressions for the three correlation functions. If we restrict our attention to the low-frequency limit, then only the stress tensor correlation function $\langle[\tau_{xz},\tau_{xz}]\rangle_{g,\omega_0}$ needs to be calculated. After much work, we find that the imaginary part of the analytically continued stress tensor correlation function is given by

$$\mathrm{Im}\langle[\tau_{xz},\tau_{xz}]\rangle_{q,\omega_0\to i\omega}$$

$$=\frac{p_F{}^4}{3\pi q}(ql)^{-1}[1-g(ql)]\omega\left\{1-\frac{\langle|\Delta_{\rm ph}(r)|^2\rangle}{[1+n_i/4T_KN(0)]^2}\frac{1}{2(2\pi T)^2}\right.$$

$$\times\left\{\psi''(\tfrac{1}{2}+\rho)-\rho^{-1}\psi'(\tfrac{1}{2}+\rho)-\frac{n_i}{2T_KN^2(0)\lambda}\left[\frac{4T_K}{\pi}\right]^2\right.$$

$$\times\left[\frac{1}{2\rho}\left[\frac{1}{(a+b)^2}+\frac{1}{(a-b)^2}\right]\psi'(\tfrac{1}{2}+\rho)\right.$$

$$-\frac{1}{(a-b)^2}\psi''\left[\frac{1}{2}+\rho\right]+\frac{1}{2\rho}\left[\frac{1}{(a+b)^2}-\frac{1}{(a-b)^2}\right]\psi'(\tfrac{1}{2}+\rho_K)$$

$$\left.\left.\left.-\left[\frac{2}{(a-b)^2}-\frac{1}{(a+b)^2}-\frac{1}{(a^2-b^2)}\right]\psi''(\tfrac{1}{2}+\rho_K)\right]\right\}\right\}, \qquad (26)$$

where $a=4T_K/\pi$, $b=(\tau_{\rm tr}v^2q^2/6)/[1+n_i/4T_KN(0)]$,$\rho_K=a/2\pi T$, and $\rho=b/2\pi T$. A simplication of Eq. (29) occurs if $a>b$. When the simplified form of Eq. (26) is substituted into the definition of the transverse attenuation coefficient, Eq. (2), we get

$$\frac{\alpha_B{}^T}{\alpha_N{}^T}=1-\frac{1-n_i/2T_nN^2(0)\lambda}{[1-n_i/4T_KN(0)]^2}\frac{\langle|\Delta_{\rm ph}(r)|^2\rangle}{2(2\pi T)^2}$$

$$\times\left\{\psi''(\tfrac{1}{2}+\rho)-\rho^{-1}\psi'(\tfrac{1}{2}+\rho)-\frac{n_i}{n_{\rm cr}-n_i/2}\left[\left[\frac{b}{a}\right][-\psi''(\tfrac{1}{2}+\rho)-3\psi''(\tfrac{1}{2}+\rho_K)-\rho^{-1}\psi'(\tfrac{1}{2}+\rho_K)]\right.\right.$$

$$\left.\left.+\left[\frac{b}{a}\right]^2\{-\psi''(\tfrac{1}{2}+\rho_K)-\tfrac{3}{2}[\psi''(\tfrac{1}{2}+\rho)-\rho^{-1}\psi'(\tfrac{1}{2}+\rho_K)]\}\right]\right\}, \qquad (27)$$

where $\langle|\Delta_{\rm ph}(r)|^2\rangle$ is given by Eq. (23). For very large T_K, where $b/a\to 0$, Eq. (27) reduces to the attenuation coefficient for dirty superconductors in the gapless state near T_c obtained by Maki.[20]

V. DISCUSSION

Both Eqs. (14) and (27) predict that the decrease in the attenuation of transverse sound wave as the alloy becomes superconducting should become less as more transition-metal impurities are added to the alloy. This tendency is seen in the experimental data of Lou and Bömmel[11] for the attenuation of 20-MHz transverse waves in superconducting *Zn*Mn alloys. In Fig. 5 of their paper, we see that the normalized transverse attenuation coefficient for the 1.25 ppm doped *Zn*Mn superconductor lies below the attenuation for the 8.5 and 16 ppm doped superconductors. The frequency of the sound wave is such that the low-frequency limit is satisfied, i.e., $ql \ll 1$ and so $g(ql)\sim 1$.

Lou and Bömmel found that the simple Abrikosov-Gor'kov theory for paramagnetic impurities in superconductors could reasonably account for their results. However, to achieve fits to their data to the attenuation coefficient obtained within the framework of the AG theory, they had to subtract a constant residual attenuation introduced *ad hoc* into their analysis and to change the value of the ratio $\Delta(0)/k_BT_c$ (from 1.50 to 1.70 as n_i was increased from 1.5 to 8.5 for example). Without having to make these additional adjustments, Machida[3] showed that the attenuation coefficients obtained on the basis of the Shiba[2] description of the magnetic impurities in superconductors gave better fits to the data than did the AG based coefficient. In his study, Machida assumed that the position of the impurity state within the energy gap was at 0.25 Δ_g. The fits, however, became progressively worse as the impurity concentration was increased.

Matsui and Masuda[5] have also analyzed the data of Lou and Bömmel. They based their analysis on the Müller-Hartmann and Zittartz theory[6] for the Kondo effect in the superconductors. They found that the MHZ based coefficients gave better fits to the data than the AG based coefficients and was as good as the fits obtained by

Machida.[3] A careful inspection of their fit indicated certain discrepancies. First of all, the MHZ based coefficients could not be made to fit the high-T and low-T data at the same time (the same being true of the fits of Machida). The value of T_K (0.24 K) used to achieve a fit to the data for the 16 ppm doped superconductor leads to a predicted attenuation for the 8.4-ppm superconductor lower than the ones observed. The fits achieved by Matsui and Masuda would not have been obtained if T_K was taken to be 0.9 K, the value obtained by Kaštner and Wassermann[21] based on their analysis of the transition temperature of ZnMn superconductors using the improved Müller-Hartmann, Schuh, and Zittartz theory.[22] The value 0.9 K is close to the value obtained from the normal phase properties of ZnMn, i.e., 1.0 K. These values of T_K would bring into question, the use of the MHZ theory to describe the Kondo effect in the ZnMn superconductors since the MHZ theory is based on the Suhl-Nagaoka approximation which is valid for $T \gg T_K$. Further complicating the situation is the fact that the behavior of the normal state properties of the higher doped ZnMn alloys are such to indicate that ZnMn is very near to the magnetic-to-nonmagnetic transition and it is uncertain as to which side the ZnMn alloys belong.

If the ZnMn alloys are on the nonmagnetic side of the transition and the behavior of the alloys is determined by the local spin fluctuations, then the expressions developed in Sec. III should be used to interpret the data. Using the transition temperatures reported by Sanchez,[23] we find that a least-square best fit of the data by Schlottmann's[9] expression for the decrease in T_c due to the rapid spin fluctuations is achieved with a critical concentration of 39 ppm, which is in reasonable agreement with the observed value of 25 ppm. To obtain the best fit, we used the following values for the other parameters appearing in his expression, $N_d(0)=0.455$ states/eV atom, $N(0)=0.139$ states/eV atom and $\Gamma_d=0.7$ eV. The value of $\alpha=\tau_{tr}v^2q^2/6$ was obtained by fitting Eq. (14) to the experimental value of α_S^T/α_N^T for the 1.25 ppm doped ZnMn alloy at $T=0.79$ K. Once this value was determined, Eq. (14) was used to calculate the expected values of the normalized transverse attenuation coefficients for the 16 and 8.5 ppm doped ZnMn superconductor near T_c. The results of our calculations are shown in Fig. 1 along with a few of the data points taken from Fig. 5 of Ref. 11. As we see, the calculated values are in good agreement with the observed values.

If we assume that ZnMn is a Kondo alloy with $T_K=1$ K (and not 0.24 K), then the formulas developed in Sec. IV should be applicable. However, the low value of T_K (which is nevertheless greater than T_{co} of the pure zinc superconductor) leads to a critical concentration much less than the 25 ppm observed experimentally. The small value [2 ppm obtained by substituting $N(0)\lambda=0.18$ (value for superconducting zinc) into Eq. (24)] indicate that for intermediate energies, $4T_K/\pi \gtrsim \omega_n \sim T_c$, the MIN predicts a too rapid of a weakening of the BCS electron-phonon coupling constant. MIN pointed out that there is some ambiguity in the form of $\alpha(\omega_n)$ in the region $\omega_n \sim T_K$. The small value of n_{cr} just points this out.

As it stands, Eq. (27), with the values of the parameters being those for ZnMn, cannot account for the observed decreases in the attenuation of the transverse waves in the ZnMn alloys as they become superconducting. This is probably due to the incorrectness of the MIN interpolation scheme at $\omega_n \sim T_K$.

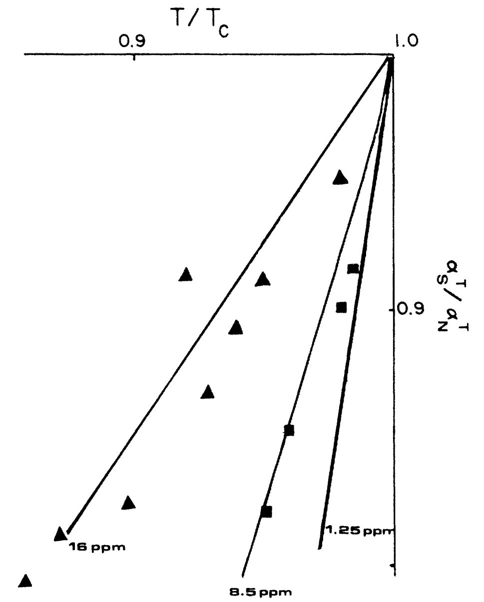

FIG. 1. Temperature dependence of the transverse ultrasonic attenuation coefficient for ZnMn superconductors. The curves show the temperature behavior of the attenuation coefficient as predicted by Eq. (14) which is based on the local spin-fluctuation description of the ZnMn superconductors. The curves for the 16 and 8.5 ppm doped ZnMn superconductors were obtained by substituting the normal state values of several parameters into Eq. (14). Values of other parameters in the expression were obtained by fitting Eq. (14) to the attenuation data for 1.25-ppm superconductor. ▲ and ■ are some experimental points for the transverse attenuation coefficients of the 16- and 8.5-ppm superconductors, respectively. These points were taken from Fig. 5 of Ref. 11.

[1]L. P. Kadanoff and I. I. Falko, Phys. Rev. **136**, A1170 (1964).
[2]H. Shiba, Prog. Theor. Phys. **50**, 50 (1973).
[3]K. Machida, Prog. Theor. Phys. **54**, 1251 (1975).
[4]B. Leon and A. D. S. Nagi, J. Phys. F **4**, L13 (1974).
[5]K. Matsui and Y. Masuda, J. Low Temp. Phys. **31**, 101 (1978).
[6]E. Müller-Hartmann and J. Zittartz, Z. Phys. **254**, 23 (1970).
[7]P. J. Ford, C. Rizzuto, and E. Salamoni, Phys. Rev. B **6**, 1851 (1972).
[8]K. Maki, Phys. Rev. **148**, 370 (1966).
[9]P. Schlottmann, Solid State Commun. **21**, 663 (1973).
[10]T. Matsuura, S. Ichinose, and Y. Nagaoka, Prog. Theor. Phys. **57**, 713 (1977).
[11]D. Y. Lou and H. E. Bömmel, Phys. Rev. B **9**, 3739 (1974).
[12]K. Maki, Physics **1**, 21 (1964).
[13]M. Cyrot and K. Maki, Phys. Rev. **156**, 433 (1967).
[14]G. Iche, J. Low Temp. Phys. **11**, 215 (1973).

[15]I. M. Tang, J. Low Temp. Phys. **60**, 9 (1985).
[16]K. Yosida and K. Yamada, Prog. Theor. Phys. **53**, 1286 (1975).
[17]P. Nozieres, J. Low Temp. Phys. **17**, 31 (1974).
[18]A. Sakurai, Phys. Rev. B **17**, 1195 (1978).
[19]S. Ichinose, Prog. Theor. Phys. **58**, 404 (1977); **58**, 733 (1977).
[20]K. Maki, *Superconductivity, Vol. II,* edited by R. D. Parks (Dekker, New York, 1969), Chap. 18.
[21]J. Kästner and E. F. Wassermann, J. Low Temp. Phys. **29**, 411 (1977).
[22]E. Müller-Hartmann, B. Schuh, and J. Zittartz, Solid State Commun. **19**, 439 (1976).
[23]D. H. Sanchez, J. Low Temp. Phys. **17**, 101 (1974).